

Design, Modelling and Analysis of Two Phase Interleaved Buck DC-DC Converter

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Abstract: -- The two phase interleaved buck converter (IBC) for low input voltage (battery or photovoltaic) applications. The interleaved multi-channel converter is very popular these days because of its ability to reduce current ripples. In high power applications, parallel operation of buck converter is recommended. The steady state performance of the interleaved buck converter is investigated by MATLAB/simulation. The IBC is designed for low input voltage application. The modeling and analysis is carried out considering the parasitic elements of all the elements and devices. The IBC is modeled and analyzed using state-space averaging technique and analysis is carried using small signal analysis. The analog PID controller is designed based on the design parameters of the converter and simulated in open loop and closed loop.

Index Terms—Interleaved Buck converter, small signal analysis, state space technique, continuous conduction mode

I. INTRODUCTION

The demand for DC-DC converter has increased as the front end stage for the battery sources and the renewable energy applications. Two Phase Interleaved DC-DC Buck converter is introduced to meet the increased demands such as low current ripple, high efficiency, faster dynamics, light weight and higher power density. Interleaving also called multi-phasing, is a technique that is useful for reducing the size of filter components [5]. IBC performs remarkably in terms of magnetics, input and output current ripple and part count [4]. The Voltage Regulator Modules (VRMs) are used to power the μ p (microprocessors) in the computers, make use of the conventional or synchronous buck converters. The VRMs require output voltage lower than 1 volt and output current higher than 100 amperes. This makes the existing topology inadequate. The interleaved dc-dc converters are suitable for the low voltage and high current output topology.. IBC will act as a pre-regulator in the PV system. Interleaved buck converter is used to convert the unregulated DC input voltage into a controlled DC output at a desired voltage level [2].

The Two Phase Interleaved Buck converter was modeled considering all the parasitic elements of the converter using state space average technique [1,2] and analyzed using small signal analysis. The model is simulated in MATLAB/Simulation in open loop and closed loop with analog PID controller.

This paper presents the design of IBC and also

modeling and analysis of IBC. Section II presents working principle of IBC. Section III describes the steady state characteristics of buck converter and also design of IBC. Section IV presents modeling and analysis of two phase IBC. Section V gives the simulation results. Finally section VI concludes the paper.

II. TWO PHASE INTERLEAVED BUCK CONVERTER

The two phase interleaved buck converter is as shown in “Fig.1,”. the two converters are essentially connected in parallel but operate in an interleaved mode. The first converter is composed of inductor L_1 , Switch S_1 , and Diode D_1 , where as the second converter consists of L_2 , S_2 , and D_2 . The IBC share the same filter capacitor C at the output. It is assumed that the parameters of the two converter are identical. The gating arrangements and the inductor current waveforms of the convert is shown in “Fig.2,”. this figure represents waveforms of inductor current and input current and the converter is operated with duty ratio d ($0 < d \leq 0.5$) in this duty ratio any one switch will be conducting at time and also at specific period of time both the inductors will be discharging. With two phase IBC designs, the gating signals g_1 and g_2 for switch S_1 and S_2 are identical but shifted by 180° . The total input current I_{in} , which is the sum of the two inductor currents i_{L1} and i_{L2} are shown in “Fig.2”.

Abbreviations and Acronyms

V_{in} – Input Voltage, V_{dc} – output Voltage, I_L – Inductor Current, I_{in} – Input Current, i_{L1} , i_{L2} – Current

flowing in inductors L_1 and L_2 , I_o – output Current,
 S_1, S_2 – MOSFETs/Switches
 g_1, g_2 –Gating signals, D_1, D_2 – Diodes, C – Capacitor d/D –
 Duty ratio, $\Delta I_{L1}, \Delta I_{L2}$ – L_1 and L_2 Inductor ripple current
 f_{sw} – Switching frequency, P_{in} – input power
 $i_{D,on}, i_{D,off}$ – Diode Current during ON/OFF
 $i_{M,on}, i_{M,off}$ – MOSFET (switch) current during ON/OFF
 r_{L1}, r_{L2} – Resistance of Inductors L_1 and L_2
 r_{S1}, r_{S2} – On state resistance of switches S_1 and S_2
 R – Load Resistance
 V_{d1}, V_{d2} – Voltage across D_1 and D_2
 r_c – Resistance of capacitance C

