ISSN (Online) 2394-2320

International Journal of Engineering Research in Computer Science and Engineering (IJERCSE)

Vol 4, Issue 5, May 2017

Peristaltic Flow of a Conducting Newtonian Fluid in an Inclined Channel under the Effects of Hall Current

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Abstract:- **In this paper, the effect of hall on the peristaltic flow of a conducting fluid in an inclined two dimensional channel under the assumption of long wavelength is investigated. A closed form solution is obtained for axial velocity and pressure gradient. The effect of various emerging parameters on the time-averaged volume flow is analyzed with the help of graphs.**

Keywords: **Newtoninan fluid, Hall, Hartmann number, peristaltic flow, Froude number**

I. INTRODUCTION

The word peristalsis stems from the Greek word Peristaltikos, which means clasping and compressing. Peristaltic pumping is a form of fluid transport generated in the fluid contained in a distensible tube when a progressive wave travels along the wall of the tube. It is an inherent property of many syncytial smooth muscle tubes; stimulation at any point can cause a contractile ring to appear in the circular muscle of the gut, and this ring then spreads along the tube. In such a way, peristalsis occurs in the gastrointestinal tract, the bile ducts, other glandular ducts throughout the body, the ureters, and many other smooth muscle tubes of the body, Guyton and Hall (2003). The study of the mechanism of peristalsis in both mechanical and physiological situations has recently become the object of scientific research, since the first investigation of Latham (1966). Several theoretical and experimental attempts have been made to understand peristaltic action in different situations. A review of much of the early literature is presented in an article by Jaffrin and Shapiro (1971). A summary of most of the experimental and theoretical investigations reported with details of the geometry, fluid Reynolds number, wavelength parameter wave amplitude parameter and wave shape has been given by Srivastava and Srivastava (1984).

The magnetohydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids, (e.g., the blood flow in arteries). Agrawal and Anwaruddin (1984) investigated the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations. Mekheimer (2004) studied the peristaltic transport of blood under effect of a magnetic field in non uniform channels. Hayat et al. (2007) have first investigated the Hall effects on the peristaltic flow of a Maxwell fluid trough a porous medium in channel. Hall Effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer was studied by Eldabe (2015). Recently, Subba Narasimhudu and Subba Reddy (2017) have investigated the effects of Hall on the peristaltic flow of a hyperbolic tangent fluid in a channel.

In view of these, the effect of hall on the peristaltic flow of a conducting fluid in an inclined two dimensional channel under the assumption of long wavelength is investigated. A closed form solution is obtained for axial velocity and pressure gradient. The effect of various emerging parameters on the time-averaged volume flow is analyzed with the help of graphs.

II. MATHEMATICAL FORMULATION

We consider the peristaltic pumping of a conducting Newtonian fluid flow in an inclined channel of half-width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half-width of the channel as shown in the Fig.1. The wall deformation is given by

$$H\left(X,t\right)=a+b\sin\left[\frac{2\pi}{\lambda}\left(X-ct\right)\right]$$

(2.1)

where b is the amplitude, λ the wavelength and c is the wave speed.

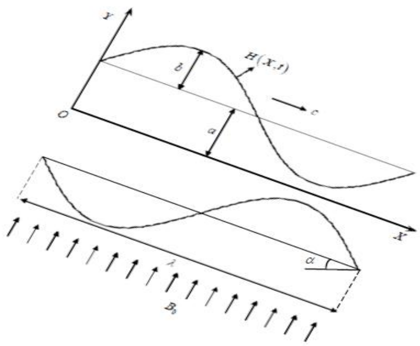

Fig. 1 Physical Model

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X,Y). The transformation between these two frames is given by

$x=X-c\,t,\ y=Y,\ u=U-c,\ v=V$ and $p(x)=P(X,t),$ (2.2)

where $(u,\ v)$ and $(U,\ V)$ are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow in wave frame are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (2.23)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma B_0^2}{1+m^2}\left(mv-\left(u+c\right)\right)+\rho g\sin\alpha \quad (2.4)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B_0^2}{1+m^2}\left(m\left(u+c\right)+v\right)-\rho g\cos\alpha \quad (2.5)$$

where ρ is the density σ is the electrical conductivity, B_0 is the magnetic field strength and m is the Hall parameter.

