

BER Analysis of LDPC Coded System in AWGN Channel

^[1]Anumol P Joy, ^[2]Gils Mary George, ^[3]Greeshma Paul, ^[4]Jittu Deena Gladly

^{[1][2][3][4]}Department of Electronics and Communication Engineering, Amal Jyothi college of Engineering Kanjirappally, Kottayam, India

Abstract: Wireless communication is one of the fastest growing segments in communication industry. However, it is faced with the challenge of providing high-speed communications due to channel impediments which change over time in unpredictable ways. This paper deals with analysis of bit error rate using different modulation techniques and finding a better mechanism for reliable communication. The results obtained shows that the proposed technique is robust for signal transmission over mobile radio channels.

I. INTRODUCTION

Since the final decades of 20th century, the need of wireless communication has grown tremendously with a high speed wireless data access i.e. in Mbps with fewer errors to a very long distance. In order to avoid the burst errors that occur in physical channel different Forward Error Correction (FEC) methods on the data are required. The FEC code used in this paper is LDPC. Low-density parity-check (LDPC) codes, which were first proposed by Robert Gallager in 1960, have attracted considerable attention over the past decade due to their capacity-achieving error performance and low complexity of the associated iterative decoding algorithms. A low density parity-check (LDPC) code is a linear error correcting code, a method of transmitting a message over a noisy transmission line. LDPC codes are capacity-approaching codes, which means that practical constructions exist that allows the noise threshold to be set very close to the threshold maximum (the Shannon limit) for a symmetric memory less channel. The noise threshold defines upper bound for the channel noise, up to which the probability of lost information can be made as small as desired. This code can be applied to provide a more efficient system with minimum BER. This paper analyses the system performance by applying LDPC coding technique. And the performance is evaluated by calculating the BER for different values of Signal to Noise Ratio (SNR).

LDPC codes are finding increasing use in applications requiring reliable and highly efficient information transfer over bandwidth. The LDPC codes are defined using sparse parity-check matrix. Below is a graph fragment of an example of LDPC code using Forney's factor graph notation. In this graph n variable nodes in the top are connected to $(n-k)$ constraint nodes in the bottom of the

graph. This is a popular way of graphically representing an (n, k) LDPC code.

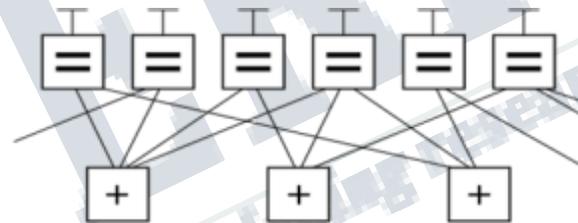


Figure 1: Forney's factor graph notation

This paper deals with the Bit Error Rate (BER) calculation of LDPC code, by applying different modulation schemes such as BPSK and QPSK on it. BER is the number of bit error per unit time. In a communication system the receiver side BER may be affected by transmission channel noise, interference, distortion, attenuation etc. In a noisy channel, BER is often expressed as the function of the normalized carrier-to-noise ratio measure denoted by E_b/N_0 . For example, in the case of QPSK modulation and AWGN channel, the BER as function of the E_b/N_0 is given by:

$$BER = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right)$$

Performance of a code is measured through its bit error rate Vs. signal to noise ratio in dB. The curve of a good code will show a dramatic drop in BER as SNR improves. The best codes have a cliff drop at an SNR slightly higher than the Shannon's limit.

II. SYSTEM MODEL

The proposed model of the LDPC system is shown in Figure 1. A stream of bits are generated by the message source,

which is then encoded into code word using LDPC encoder. The BPSK modulator maps the input binary signals, to an analog signal for transmission. The modulated signal is then passed through AWGN channel, where the addition of noise occurs normally. The LDPC decoder, which is implemented at the end-user, decodes the data and is then passed to the destination after retrieving the estimated message from the estimated code word.

A. Message Source

The message source is the end-user transmitting the data. The stimulation utilized a random message generator.