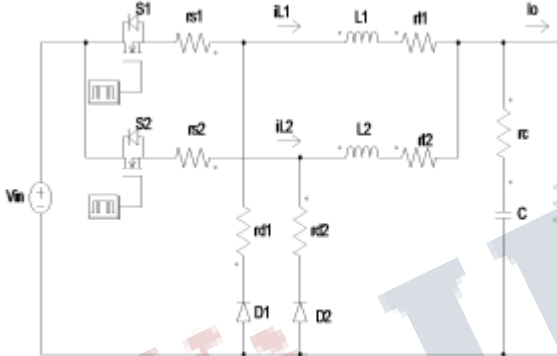


Fig 1. Two Phase Interleaved Buck Converter

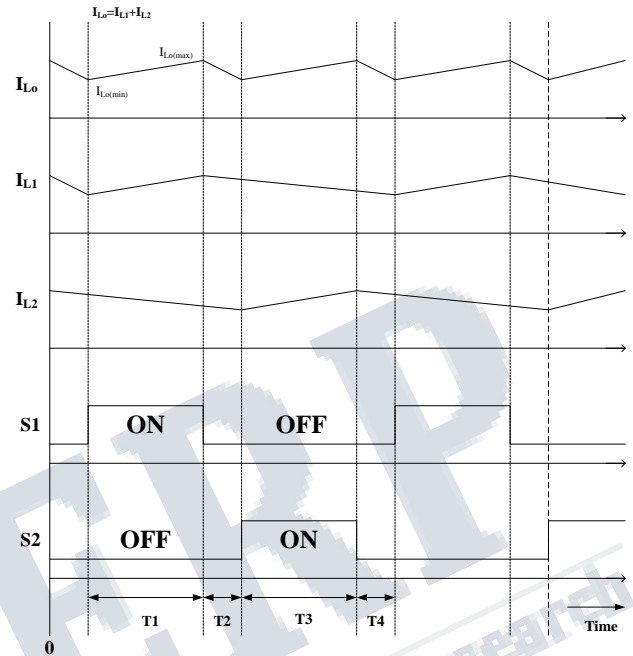


Fig 2 Typical Waveforms of Inductor Currents

STEADY STATE CHARACTERISTICS AND DESIGN OF IBC

BUCK RATIO: THE BUCK RATIO OF THE CONVERTER IS A FUNCTION OF THE DUTY RATIO. $\frac{V_{dc}}{V_{in}} = D$

INPUT CURRENT: THE INPUT CURRENT CAN BE CALCULATED BY THE INPUT POWER AND THE INPUT VOLTAGE. $I_{in} = \frac{P_{in}}{V_{in}}$

INDUCTOR CURRENT RIPPLE AMPLITUDE: $\Delta I_{L1,L2} = \frac{(V_s - V_o)D}{f_{sw}L}$

Design of IBC: Input data for designing the two phase interleaved buck converter output of 5V from an input of 24V source inductor current is continuous and output ripple of less

than 10%. The output power of 25Watts, switching frequency 25 kHz

Duty ratio (D)

$$D = \frac{V_o}{V_s} = \frac{5}{24} = 0.208$$

$$R = \frac{V_o^2}{P_o} = \frac{(5)^2}{25} = 1\Omega$$

Inductor (L)

$$L_{\min} = \frac{(1-D)R}{2 \times f_{sw}} = 16\mu H$$

$$L = 1.25 \times L_{\min} = 20\mu H$$

$$L_1 = L_2 = \frac{L}{2} = 10\mu H$$

Load Current

$$I_L = \frac{V_o}{R} = 5A$$

$$I_{L1} = I_{L2} = \frac{I_L}{2} = 2.5A$$

Ripple Inductor Current

$$\Delta i_L = \frac{(V_s - V_o)D}{L \times f} = 7.904A$$

Capacitor (C)

$$C = \frac{(1-D)}{8L \left(\frac{\Delta V_o}{V_o} \right) f^2} = 80\mu F$$

Maximum and Minimum Inductor Current

$$i_{L\max} = I_L + \frac{\Delta i_L}{2} = 8.952A$$

$$i_{L\min} = I_L - \frac{\Delta i_L}{2} = 1.048A$$

Equivalent series resistance (ESR)

