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# Modeling and control of Ball and Beam system

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*Abstract:* — **In this paper mathematical modeling, control and dynamics of ball and beam system have been studied and presented. The ball and beam problem basic backbone of control system study due to its highly nonlinear behavior property and used in study of equivalent physical model. Due to Its equivalency property this model is safe for study of controlling the equivalent physical system in industrial and teaching applications. Ball and beam system consists of subsystem and process assembled for controlling the ball on beam using controlling the output process. The basic purpose of this model is control the ball on beam using proportional plus derivative controller design.**

*Index Terms*— **Ball and beam system, control, dynamics, modeling, nonlinear, process.**

## 1.INTRODUCTION

Ball and beam system is a basic class of common physical system. Ball and beam system is integral part of control system laboratory in universities and industries. It is a simple and lighter mechanical system for control system study. Generally to its physical linked to stabilizing the automatic system as like horizontal stabilizing the aero plane during landing and take-off in turbulent air flow, in aerospace, in power production and in the chemical process industries etc. It is an unstable system, therefore many control techniques to use study of this system to controlling purpose by suitable feedback process. In this system automatically regulate the position of the ball on beam by tilting the angle of beam by motor due to back forth movement.
Characteristic of this system has two degree of freedom. A lever arm is connected to the beam at one end and motor gear at the other. As the motor gear rotates by angle theta, the lever changes the angle of the beam by alpha. When the angle is changed from the vertical position, gravity causes the ball to roll along the path of the beam. Therefore, a suitable controller will be designed for this system so that ball's position can be calculated.

## II. MATHEMATICAL MODELING AND DYNAMICS

### *Mathematical modeling*

Mathematical modeling of any physical system shows geometrical, numerical characteristics of the system for study and understanding of the system.

### *System equations*

System equation of the ball and beam system provides nature, behavior, property of the system. Ball and beam system equation gives relation between motor gear and beam angle. This equation shows system is linear or nonlinear manner.

### *Transfer function*

Transfer function of ball and beam system provides relation between output function and input function in Laplace order. Ball and beam transfer function is double integrator nature. This nature shows that the system is unstable in open –loop and marginally stable and will provide a challenging control problem. Thus its transfer function can be obtained as combination of transfer functions of subsystem namely (i) beam tilt to ball position and (ii) motor input voltage to its angle of rotation and tilt angle of the beam.

## III. GEOMETRICAL ANALYSIS AND TRANSFER FUNCTION OF BALL AND BEAM SYSTEM

Ball and beam system is a classical control problem. The process is highly nonlinear and unstable with one input signal and several output signal. The aim is to control a ball on beam by movement of motor. In order to derive relation between ball position and beam angle. Let us consider the Fig.1 in which position of the ball at any instant of time is x, the inclination of beam is considered along x- axis. The motion of the ball on beam is translational and rotational. Now, let the translational acceleration of the ball is    for which force is given by

$$F_{tx} = m\ddot{x} = m\frac{d^2x}{dt^2} \qquad (1)$$

where $m = mass\ of\ the\ ball.$

The rotational torque $T$ of the ball $= \mathrm{J}\frac{d\omega}{dt}$

where $\omega = angular\ velocity$

and $J = moment\ of\ inertia\ of\ the\ ball,$

and rotational force $F_{rx} = \frac{T}{R} = \frac{J}{R^2}\ddot{x}$

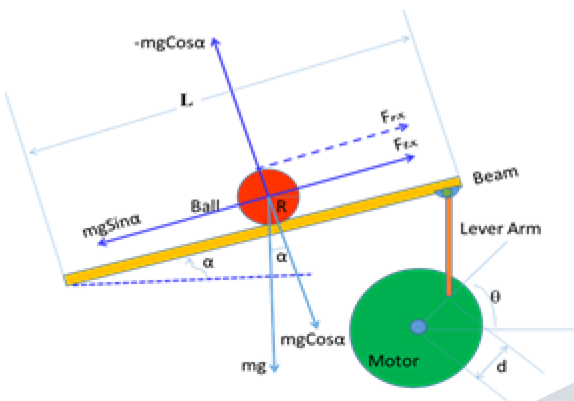


***Fig.1. Ball and Beam system with Newton balances of forces As shown in Fig.2.We have***

$$F_{rx} + F_{tx} = -mgsin\alpha$$
$$\Rightarrow \left(\frac{J}{R^2} + m\right)\ddot{x} = -mgsin\alpha \qquad (2)$$