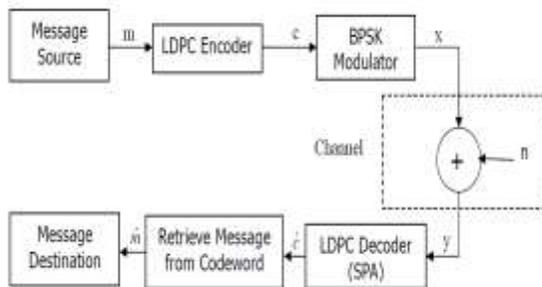


Figure 2: LDPC system model

B. LDPC encoder

LDPC codes are linear block codes. To correct more error and to reduce the extent of decoding complexity involved in doing so, the parity check matrix must be sparse in nature. Hence LDPC codes are defined by a very sparse natured parity check matrix in a finite Galois Field (GF). Non-binary LDPC codes are defined by an $M \times N$ parity check matrix H where elements of H are defined over the finite field GF. The parity check matrix is generated with each non-zero element can take on $(q-1)$ values. As the H is not in systematic form initially, to make it systematic we write it as

$$H = [P \mid I_m]$$

Where I_m is the $M \times M$ identity matrix and P has dimension $M \times K$ where $K = M - N$. All the arithmetic operations are carried out over the GF. Encoder convert input message frame consisting of K information bits U from GF. Frame U is encoded by LDPC encoder to give code word V of length N symbols.

$$V = GU$$

Here the matrix multiplication is performed over the finite field GF and finally encoder generates code word V of length $N = nP$ bits.

C. BPSK Modulator

The BPSK (Binary Phase Shift Keying) maps the input binary signals to an analog signal for transmission. It is the simplest form of Phase Shift Keying. It uses two phases which are separated by 180 degree and so can be termed as 2-PSK. In simulation BPSK signal is represented by the mapping:

$$\{0, 1\} \rightarrow \{\sqrt{E_b}, -\sqrt{E_b}\}$$

This modulation is most robust of all PSKs since it takes the highest level of noise or distortion to make the demodulator reach an incorrect decision.

D. Channel

The channel is the medium of which information is transmitted from transmitter to the receiver. In mobile communication, this is a wireless channel, and for other applications this could be copper or fibre optics. The addition of noise normally occurs in the channel. In simulations, the channel is modelled as an AWGN (Additive White Gaussian Noise) channel. Additive White Gaussian Noise is a channel model in which only impairment to communication is a linear addition of wideband or white noise with a constant spectral density and a Gaussian distribution of amplitude. This model does not account for fading, frequency selectivity, interference, non-linearity or dispersion. AWGN is common to every communication channels, which is statistically random radio noise characterized by a wide frequency range with regards to signal in communication channel. The assumptions are noise is additive, white and noise samples have a Gaussian distribution. The resulting noise added to the system follows the zero-mean normal distribution, with variance $N_0/2$ and N_0 is the signal sided noise power spectral density.

E. LDPC Decoder

The decoder is implemented at the end-user receiving the information. There are generally two classes of decoding techniques: Hard and Soft decision decoding. Hard decision decoding involves making a decision on the value of a bit at the point of reception, such as a MAP (Maximum A posteriori Probability) decoder. Such a decoder forms a decision based on a boundary that minimizes the probability of bit error. A decision for a soft-decision decoder is not clear. Soft decoding requires processing of the received code word vector prior to making a decision on the value of the bits. There is a large amount of research into various methods of soft-decision decoding. This is discussed much more detail in section III.

F. Retrieve message from code word

This simple process retrieves the estimated message from the estimated code word. In the simulation this is done via a simple function call following estimating code word.

G. Message Destination

The message destination is the end-user receiving the data. In a mobile communications environment, this would be the user receiving the voice information of the other user. In simulations, there is no message destination; rather the received message is compared to the transmitted message in order to detect whether a transmission error occurred.

III. DECODING

Different authors come up independently with more or less the same iterative decoding algorithm. They call it different names: the sum-product algorithm, the belief propagation algorithm and the message passing algorithm. There are two derivations of these algorithms:

Hard decision and Soft decision schemes.