$$\Delta V_{o(ESR)} = \Delta i_c r_c$$

$$\Delta i_c = i_{L\max}$$

$$r_c = \frac{\Delta V_{o(ESR)}}{\Delta i_c} = 0.066\Omega$$

$$\text{Efficiency } \eta = \frac{P_o}{P_o + P_{LOSS}} = 97.59\%$$

III. MODELLING AND ANALYSIS OF TWO PHASE BUCK CONVERTER

A. Modeling

Analysis of two phase interleaved buck converter using state space averaging technique. The two phase interleaved buck converter is analyzed for duty cycle (0<d≤0.5). The circuit can be analyzed in four modes of operation by considering parasitic elements.

MODE – 1 Switch S₁ is closed and S₂ is open, in this period inductor current i_{L1} is rising and i_{L2} is falling as shown in “Fig.2,”

Analyze the “Fig.3,” by applying KVL and KCL we get the equation in matrix form

State variables are $\begin{bmatrix} i_{L1} \\ i_{L2} \\ V_C \end{bmatrix}$

$$A1 = \begin{bmatrix} \frac{1}{L_1} \left[(-r_{L1} - r_{S1}) - \frac{R_{Load} r_c}{(R_{Load} + r_c)} \right] & 0 & -\frac{1}{L_1} \left(\frac{R_{Load}}{(R_{Load} + r_c)} \right) \\ 0 & \frac{1}{L_2} \left[(-r_{L2} - r_{D2}) + \frac{R_{Load} r_c}{(R_{Load} + r_c)} \right] & \frac{1}{L_2} \left(\frac{R_{Load}}{(R_{Load} + r_c)} \right) \\ \frac{R_{Load}}{2C(R_{Load} + r_c)} & \frac{R_{Load}}{2C(R_{Load} + r_c)} & \frac{-1}{C(R_{Load} + r_c)} \end{bmatrix}$$

$$B1 = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 \\ 0 & 0 & \frac{-1}{L_2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C1 = \begin{bmatrix} \frac{R_{Load} r_c}{(R_{Load} + r_c)} & \frac{R_{Load} r_c}{(R_{Load} + r_c)} & \frac{R_{Load}}{(R_{Load} + r_c)} \end{bmatrix}$$

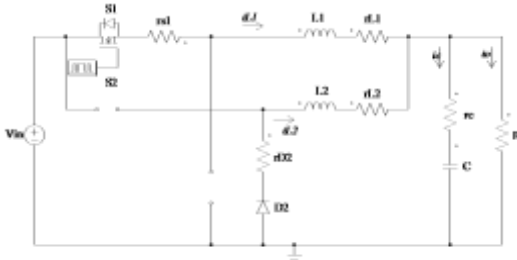


Fig 3 Mode 1 Operation

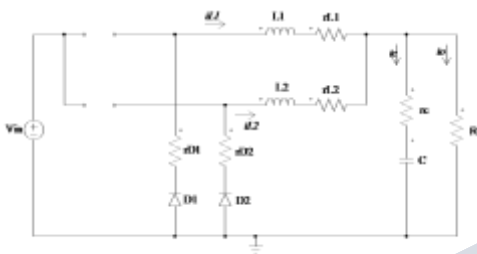


Fig 4 Mode 2 Operation

Mode – 2 Switch S_1 and S_2 is open, in this period both the inductor current i_{L1} and i_{L2} is falling as shown in “Fig.2,”

$$A2 = \begin{bmatrix} \frac{1}{L_1} \left[(-r_{L1} - r_{D1}) + \frac{R_{Load} r_C}{(R_{Load} + r_C)} \right] & 0 & \frac{1}{L_1} \left(\frac{R_{Load}}{(R_{Load} + r_C)} \right) \\ 0 & \frac{1}{L_2} \left[(-r_{L2} - r_{D2}) + \frac{R_{Load} r_C}{(R_{Load} + r_C)} \right] & \frac{1}{L_2} \left(\frac{R_{Load}}{(R_{Load} + r_C)} \right) \\ \frac{R_{Load}}{2C(R_{Load} + r_C)} & \frac{R_{Load}}{2C(R_{Load} + r_C)} & \frac{-1}{C(R_{Load} + r_C)} \end{bmatrix}$$

$$B2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C2 = \begin{bmatrix} \frac{R_{Load} r_C}{(R_{Load} + r_C)} & \frac{R_{Load} r_C}{(R_{Load} + r_C)} & \frac{R_{Load}}{(R_{Load} + r_C)} \end{bmatrix}$$