Now, tilt of the beam is controlled by rotational angle of the gear disc connected to motor. The liver arm elevates and brings down beam as per value of as shown in Fig.1. In the simple way the relation between and can be obtained using geometry of the lever arm section as shown in Fig.2. As is obvious from diagram of ball and beam system that the up and down motion in beam is produced due to its rotations about . The said rotation of beam is produced by rotation of motor at . The beam is connected at to lever mechanism consisting of lever arms and .The rotation in motor produces rotation in and vertical shift in arm which accordingly moves beam . The relation between angular displacement in and corresponding deflection in beam is required. Let the motor arm reaches at and end of beam moves to such that and . Obviously,

; ;

$OX = d\cos\theta; TX = d\sin\theta;$

$XS = OS - OX = d(1 - \cos\theta)$

$O_1M = L\cos\alpha; QM = L\sin\alpha; MP = L(1 - \cos\alpha)$

$TN = A\sin\beta; QN = A\cos\beta; MN = A\cos\beta - L\sin\alpha$

$TX = PS - PG = MZ - MN$

$OX = d\cos\theta; TX = d\sin\theta$

$XS = OS - OX = d(1 - \cos\theta)$

$O_1M = L\cos\alpha; QM = L\sin\alpha; MP = L(1 - \cos\alpha)$

$TN = A\sin\beta; QN = A\cos\beta; MN = A\cos\beta - L\sin\alpha$

$TX = PS - PG = MZ - MN \Rightarrow L\sin\alpha =$

$d\sin\theta - A(1 - \cos\beta) \Rightarrow \alpha = arc\sin\left[\frac{d}{L}\sin\theta - \frac{A}{L}(1 - \cos\beta)\right]$ (3) Now

$TN + NG = XS$

$\Rightarrow A\ \sin\beta\ = d(1 - \cos\theta) - L(1 - \cos\alpha) \qquad (4)$

The general solution for and is complex [10] for the given input angle . However, under restriction that the dependence of on is very weak and thus from (4) we can get

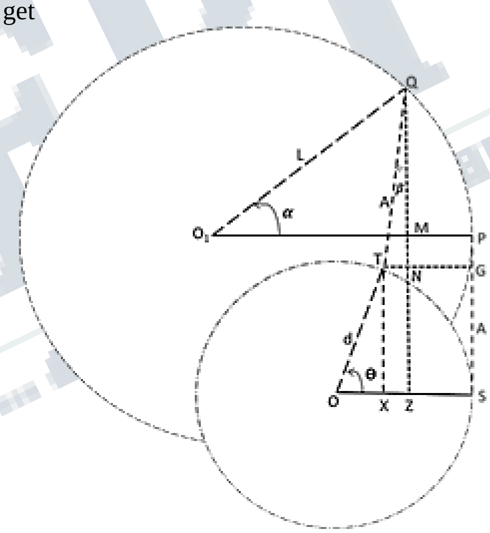


***Fig.2. Relation between motor angle and beam angle***

$$\Rightarrow \beta = arc\ \sin\left[\frac{d}{A}(1 - \cos\theta)\right]$$
$$\Rightarrow \beta = arc\ \sin\left[\frac{d}{A}(1 - \cos\theta)\right]$$

(5)

and

$$\alpha \approx arc\ \sin\left[\frac{d}{L}\ \sin\theta\right]$$

(6)

$$\alpha \approx \frac{d}{L}\theta\ , when\ \theta\ is\ small$$

(7)

$$\dot{\alpha} = \frac{\frac{d}{L}\cos\theta}{\sqrt{1-\frac{d^2}{L^2}(\sin\theta)^2}}\,\dot{\theta} \quad \& \quad \dot{\beta} = \frac{\frac{d}{A}\sin\theta}{\sqrt{1-\frac{d^2}{L^2}(1-\cos\theta)^2}}\,\dot{\theta}$$

the beam angle is proportional to motor angle which in turn proportional to input voltage.
from eq. (2) and (7), we get

$$\left(\frac{J}{R^2} + m\right)\ddot{x} = -mg\,\frac{d}{L}\,\theta, \qquad (\because \sin\alpha \cong \alpha)$$

taking the laplace transform of both sides assuming initial condition zero. we get

