ISSN (Online) 2394-2320

International Journal of Engineering Research in Computer Science and Engineering (IJERCSE)

Vol 9, Issue 10, October 2022

On The Construction Bilateral Multistep Methods and Its Application to Solve Volterra Integral Equation

[1] Vusala Nuriyeva

[1] Department of Computational Mathematics, Baku State University, Baku, Azerbaijan.
Corresponding Author Email: [1] math.lover4baku@gmail.com

Abstract— ***As is known,the investigation of many problems of natural sciences are reduced to solving some integral equations. Among them the most important question is the investigation numerical sllution of the Volterra integral equation.It is known that one of the popular numerical methods for solving Volterra integral equation is multistep methods of its constant coefficients. By taking into account this is considering to construct simple numerical methods for solving Volterra-integral equation.***

Index Terms – multistep methods, hybrid methods, Volterra integral equation, stability and degree, bilateral methods, explicit and implicit methods.

I. INTRODUCTION

There are some classes of numerical methods for solving Volterra integral equations of the second kind. In solving practical problems usually arises question to determine the reliability of the obtained values of the solution of investigated problem, about that how can be so account that order of accuracy for numerical methods are define by some asymptotic equality. For solving this problem, here suggested to constructed the bilateral methods and recommended that to solving of Volterra integral equation. For this aim have used multistep explicit and implicit methods, which are more exact than the others. There was constructed some bilateral method any of which has applied to solve model Volterra integral equation. The receiving results to corresponding of theoretical. It is known that the integral equation with the variable bounders fundamentally has investigated by Vito Volterra. He, Volterra for determine the approximately numerical solution has recommended using quadrature formulas. Noted that all the methods has its disadvantages and advantages. For the determining the main disadvantages of quadrature formulas let us consider the following Volterra integral equation of the second kind:

$$y(x) = f(x) + \int_{x_0}^{x} K(x, s, y(s))ds, \quad x_0 \le x \le X. \tag{1}$$

Noted that if the functions $f(x)$ and $K(x, y, s)$ are known, then the equation (1) takes as the given. Suppose that the equation has the unique solution, which is defined in the segment $[x_0; X]$.For the construction numerical methods for solving equation of (1), let us use the mesh-points $x_{i+1} = x_i + h, \quad (i = 0,1,2,\ldots)$, the segment $[x_0; X]$ divide to N -equal parts. Here 0 < h-is the step-size. It is known that the quadrature method for solving equation of (1) can be presented as following:

$$y_i = f_i + h\sum_{i=0}^{n} \beta_i \; K(x_i, x_i, y_i), \; y(x_0) = (f_0),$$

$$i = 1,2,\ldots N \; (2)$$

here $f_i = f(x_i)$ -approximately values for the exact value $y(x_i)$ of the solution of equation (1). This method for the calculation of y_{i+1} can be written as follows:

$$y_{i+1} = f_{i+1} + h\sum_{j=0}^{i+1} \beta_i \; K(x_{i+1}, x_j, y_j), \; (3)$$

By simple comparison of the equality (2) and (3) receive that for each values of the sum must be calculated again. If suppose that $K(x, y, s) = \phi(s, y)$, then formula (3) can be presented as:

$$y_i = f_i + h\sum_{j=0}^{i} \beta_i \phi(x_j, y_j),$$

and

$$y_{i+1} = f_{i+1} + h\sum_{j=0}^{i+1} \beta_i (x_{i+1}, x_j, y_j).$$

Here for the determination y_{i+1}, receive the nonlinear algebraic equation. In [1], for solving equation (1) constructed method, which has the same properties as the method (4). The named method in one version can be presented as the following:

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = \sum_{i=0}^{k} f_i y_{n+i} + h\sum_{i=0}^{k}\sum_{j=0}^{k} \beta_{i,j} K(x_{n+j}, x_{n+i}, y_{n+i}) \tag{5}$$

For example, trapezoidal rule receiving from the method (5) can be presented as:

$$\hat{y}_{n+1} = y_n h(2K(x_{n+1}, x_{n+1}, y_{n+1}) + K(x_{n+1}, x_n, y_n) + K(x_n, x_n, y_n)\ )/4$$

This method is implicit, therefor for using that suggested to use predictor-corrector method and

as the predictor method proposed to use following Euler method:

$$\hat{y}_{n+1} = y_n + h(K(x_{n+1}, x_n, y_n) + K(x_n, x_n, y_n))/2 \qquad (6)$$

By using this method in the trapezoidal rule, receive:

$$y_{n+1} = y_n + h(2K(x_{n+1}, x_{n+1}, \hat{y}_{n+1}) + K(x_{n+1}, x_n, y_n) + K(x_n, x_n, y_n))/4 \qquad (7)$$