Mode – 3 Switch S_1 is open and S_2 is closed, in this period inductor current i_{L1} is falling and i_{L2} is rising as shown in “Fig.2,”

$$A3 = \begin{bmatrix} \frac{1}{L_1} \left[(-r_{L1} - r_{D1}) + \frac{R_{Load} r_C}{(R_{Load} + r_C)} \right] & 0 & \frac{1}{L_1} \left(\frac{R_{Load}}{(R_{Load} + r_C)} \right) \\ 0 & \frac{1}{L_2} \left[(-r_{L2} - r_{D2}) - \frac{R_{Load} r_C}{(R_{Load} + r_C)} \right] & \frac{-1}{L_2} \left(\frac{R_{Load}}{(R_{Load} + r_C)} \right) \\ \frac{R_{Load}}{2C(R_{Load} + r_C)} & \frac{R_{Load}}{2C(R_{Load} + r_C)} & \frac{-1}{C(R_{Load} + r_C)} \end{bmatrix}$$

$$B3 = \begin{bmatrix} 0 & -1 & 0 \\ \frac{1}{L_2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C3 = \begin{bmatrix} \frac{R_{Load} r_C}{(R_{Load} + r_C)} & \frac{R_{Load} r_C}{(R_{Load} + r_C)} & \frac{R_{Load}}{(R_{Load} + r_C)} \end{bmatrix}$$

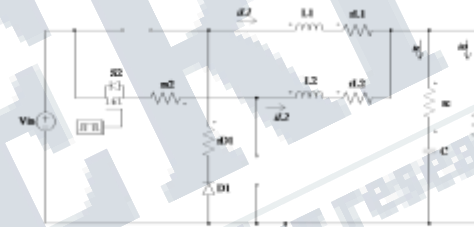


Fig 5 Mode 3 Operation

Mode – 4 Switch S_1 and S_2 is open, in this period both the inductor current i_{L1} and i_{L2} is falling as shown in “Fig.2,”

$$A4 = \begin{bmatrix} \frac{1}{L_1} \left[(-r_{L1} - r_{D1}) + \frac{R_{Load} r_C}{(R_{Load} + r_C)} \right] & 0 & \frac{1}{L_1} \left(\frac{R_{Load}}{(R_{Load} + r_C)} \right) \\ 0 & \frac{1}{L_2} \left[(-r_{L2} - r_{D2}) + \frac{R_{Load} r_C}{(R_{Load} + r_C)} \right] & \frac{1}{L_2} \left(\frac{R_{Load}}{(R_{Load} + r_C)} \right) \\ \frac{R_{Load}}{2C(R_{Load} + r_C)} & \frac{R_{Load}}{2C(R_{Load} + r_C)} & \frac{-1}{C(R_{Load} + r_C)} \end{bmatrix}$$

$$B4 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C4 = \begin{bmatrix} \frac{R_{Load} r_C}{(R_{Load} + r_C)} & \frac{R_{Load} r_C}{(R_{Load} + r_C)} & \frac{R_{Load}}{(R_{Load} + r_C)} \end{bmatrix}$$

Analysis

Based on the assumption that all switching cells carry equal average current and are operating at the same duty ratio in a switching cycle of interest,

We obtain

$$d_{2i-1} = d \dots \dots \dots (1)$$

$$d_{2i} = \frac{1}{N} - d \dots \dots \dots (2)$$

$$i = 1, 2, \dots, N$$

d - Duty ratio

If d is constant from cycle to cycle, it is defined as the steady-state duty ratio, D .