$$\left(\frac{J}{R^2} + m\right)s^2 X(s) = -mg\,\frac{d}{L}\theta(s)$$

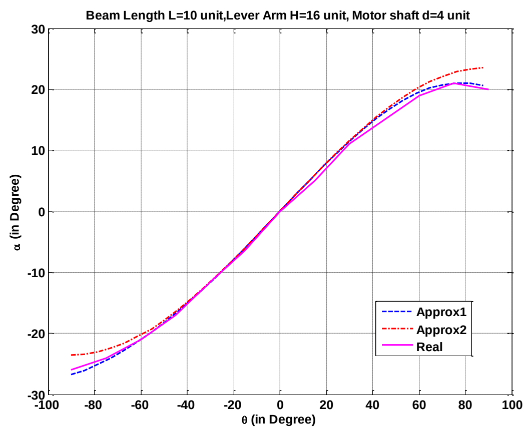


***fig.4. relation between $\theta$ and $\alpha$ as per different approximated relation which shows that the relation is linear for smaller value of motor angle***

$$\frac{X(s)}{\theta(s)} = G_B(s) = \frac{-mg}{\left(\frac{J}{R^2}+m\right)}\frac{d}{L} \times \frac{1}{s^2} \tag{8}$$

obviously, transfer function is double integrator and thus provides challenging control problem. we know that the moment of inertia of ball is $J = \frac{2}{5}mR^2$. thus the transfer function $G_B(s)$ can be finally given by

$$G_B(s) = \frac{X(S)}{\theta(s)} = -K\left(\frac{1}{S^2}\right) \tag{9}$$

where constant $K = 7\frac{d}{L}$.

since the tilt angle $\alpha$ of the beam is controlled by the motor angle $\theta$ and $\theta$ is controlled by the input voltage, the transfer function of motor is also essential to establish relation between $\theta$ and input voltage. the dc servomotor linked to beam pivot decides tilt of the beam which in turn controls motion of ball on the beam [9]. as discussed above the tilt angle $\alpha$ and angular rotation $\theta$ of the motor are related by eq.7.the angle of rotation $\theta$ of the motor decided by the applied input voltage $E_a(s)$.the transfer function $G_m(s)$ of the motor is obtained as follows.

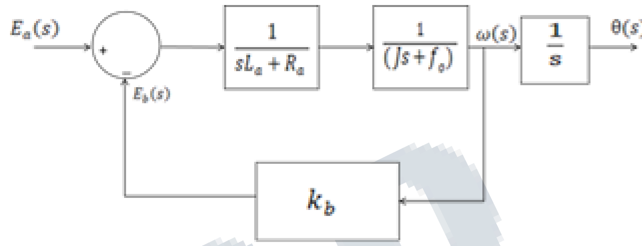


***fig.5. modified block diagram of dc motor***

the overall transfer function of the block diagram of fig.5 is given by

$$G_m(s) = \frac{\theta(s)}{E_a(s)}$$

$$G_m(s) = \frac{k_T/R_a}{s(Js+f)}, \tag{10}$$

where $f = f_o + \frac{k_T k_b}{R_a}$.

appearance of term $k_b$ in $f = f_o + \frac{k_T k_b}{R_a}$ shows that the in built feedback loop for back e.m.f. enhances viscous friction and thus known as electric friction.

$$G_m(s) = \frac{k_T/R_a f}{s\left(\frac{J}{f}s+1\right)} = \frac{k_m}{s(\tau s+1)} \tag{11}$$

where

$k_m = {k_T}/{R_a f} = motor\ gain\ constant,$

$\frac{J}{f} = \tau = motor\ time\ constant.$

further it can be shown that $k_T = k_b$ in mks units. also, the transfer function for the speed can be given by

$$\frac{\omega(s)}{E_a(s)} = \frac{k_m}{(\tau s+1)} \tag{12}$$

where $k_m$ and $\tau$ are gain and time constants depending on motor parameters such as armature resistance, back e.m.f. constant and motor load equivalent moment of inertia.
this block diagram makes it obvious that dc motor is an

integrating device. in the servo system this dc motor is connected to load via gear system and a position sensor such as potentiometer is attached to the output shaft.
the overall open loop transfer function g(s) of the ball and beam system can obtained by combining motor function $g_m(s)$ and $g_b(s)$, tf of ball and beam system as shown in fig.6.