The dimensional boundary conditions are

$$u=-c \quad \text{at} \quad y=H \quad (2.6)$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \quad (2.7)$$

Introducing the non-dimensional quantities

$$\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a},\bar{u}=\frac{u}{c},\bar{v}=\frac{v}{c\delta},\delta=\frac{a}{\lambda},\bar{p}=\frac{pa^2}{\mu c\lambda},\bar{t}=\frac{ct}{\lambda},h=\frac{H}{a},\phi=\frac{b}{a},\bar{q}=\frac{q}{ac},M^2=\frac{\sigma a^2B_0^2}{\mu}$$

Into equations (2.3) to (2.5), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2.8)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{M^2}{1+m^2}\left(m\delta v-\left(u+1\right)\right)+\frac{\mathrm{Re}}{Fr}\sin\alpha \quad (2.9)$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\delta M^2}{1+m^2}\left(m\left(u+1\right)+\delta v\right)+\frac{\delta\,\mathrm{Re}}{Fr}\cos\alpha \quad (2.10)$$

where $Fr=\frac{c^2}{ag}$ is the Froude number, M is the Hartmann number and Re is the Reynolds number.

Using long wavelength (i.e., $\delta<<1$) approximation, the equations (2.9) and (2.10) become

$$\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}u=\frac{\partial p}{\partial x}-\frac{\mathrm{Re}}{Fr}\sin\alpha+\frac{M^2}{1+m^2} \quad (2.11)$$

$$\frac{\partial p}{\partial y}=0 \quad (2.12)$$

From Eq. (2.12), it is clear that p is independent of y. Therefore Eq. (2.11) can be rewritten as

$$\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}u=\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\sin\alpha+\frac{M^2}{1+m^2} \quad (2.13)$$

The corresponding non-dimensional boundary conditions are given as

$$u=-1 \quad \text{at} \quad y=h \quad (2.14)$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \quad (2.15)$$

Knowing the velocity, the volume flow rate q in a wave frame of reference is given by

$$q=\int_0^h u\,dy. \quad (2.16)$$

The instantaneous flow $Q(X,t)$ in the laboratory frame is

$$Q(X,t)=\int_0^h U\,dY=\int_0^h\left(u+1\right)dy=q+h \quad (2.17)$$

The time averaged volume flow rate $\overline{Q}$ over one period $T\left(=\dfrac{\lambda}{c}\right)$ of the peristaltic wave is given by

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt=q+1 \qquad (2.18)$$

III. SOLUTION

Solving Eq. (2.13) together with the boundary conditions (2.14) and (2.15), we get

$$u=\frac{1}{\beta^2}\left(\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left[\frac{\cosh\beta y}{\cosh\beta h}-1\right]-1 \qquad (3.1)$$

Where $\beta=M/\sqrt{1+m^2}$.

The volume flow rate q in a wave frame of reference is given by

$$q=\frac{1}{\beta^3}\left(\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\sin\alpha\right)\left[\frac{\sinh\beta h-\beta h\cosh\beta h}{\cosh\beta h}\right]-h \qquad (3.2)$$

From Eq. (3.2), we write

$$\frac{dp}{dx}=\frac{(q+h)\beta^3\cosh\beta h}{\sinh\beta h-\alpha h\cosh\beta h}+\frac{\mathrm{Re}}{Fr}\sin\alpha \qquad (3.3)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \qquad (3.4)$$

Note that, as $\alpha\to 0$, $M\to 0$ and $m\to 0$ our results coincide with the results of Shapiro et al. (1969).

IV. RESULTS AND DISCUSSION

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hartmann number M with $\phi=0.5$, $\mathrm{Re}=5$, $Fr=2$, $\alpha=\dfrac{\pi}{4}$ and $m=0.2$ is depicted in Fig. 2. It is found that, the time-averaged flow rate $\overline{Q}$ increases in the pumping region $(\Delta p>0)$ with increasing M , while it decreases in both the free-pumping $(\Delta p=0)$ and co-pumping $(\Delta p<0)$ regions with increasing M .

Fig. 3 illustrates The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi=0.5$, $\mathrm{Re}=5$, $Fr=2$, $\alpha=\dfrac{\pi}{4}$ and $M=1$. It is observed that, the time-averaged flow rate $\overline{Q}$ decreases in the pumping region with an increase in m, while it increases in both the free-pumping and co-pumping regions with increasing m.