A. Hard Decision Decoder

Consider a (4, 8) linear block code to illustrate the hard decision decoder. The code is represented in Figure 3, its corresponding parity matrix is:

$$H = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

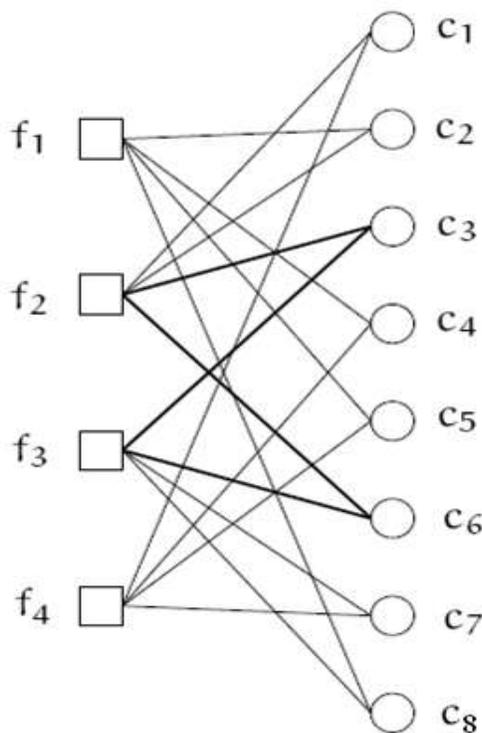


Figure3: Belief Propagation example code

An error free code word of H, $c = [0 0 1 0 1 0 1]$. Suppose we receive $y = [1 1 0 1 0 1 0 1]$. The algorithm is as follows:

1. In the first step, all message nodes send a message to their connected check nodes. In this case, the

message is the bit they believe to be correct for them. For example message node c2 receives a 1, so it sends a message containing 1 to check nodes f1 and f2.

2. In second step, every check nodes calculate a response to their connected message nodes using the messages they receive from step 1. The response message in this case is the value that the check node believes the message node has based on the information of other check nodes connected to that check node. In table 1, check node f1 receives 1 from c4, 0 from c5, 1 from c8 thus it believes c5 has 1 and sends 1 back to c5. At this point, if the values that the check node calculates match the value they receive, the algorithm terminates.
3. In this step, the message node uses the messages that they get from the check node to decide if the bit at their position is a 0 or a 1 by majority rule. The message nodes then send their hard decision to their connected check nodes. Table 2 illustrates this step. To make it clear, let us see the message node c2. It receives two 0's from check nodes f1 and f2. Together with what it already has, it decides its real value is 0. It then sends this information back to check nodes f1 and f2.
4. Repeat step 2 until either exit at step 2 or a certain number of iterations has been passed. In this example, the algorithm terminates right after the first iteration as all parity check equations have been satisfied.

Table 1
Check Nodes Activities for Hard Decision Decoder

check nodes	activities					
f ₁	receive	c ₂ → 1	c ₄ → 1	c ₅ → 0	c ₈ → 1	
	send	0 → c ₂	0 → c ₄	1 → c ₅	0 → c ₈	
f ₂	receive	c ₁ → 1	c ₂ → 1	c ₃ → 0	c ₆ → 1	
	send	0 → c ₁	0 → c ₂	1 → c ₃	0 → c ₆	
f ₃	receive	c ₃ → 0	c ₆ → 1	c ₇ → 0	c ₈ → 1	
	send	0 → c ₃	1 → c ₆	0 → c ₇	1 → c ₈	
f ₄	receive	c ₁ → 1	c ₄ → 1	c ₅ → 0	c ₇ → 0	
	send	1 → c ₁	1 → c ₄	0 → c ₅	0 → c ₇	

Table 2
Message Nodes Decision for Hard Decision Decoder

message nodes	y_i	messages from check nodes		decision
c_1	1	$f_1 \rightarrow 0$	$f_4 \rightarrow 1$	1
c_2	1	$f_1 \rightarrow 0$	$f_2 \rightarrow 0$	0
c_3	0	$f_2 \rightarrow 1$	$f_3 \rightarrow 0$	0
c_4	1	$f_1 \rightarrow 0$	$f_4 \rightarrow 1$	1
c_5	0	$f_1 \rightarrow 1$	$f_4 \rightarrow 0$	0
c_6	1	$f_2 \rightarrow 0$	$f_3 \rightarrow 1$	1
c_7	0	$f_3 \rightarrow 0$	$f_4 \rightarrow 0$	0
c_8	1	$f_1 \rightarrow 1$	$f_3 \rightarrow 1$	1