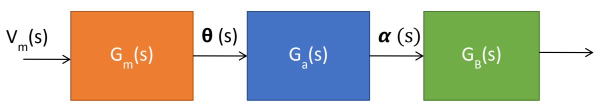


***fig.6. tf of ball and beam system***
***the transfer function of beam angle α with respect to θ obtained from given equation***

$$\alpha \approx \frac{d}{L}\theta \text{ , when } \theta \text{ is small}$$

system parameters used in simulation
m = 0.111 kg; r = 0.015 m; g = -9.8m/s^2;
l = 1.0h; d = 0.03m;
j = 9.99e-6 kgm^2;

therefore,

$$G_a(s) = \frac{\alpha(s)}{\theta(s)} = \frac{d}{L}$$

here in the considered model beam angle $\alpha$ is changed by lever system consisting of lever arm $PS$ connected to motor shaft $os$.
therefore the overall transfer function g(s) of the ball and beam system is given by

$$G(s) = G_m(s).G_a(s).G_B(s)$$

$$= \frac{X(s)}{\theta(s)} = \frac{K_m K}{s^3(1+\tau s)}\frac{d}{L} \quad (13)$$

Nature of transfer function

***Beam tilt to ball position tf $G_B(s)$***

This transfer function as obtained in eq.(7) and (8) decides position of the ball on the beam for the given tilt angle of the beam. the step response of this transfer function is shown in fig.7 which shows that for the any given tilt of the beam the ball will roll down under gravity and go out of the beam with velocity depending upon tilt of the beam as shown in fig.8. thus to control and regulate motion of the ball on the beam some control mechanism is required which will change tilt of the beam in such way that will stop fall down of ball as well as regulate motion of the ball. this tf contains two poles at origin.

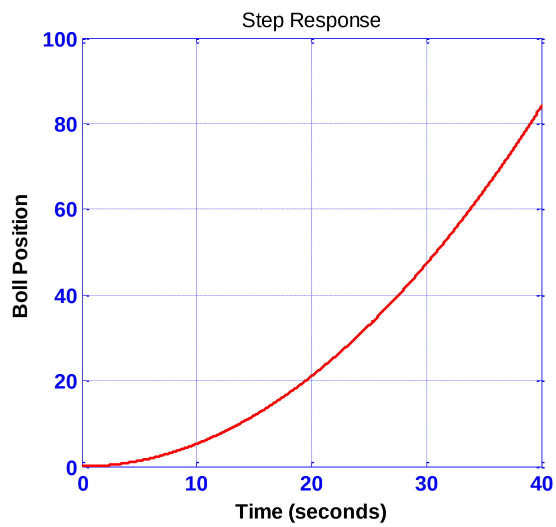


***Fig.7.step response of ball and beam system which shows that for the constant tilt angle the ball will move away from if tilt is not changed.***

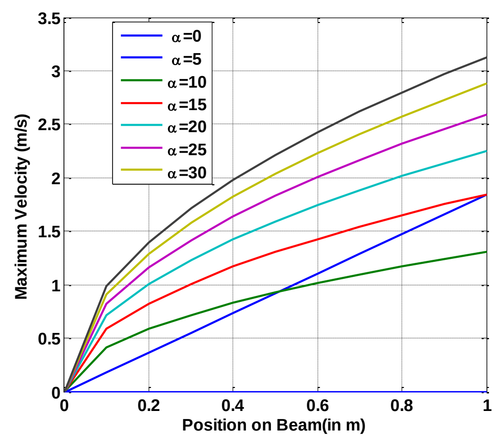


fig.8.velocity of ball at different positions of beam for different values of beam tilt α (in degree)
This transfer function includes relation between motor arm rotation θ and transmission of θ to beam angle α. the input voltage rotates motor and the angular displacement of motor arm is transmitted to beam rotation. obviously this contains two parts namely (i) input voltage to motor arm rotation and motor arm to beam rotation. the relation between θ and beam angle α is also complicated and has been obtained in eq.(3), (6) and (7). this relation is shown in fig.9 for different d/l ratio and it is evident that for smaller range of θ, beam

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angle varies linearly. the value of beam tilt depends on the contact point of the lever arm with beam also. for the given setup of motor arm length and lever arm such variation in beam tilt is shown in fig.10 as per eq.(6). the linearity as well as range of relation reduces with increasing contact length.