Average state space model over one particular cycle can be written as

$$\dot{x} = Ax + BV_s \dots \dots \dots (3)$$

By applying (2) to the waveforms in figure , we obtain

$$A = \sum_{j=1}^{2N} d_j A_j \dots \dots \dots (4)$$

$$B = \sum_{j=1}^{2N} d_j B_j \dots \dots \dots (5)$$

$$C = \sum_{j=1}^{2N} d_j C_j \dots \dots \dots (6)$$

Where A_j and B_j are the state matrix and the control vector of the interval $d_j T_s$ respectively, and $j=1, 2, \dots, 2N$.

We now assume that $d = D$. Using equations (1) and (2) in equation (4), we can write

$$A = D \sum_{i=1}^N A_{2i-1} + \left(\frac{1}{N} - D \right) \sum_{i=1}^N A_{2i} \dots \dots \dots (7)$$

similarly we can write (5)

$$B = D \sum_{i=1}^N B_{2i-1} + \left(\frac{1}{N} - D \right) \sum_{i=1}^N B_{2i} \dots \dots \dots (8)$$

similarly we can write (6)

$$C = D \sum_{i=1}^N C_{2i-1} + \left(\frac{1}{N} - D \right) \sum_{i=1}^N C_{2i} \dots \dots \dots (9)$$

Substituting $N=2$ in the above equations we get

$$\begin{aligned} A &= (A_1 + A_3)(1-D) + (2D-1)A_2 \\ B &= (B_1 + B_3)(1-D) + (2D-1)B_2 \dots \dots \dots (10) \\ C &= (C_1 + C_3)(1-D) + (2D-1)C_2 \end{aligned}$$

To investigate the small-signal behavior, we now assume that d varies from cycle to cycle. Equation (7) (8) and

(9) are the perturbations in the input voltage, in the duty ratio and in the states are introduced to (3).

By neglecting the non-linear second-order term, the perturbed state-space equation for an N-phase interleaved converter is obtained as

$$\dot{x} = AX + BV + \overset{\square}{A}x + \overset{\square}{B}V_s + \left[\sum_{i=1}^N (A_{2i-1} - A_{2i})X + \sum_{i=1}^N (B_{2i-1} - B_{2i})V_s \right] \overset{\square}{d}$$

When all perturbations are set to zero, the steady-state model is obtained as

$$X = -A^{-1}BV_s$$

Consequently, the small signal model is found to be as

$$\dot{x} = \overset{\square}{A}x + \overset{\square}{B}V_s + \left[\sum_{i=1}^N (A_{2i-1} - A_{2i})X + \sum_{i=1}^N (B_{2i-1} - B_{2i})V_s \right] \overset{\square}{d}$$

$$\dot{x} = \overset{\square}{A}x + \overset{\square}{B}V_s + [(A_1 + A_3 - 2A_2)X + (B_1 + B_3 - 2B_2)V_s] \overset{\square}{d}$$

Applying laplace transform

$$s \overset{\square}{x}(s) = \overset{\square}{A} \overset{\square}{x}(s) + \overset{\square}{B} \overset{\square}{V}_s(s) +$$

$$[(A_1 + A_3 - 2A_2)X + (B_1 + B_3 - 2B_2)V_s] \overset{\square}{d}(s)$$

$$\overset{\square}{x}(s) = [sI - \overset{\square}{A}]^{-1}$$

$$\left[\overset{\square}{B} \overset{\square}{V}_s(s) + [(A_1 + A_3 - 2A_2)X + (B_1 + B_3 - 2B_2)V_s] \overset{\square}{d}(s) \right]$$

let us now assume

$$K = [(A_1 + A_3 - 2A_2)X]$$

$$T = [(B_1 + B_3 - 2B_2)V_s]$$

$$\overset{\square}{x}(s) = [sI - \overset{\square}{A}]^{-1} \left[\overset{\square}{B} \overset{\square}{V}_s(s) + [(K)X + (T)V_s] \overset{\square}{d}(s) \right]$$

$$\overset{\square}{v}_o(s) = C^T \overset{\square}{x}(s)$$

$$\overset{\square}{v}_o(s) = C^T [sI - \overset{\square}{A}]^{-1} \left[\overset{\square}{B} \overset{\square}{V}_s(s) + [(K)X + (T)V_s] \overset{\square}{d}(s) \right]$$