B. Soft Decision Decoder

A soft decision decoder is a class of algorithm used to decode data that has been encoded with an error correcting code. Whereas, a hard decision code operates on data that takes on a fixed set of possible values, the input to the soft decision decoder may take on a whole range of values in-between. This extra information indicates the reliability of each input data point, and is used to form better estimates of the original data. Therefore; soft decision decoder will typically perform better in the presence of corrupted data than its hard decision counterpart.

IV. ANALYSIS

In this paper, BER performance of LDPC coded system on AWGN channel using different modulation schemes is analysed. BPSK modulation and QPSK modulation techniques has been used.

1. BPSK Modulation

BPSK is one form of Phase Shift Keying and is also termed as Phase reversal keying. It consists of two points located 180 degree apart from each other and is used for representing binary information. The location of points on Cartesian coordinate is shown below. The Cartesian coordinate plot is also referred to as constellation.

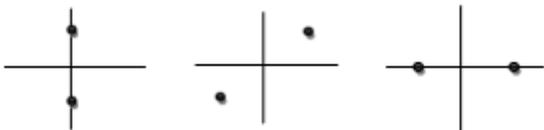


Figure 4: Cartesian coordinate plot of BPSK

$$S_b(t) = \sqrt{\frac{2E_b}{T_b}} \times \cos(2\pi f_c t + \pi(1 - n))$$

Where $n=0, 1$

For binary bit 0:

$$S_0 = \sqrt{\frac{2E_0}{T_0}} \times \cos(2\pi f_c t + \pi(1 - 0))$$

$$S_0 = \sqrt{\frac{2E_0}{T_0}} \times \cos(2\pi f_c t + \pi)$$

If the signal space is $\phi(t) = \left(\frac{2}{T}\right) \cos(2\pi f_c t)$

Then,

$$S_0 = -\sqrt{E_0} \phi(t)$$

Where E_0 =energy bit.

For binary bit 1:

$$S_1 = \sqrt{\frac{2E_1}{T_1}} \times \cos(2\pi f_c t + \pi(1 - 1))$$

$$S_1 = \sqrt{\frac{2E_1}{T_1}} \times \cos(2\pi f_c t)$$

If signal space $\phi(t) = \left(\frac{2}{T}\right) \cos(2\pi f_c t)$

Then,

$$S_1 = \sqrt{E_1} \phi(t) \text{ where } E_1 = \text{energy bit}$$

Bit Error Rate in BPSK is:

$$P_b = (1/2) \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \text{ where,}$$

E_b = energy / bit

N_0 = Noise Power Spectral Density

The system model of BPSK used is:

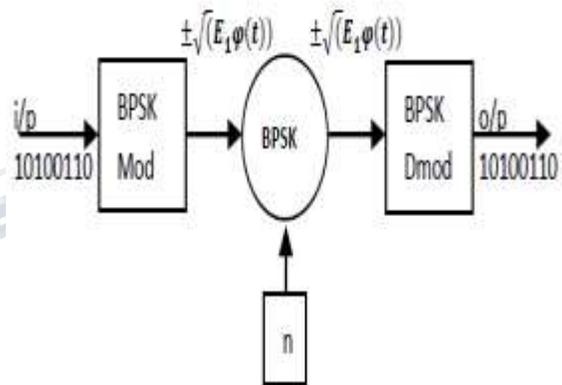


Figure 5: BPSK Model

2. QPSK Modulation

QPSK can be obtained by adding two more binary information symbols in BPSK. QPSK have twice the data rate than BPSK and it consist of four signal elements separated 90 degree apart from each other. QPSK constellation is as shown below:

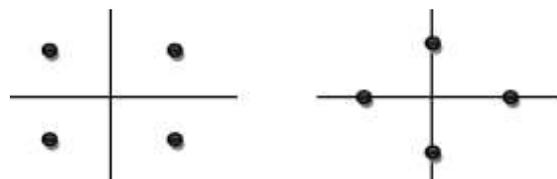


Figure 7: Constellation of QPSK

Generalized equation for QPSK is:

$$S_i(t) = \sqrt{\left(\frac{2E_s}{T}\right)} \times \cos\left(2\pi f_c t + \frac{(2n-1)\pi}{4}\right)$$

n=1, 2, 3, 4

Converting in signal space:

$$\varphi_1(t) = \sqrt{\left(\frac{2}{T}\right)} \times \cos(2\pi f_c t)$$

$$\varphi_2(t) = \sqrt{\left(\frac{2}{T}\right)} \times \sin(2\pi f_c t)$$

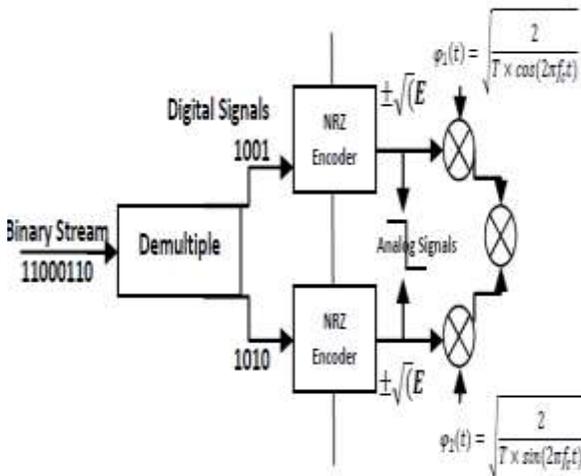


Figure 8: System model for QPSK

3. Comparison of BPSK AND QPSK

The equation below is the ratio of Energy consumed in Bit to the signal spectral noise and it gives bit error rate in QPSK:

$$P_b = Q\sqrt{\left(\frac{2E_b}{N_0}\right)}$$

In BPSK and QPSK, bit error rate is similar and thus twice amount of data can be transmitted on the same channel with same bandwidth using QPSK.

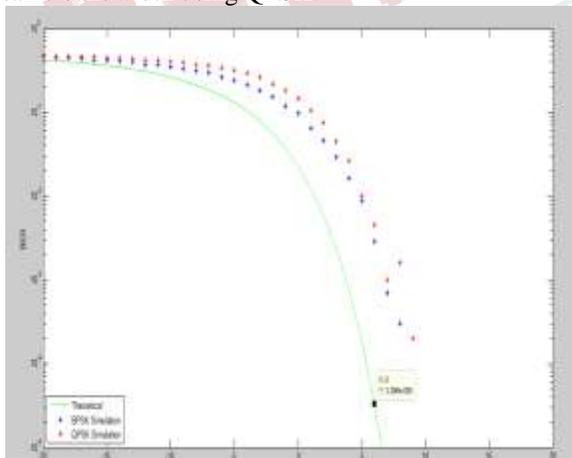


Figure 9: Comparison graph

The comparison graph is shown above. BPSK provides high immunity to noise as the signal to noise ratio for BPSK is very low.

Table 3

Result of BPSK and QPSK with LDPC Coding

SNR VALUES	BER values		
	Theoretical	BPSK	QPSK
-20	0.4207	0.4737	0.4737
-16	0.3756	0.4626	0.4626
-12	.3011	0.3719	0.4174
-9	0.239	0.3295	0.3933
-7	0.1858	0.2941	0.3585
-3	0.0784	0.1819	0.2607
1	0.0124	0.0648	0.107
4	0.000762	0.0164	0.0262
6	3.296e-.005	0.0029	0.0045

The BER performance analysis of LDPC coding and modulation schemes over AWGN channel is shown in the result. As per the result we obtained, performance of BPSK modulation with LDPC coding is best over AWGN channel.

CONCLUSION

The important concepts regarding low density parity check-matrix (LDPC) has been summarized in this paper. And it shows how LDPC can be decoded and encoded. The simulation of LDPC code for message transmission with different modulation techniques enable in analysing the Bit Error Rate. Thus, help in finding a better method for reliable communication.

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