It is easy to verify that by using the methods (6) and (7) one can be solved the integral equation

(1). As is known very simple methods for calculation of definite integrals are the Euler's explicitand implicit methods, which are called as the left and right rectangular method. It is easy todefine that method (6) is the left rectangular method but the following is the right rectangular methods: $y_{n+1} = y_n + hK(x_{n+1}, x_n, \hat{y}_{n+1})$. (8)

Thus there was constructed two simple methods for solving Volterra integral equations. And now let usdefine local truncation errors of these methods. For this, let us consider estimation the errors of these methods and suppose that K(x, s, y) = φ(s, y). In this case methods (6) and (8) can be written as $y_{n+1} = y_n + h\phi(x_n, y_n)$, $y_{n+1} = y_n + h\phi(x_{n+1}, y_{n+1})$.

If in these methods to change approximately values y_m by the corresponding exact value $y(x_m)$ then receive:

$$y(x_{n+1}) = y(x_n) + h\phi(x_n, y(x_n)) + h^2 y''(x_n)/2! + O(h^3), \qquad (9)$$

$$y(x_{n+1}) = y(x_n) + h\phi(x_n + h, y(x_n + h)) - h^2 y''(x_n)/2! + O(h^3) \qquad (10)$$

It follows from here that

$$y(x_{n+1}) = y(x_n) + h(\phi(x_n + h, y(x_n + h)) + \phi(x_n y(x_n)))/2 + O(h^3)$$

Which is the trapezoidal rule and more exact then the Euler's explicit and implicit methods. From the equality (9) and (10) it follows that if y″(x) > 0 then receive that the following is holds: $\hat{y}_{n+1} \le y(x_{n+1}) \le y_{n+1}$. Hence receive that by using the methods (6) and (8), one can be constructed the bilateral method. As was noted above by using the half-sum of equalities (9) and (10), receive the new method, which is more exact than the methods using in the construction of that. In the construction of bilateral methods one of the main properties is the presentation local truncation error. Depends on the value of the coefficient of the main term in the local truncation error, for the finding more exact value one can be used the linear combination of the values calculating by different formulas. For example, let us consider the following methods:

$$\hat{y}(x_{n+2}) = y(x_n) + 2h\phi(x_{n+1}, y_{n+1}) + h^3 c/3 + O(h^4), \qquad (11)$$

$$\bar{y}(x_{n+2}) = y(x_{n+1} + h(\phi(x_{n+2}, y_{n+2}) + \phi(x_{n+1}, y_{n+1}\ )\ )/2 - h^3 c/12 + O(h^4) \qquad (12)$$

By using these equalities receive that, if the value y_{n+2} to define in the following form: $y_{n+2} = \hat{y}_{n+2} + 4\bar{y}_{n+2}$, then receive that value calculated by the formula (13) is more exact than the calculated by the methods (11) and (12). Noted that the exact value $y(x_{n+2})$ satisfies the following condition (if y‴ (x) > 0): $\hat{y}_{n+2} < y(x_{n+2}) < y_{n+2}$, Noted that the bilateral methods gives the best results for the symmetrical methods (the absolute value of the main terms in the expansion of the local error of these methods coincides, but they have different signs. For example, the Euler's methods.). Here for receiving the results have been used some ways from works [1, 2, 3, 4, 5].

REFERENCES

[1] Mehdiyeva G.,Ibrahimov V., Imanova M., On the construction test equations and its Applying to solvingVolterra integral equation, Conference: Mathematical Methods for Information Science and Economics, 2012,pp.109-114.

[2] Mehdiyeva G., Ibrahimov V.,Imanova M., Application of A Second Derivative Multi-Step Method to Nu-merical Solution of Volterra Integral Equation of Second Kind.Pak.j.stat.oper.res., Vol.VIII, No.2, 2012,pp.245-258.

[3] Mehdiyeva G., I brahimov V.,Imanova M.,General Theory of the Application of Multistep Methods to Cal-culation of the Energy of SignalsWireless Communications, Networking and Applications,Lecture Notes in Electrical Engineering series, Springer (Thomson Reuters) , Vol.348, 2016, pp.1047-1056.

[4] Mehdiyeva G., Ibrahimov V.,Imanova M., On one application of hybrid methods for solving Volterre Integro-differential equations World Academy of Science, engineering and Technology, Dubai, (Scopus), 2012,pp.1197-1201.

[5] Juraev D., On the solution of the Cauchy problem for matrix factorizations of the Helmholtz equation in a multidimensional spatial domain Global and Stochastic Analysis, Vol.9 (2), 2022, pp.1-17.