$$\overset{\square}{v}_o(s) = C^T \overset{\square}{x}(s)$$

$$V_o = [(C_1 + C_3)(1-d) + (2d-1)C_2] X$$

$$v_o = V_o + \overset{\square}{v}_o ; x = X + \overset{\square}{x} ; d = D + \overset{\square}{d}$$

$$v_o = \left(V_o + \overset{\square}{v}_o \right) = \left[(C_1 + C_3) \left(1 - \left(D + \overset{\square}{d} \right) \right) + \left(2 \left(D + \overset{\square}{d} \right) - 1 \right) C_2 \right] \left(X + \overset{\square}{x} \right)$$

$$v_o = \left(V_o + \overset{\square}{v}_o \right) = \left[(C_1 + C_3) \left(1 - D - \overset{\square}{d} \right) + \left(2D + 2\overset{\square}{d} - 1 \right) C_2 \right] \left(X + \overset{\square}{x} \right)$$

The product of small signals is zero, after simplifying the above equation. We get

$$v_o = [(C_1 + C_3)(1 - D) + (2D - 1)C_2]x + [(2C_2 - C_1 - C_3)X]d$$

let us now assume

$$P = [(C_1 + C_3)(1 - D) + (2D - 1)C_2]$$

$$Q = (2C_2 - C_1 - C_3)$$

$$v_o = [P]x(s) + [QX]d(s)$$

We know that

$$x(s) = [sI - A]^{-1} [BV_s + [(K)X + (T)V_s]]d$$

substituting $x(s)$ in the voltage equation

$$v_o(s) = P[sI - A]^{-1} [BV_s(s) + [(K)X + (T)V_s]]d(s) + [QX]d(s)$$

substituting $BV_s(s) = 0$

$$v_o(s) = P[sI - A]^{-1} [(K)X + (T)V_s]d(s) + [QX]d(s)$$

Finally, the transfer function of output to variation in the duty ratio is expressed as

$$\frac{v_o(s)}{d(s)} = P[sI - A]^{-1} [(K)X + (T)V_s] + [QX]$$

Finally by substituting the above equations with their respective values, we obtain the transfer function as follows,

$$G(s) = \frac{s^2(177163.99) + s(1.887954 \times 10^{10}) + (7.8026 \times 10^{13})}{s^3 + s^2(23606.07) + s(1181050336) + (6.5807 \times 10^2)}$$