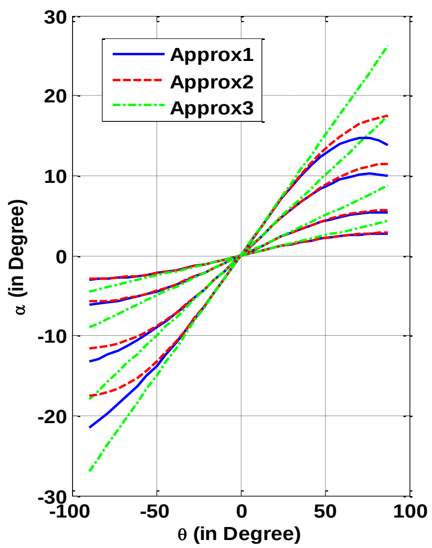


***fig.9. the relation between beam motor angle between θ and beam angle α shown by legend approx1, approx2 and approx3 for eq.(3),(6) and (7) respectively.***

the dependence of β on motor angle θ for different values of motor arm *d* and lever arm *a*, as is obtained in eq.(5) is shown in fig.11. the practical values for the same setup obtained by making diagram as shown in fig.3 were measured and are plotted in fig.12 which shows agreement with eq.(5).

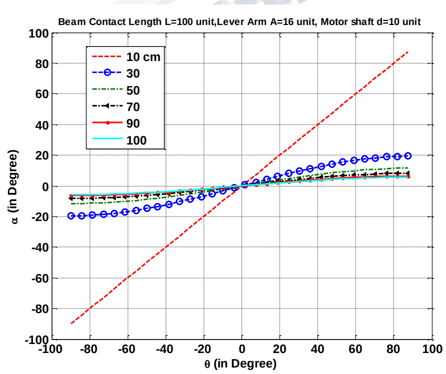


***fig.10 variation in beam tilt α with variation in motor angle θ for different contact point of lever arm on beam.***

It is obvious from these graphs that the dependence of β and motor angle θ can be ignored for ranges up to -30 to 30 degrees. The tilt in beam angle depends on point contact with lever arm but contact near the pivot O of the beam may introduce unwanted vibration in the beam. The ration of length of motor arm and beam length plays important role in deciding these relations. The graph shown in Fig.11 in red line presents approximation done by Eq.(7) for the relation between beam angle and motor angle. For the approximation of these angular relation in [10] authors have presented nonlinear relations taking into consideration three parameters namely beam length, lever arm length and motor arm length.

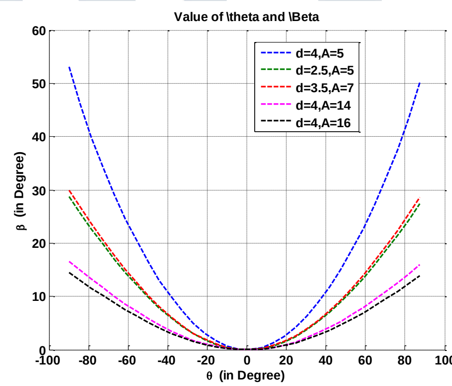


***Fig.11 Dependence of β on motor angle θ for different values of motor arm d and lever arm A.***

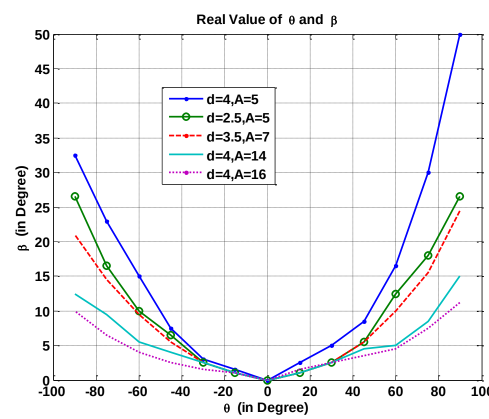


***Fig.12 Relation between of β and motor angle θ measured for different values of motor arm d and lever arm A.***

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### *B. Motor transfer function*

The motor transfer function as given by Eq. (17) and (18)

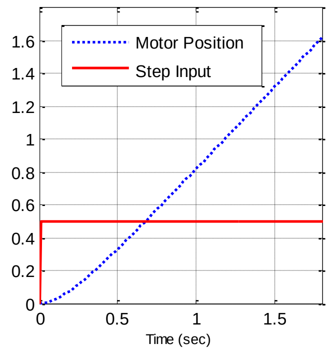


***Fig.13 step response of motor (open loop)***

The ball and beam system represents a system which requires control to direction of rotation. For the given constant input the motor shaft position is shown in Fig 13. In order to control motor PD control can be used. The motor transfer function represents 2nd order system and PD control will preserve its order. In the PD controller the control signal is given by

$$C_1(s) = K_p + K_d S$$

Where

$K_p$ = Proportiooanl gain

$K_d$ = Derivative gain

These PD parameters are tuned to give optimal performance. The step response of PD controller is shown in Fig.15