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Froude number Fr with $\phi=0.5$, $\mathrm{Re}=5$, $\alpha=\dfrac{\pi}{4}$, $M=1$ and $m=0.2$ is shown in Fig. 4. It is observed that as increase in Fr decreases the time averaged flow rate $\overline{Q}$ in all the pumping, free-pumping and co-pumping regions.

Fig. 5 shows the variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Reynolds number Re with $\phi=0.5$, $Fr=2$, $\alpha=\dfrac{\pi}{4}$, $M=1$ and $m=0.2$. It is found that, on increasing Re increases the time averaged flow rate $\overline{Q}$ in all the pumping, free-pumping and co-pumping regions.

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of inclination angle α with $\phi=0.5$, $\mathrm{Re}=5$, $Fr=2$, $M=1$ and $m=0.2$ is presented in Fig. 6. It is noticed that, the time averaged flow rate $\overline{Q}$ increases with increasing α in all the pumping, free-pumping and co-pumping regions.

Fig. 6 depicts the variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $m=0.2$, $\mathrm{Re}=5$, $Fr=2$, $\alpha=\dfrac{\pi}{4}$ and $M=1$. It is observed that, the time-averaged flow rate $\overline{Q}$ increases with increasing amplitude ratio ϕ in both the pumping and free pumping regions, while it decreases with increasing amplitude ratio ϕ in the co-pumping region for chosen $\Delta p(<0)$.

V. CONCLUSIONS

In this chapter, the effect of Hall on the peristaltic flow of a conducting fluid in an inclined channel under the assumption of long wavelength approximation is investigated. The expressions for the velocity and pressure gradient are obtained analytically. It is found that, the time-averaged flow rate in the pumping region is increases with increasing Hartmann number Reynolds number, angle of inclination or amplitude ratio, whereas it decreases with increasing hall parameter or Froude number. Further it is observed that, the pumping is more for vertical channel $\left(\alpha = \frac{\pi}{2}\right)$ than that of horizontal channel $(\alpha = 0)$.

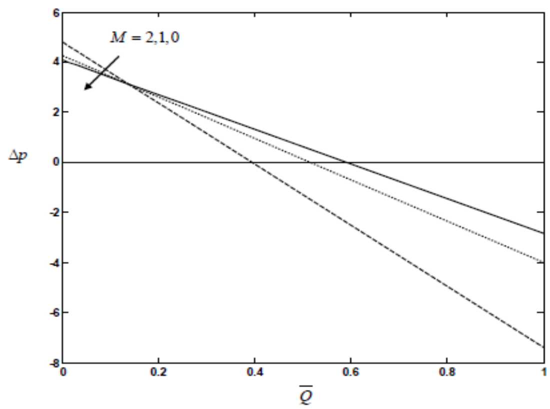

Fig. 2.

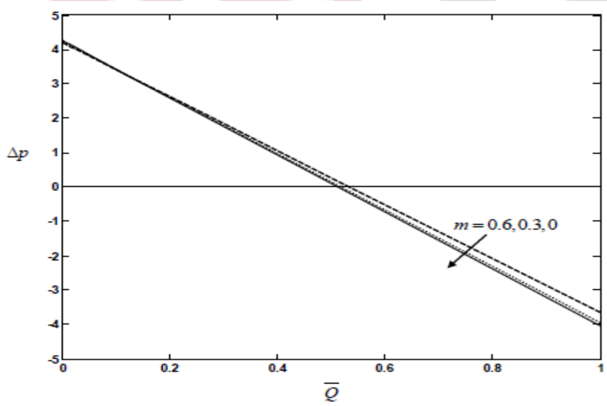

Fig. 3

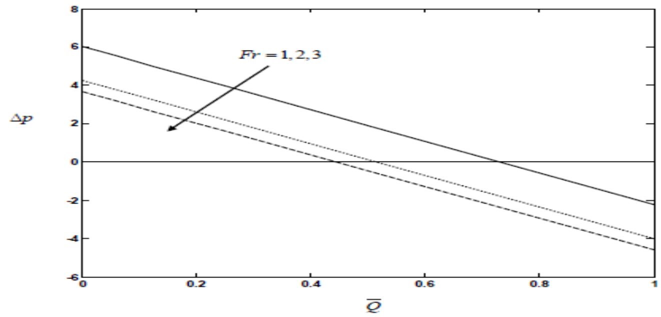

Fig. 4.