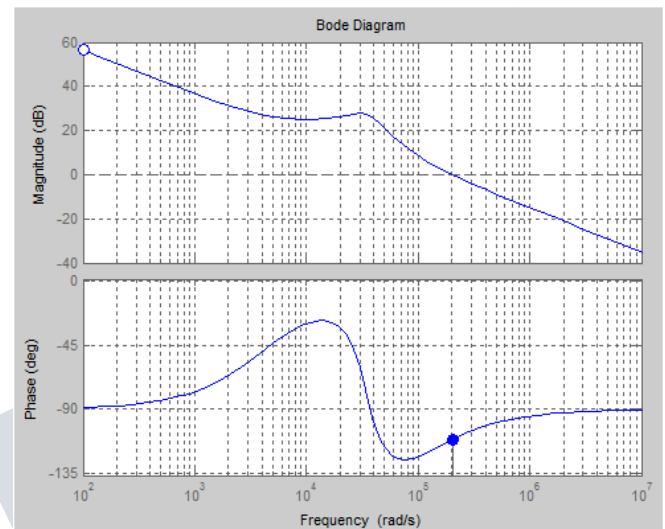


Fig 6 System Bode Plot

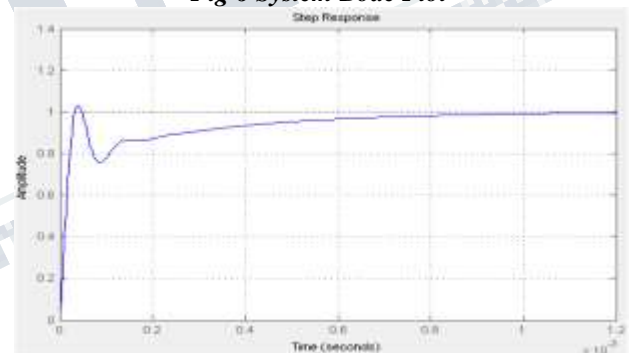


Fig 7 Closed Loop Step Response

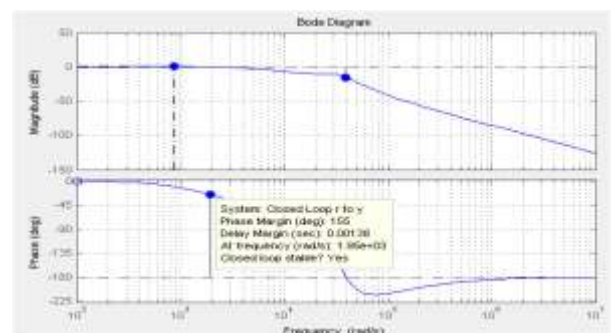


Fig 8 Closed Loop Bode

After obtaining the transfer function of the converter, the same is fed into the SISO design tool command in the MATLAB to generate bode of the system “Fig.6,” closed loop response of compensated system “Fig.7,” closed loop bode “Fig.8,” gives the analysis plot. And also using automated PID tuning we can find the K_p , K_i , and K_d values [$K_p= 0.06256$, $K_i=641.125$ and $K_d=0$]. The same values are fed in to the PID controller these values are set into the PID controller.

IV. SIMULATION RESULTS

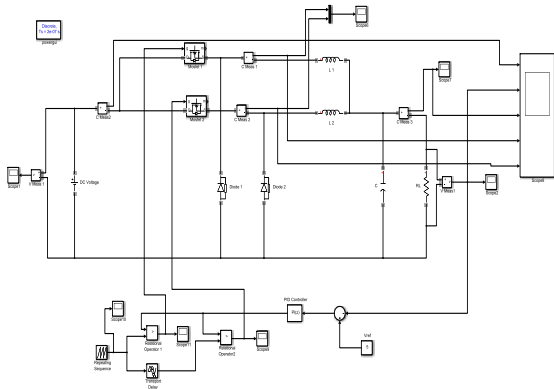


Fig 9 Two Phase IBC in Closed Loop

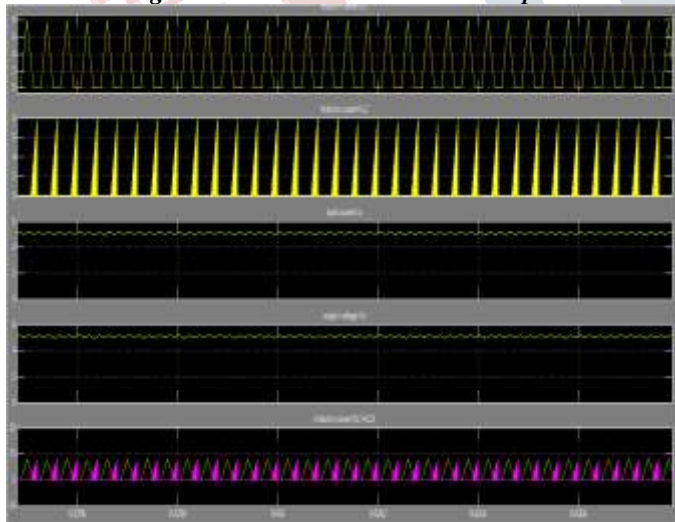


Fig 10 Voltage and Current Waveforms in Simple PID Controller

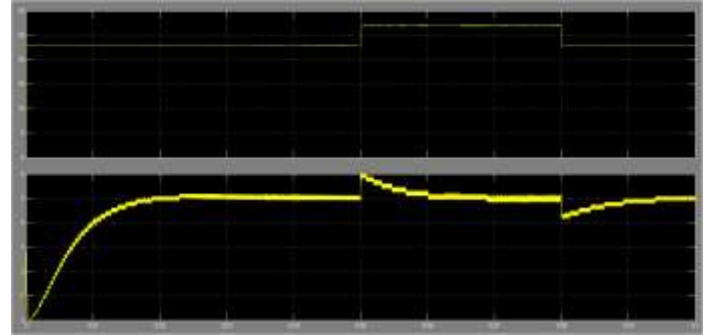


Fig 11 Output Voltage waveform in Simple PID Controller (+10% Variation in Input Voltage)