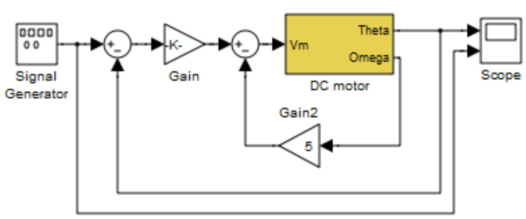


***Fig.14 SIMULINK Model of DC motor control***

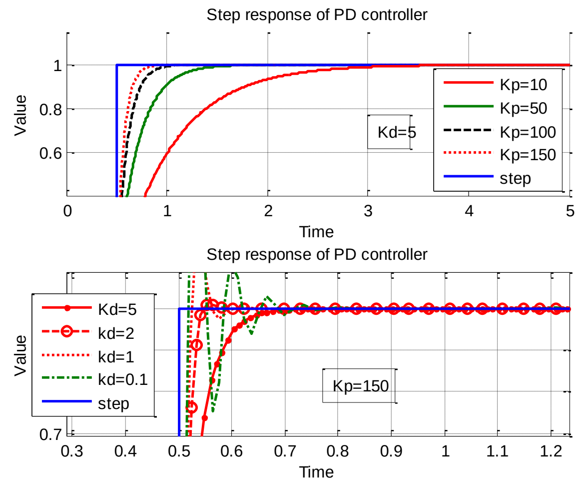


***Fig.15 Step response of PD controlled DC motor***

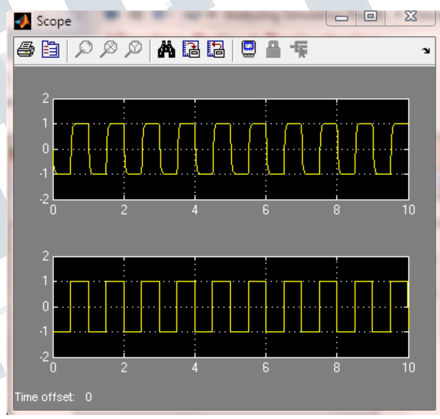


***Fig.16 Tracking of input signal pattern by PD controlled motor obtained by simulation in SIMULINK.***

### *C. Ball and beam system transfer function*

The overall transfer function as obtained above in Eq.(19) of the ball beam system represents 4th order system. The open loop behavior of the ball and beam system, as shown in Fig.7 indicates that with no control on the tilt of beam, the beam allows ball to move rapidly over it to fall out. The close examination of ball beam system from control point of view shows that for the controlled movement of ball on beam, beam angle needs to be changed according to position of the ball so that ball can be stopped from falling down. The beam angle is decided by the motor angle through lever arms. The amount of rotation in motor is governed by the input voltage supplied to motor. The functional relation for control of ball beam system is shown in

Fig.17. Obviously, the equivalent block diagram contains TF of fourth order and design of the controller for the same is very difficult. However ball beam system can be controlled by two controllers one for motor control and other for the beam angle control per ball position.

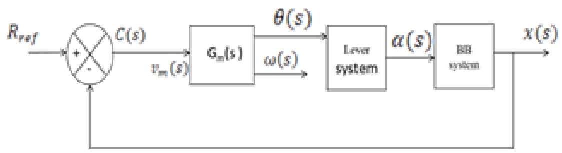


*Fig.17 Control of Ball Beam System*

The step response of ball and beam system using PD control is shown in Fig.18 for different values of Kp and Kd.

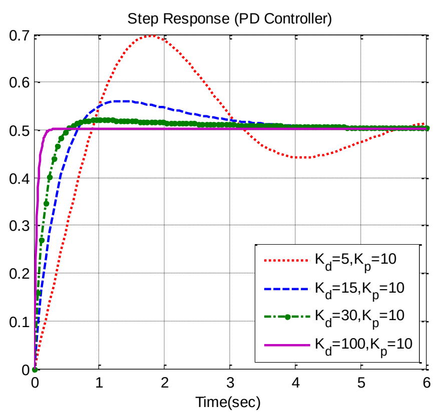


***Fig.18 Step response of PD controlled Ball beam system.***

## CONCLUSION

In this paper modeling, dynamics and control of ball and beam system has been presented. The system function for different subsystems have been obtained and detailed analysis on relation between motor and beam angle has been presented with theoretical, geometrical and experimental results. PD control has been used to control the system and result obtained by simulation in SIMULINK have been also presented.

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