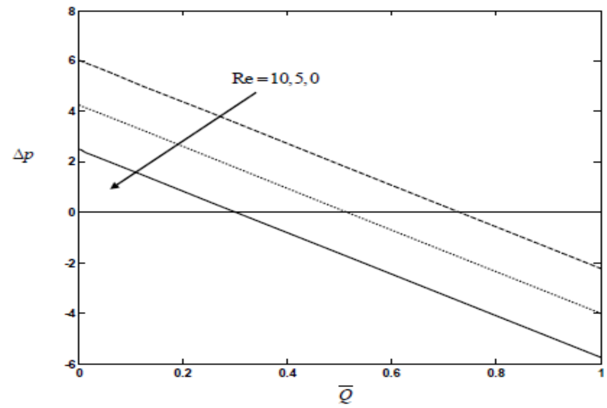

Fig. 5

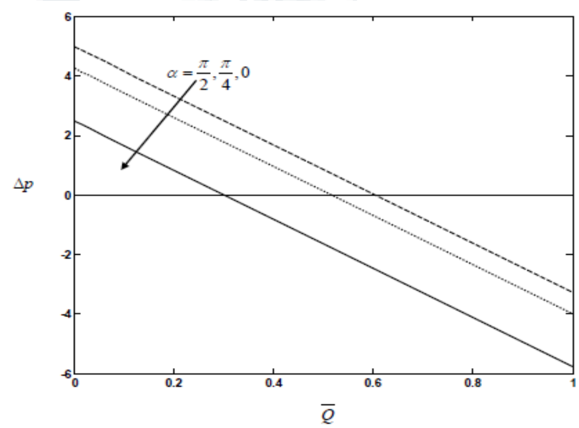

Fig. 6.

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ISSN (Online) 2394-2320

International Journal of Engineering Research in Computer Science and Engineering (IJERCSE)
Vol 4, Issue 5, May 2017

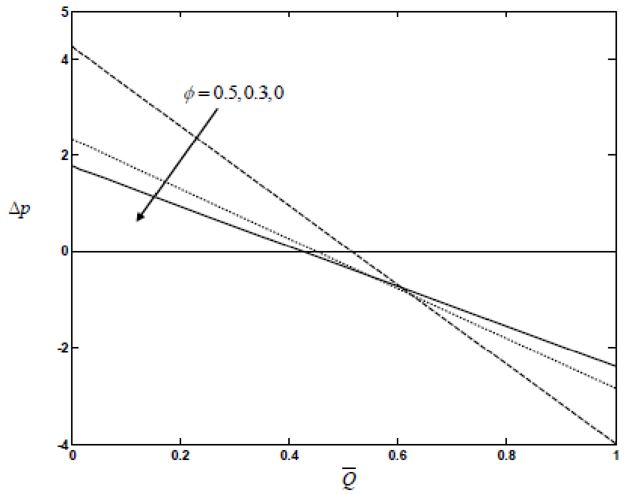

Fig. 7

Fig. 2. The variation of pressure rise Δp ***with time-averaged***

flow rate $\overline{Q}$ for different values of Hartmann number M with **Fig. 3.** The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi = 0.5$, $\mathrm{Re} = 5$, $Fr = 2$, $\alpha = \frac{\pi}{4}$ and $M = 1$.

, $\mathrm{Re} = 5$, $Fr = 2$, $\alpha = \frac{\pi}{4}$ and $m = 0.2$.

Fig. 4. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Froude number Fr with $\phi = 0.5$, $\mathrm{Re} = 5$, $\alpha = \frac{\pi}{4}$, $M = 1$ and $m = 0.2$.

Fig. 5. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Reynolds number Re with $\phi = 0.5$, $Fr = 2$, $\alpha = \frac{\pi}{4}$, $M = 1$ and $m = 0.2$.

Fig. 6. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of inclination angle α with $\phi = 0.5$, $\mathrm{Re} = 5$, $Fr = 2$, $M = 1$ and $m = 0.2$.

Fig. 7. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $m = 0.2$, $\mathrm{Re} = 5$, $Fr = 2$, $\alpha = \frac{\pi}{4}$ and $M = 1$

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