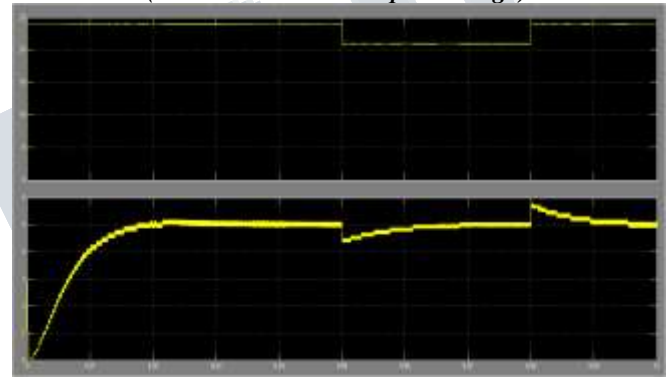


Fig 12 Output Voltage waveform in Simple PID Controller (-10% Variation in Input Voltage)

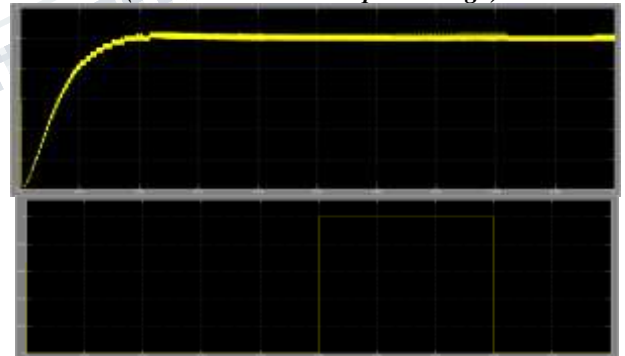


Fig 13 Output Voltage waveform in Simple PID Controller (+10% Variation in Load)

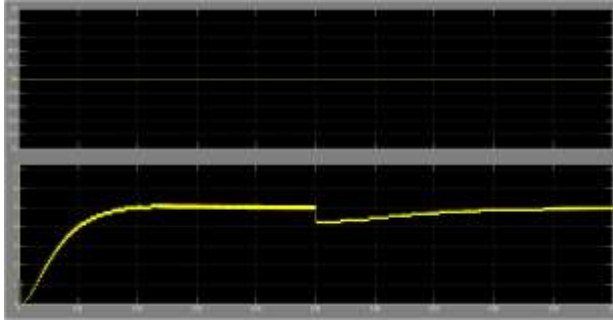


Fig 14 Output Voltage waveform (Variation in Load keeping the Input Voltage Constant)

“Fig.9,” shows the two phase IBC in closed loop using analog PID controller. “Fig.10,” shows results of analog PID controller output voltage and inductor current waveforms. The source current I_{in} is the sum of I_{L1} and I_{L2} as they are in phase opposition, therefore the ripple cancellations in the source current ripple gets reduced. An observed in the current waveforms I_{in} the ripple current ΔI_L is 7.904A. whereas the ripple current in individual inductor is 2.5A. “Fig.11,” and “Fig.12,” shows the simulation results of output voltage waveforms with $\pm 10\%$ variations in input voltage in analog PID controller. The transients at the output voltage waveforms are observed in the rise time and fall time. “Fig.13,” and “Fig.14,” shows the simulation results of output voltage waveforms with $\pm 10\%$ load variations and keeping the input voltage constant and varying the load in analog PID controller.

V. CONCLUSION

The two phase interleaved buck converter is modeled using state space averaging technique considering all parasitic elements of the converter which gives the accurate modeling of the converter. The modeling is analyzed based on small signal analysis to find the transfer function output voltage to variations in duty ratio. This results the perfect modeling of the converter and gives the accurate results. The two phase interleaved buck converter has advantages based on comparison with a single buck converter. The input current and output voltage ripple will be reduced and also the size of filtering component at the output. By increasing the number of levels of the converter, will result in the reduction of the size of the filter and source current ripple and increase the power density and efficiency of the converter.

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