

# PSO Based Fuzzy Stochastic Long-term Model for Deployment of Distributed Energy Resources in Distribution Systems with Several Objectives

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**Abstract:** —This paper presents a particle swarm optimization (PSO) based fuzzy stochastic long term approach for determining optimum location and size of distributed energy resources (DERs). The Monte Carlo simulation method is used to model the uncertainties associated with long-term load forecasting. A proper combination of several objectives is considered in the objective function. Reduction of loss and power purchased from the electricity market, loss reduction in peak load level, and reduction in voltage deviation are simultaneously considered as the objective functions. At first these objectives are fuzzified and designed to be comparable with each other, then they are introduced to a PSO algorithm in order to obtain the solution which maximizes the value of integrated objective function. The output power of DERs is scheduled for each load level. An enhanced economic model is also proposed to justify investment on DER. IEEE 30-bus radial distribution test system is used as an illustrative example to show the effectiveness of the proposed method.

**Index Terms**— Distributed energy resources (DERs), fuzzy optimization, loss reduction, particle swarm optimization (PSO), stochastic programming, voltage deviation reduction.

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## I. INTRODUCTION

The Uncertainties associated with load forecasting and equipments' unavailability affect the system operation and planning decisions. Applying a proper method for modeling these uncertainties in planning phase, one can reduce the risk of the decisions as well as the stochastic cost of operation. Ignoring the uncertainties in planning process leads to a high risk, and renders the stochastic saving gained by applying the decisions no optimal.

In this paper a new methodology to solve the complicated problem of finding optimal location and size of distributed energy resources (DERs) is presented that considers the un-certainties associated with load forecasting. In the proposed stochastic planning scheme, the stochastic characteristics of load growth are simulated using the Monte Carlo simulation of method. Each possible system state is represented by a scenario, and a scenario reduction technique is employed to decrease the number of created scenarios.

Particle swarm optimization (PSO) is a heuristic global optimization approach whose main strength is its simplicity and fast convergence [1]–[3]. This optimization method has been widely applied to solve different problems, especially locating the problems of power system.

Restructuring of power systems has caused an increasing interest in DERs. Successful application, and worldwide ten-dency to DERs have led to the emergence of new technologies in this area. Moreover, the increasing awareness on environ-mental issues have motivated the application of DERs even more [4].

Many benefits are gained by placement of DERs, yet they may cause some troubles in operation of distribution systems if they are installed without thorough consideration. Therefore, special care should be taken in locating and sizing of DERs. A wide range of benefits, from loss reduction to voltage profile improvement, can be gained by placement of DERs in distributed systems. Therefore the realm of study of distribution systems is replete with the works on solving the problem of DER placement with different objective functions. In [5] the most important benefits

**International Journal of Engineering Research in Electrical and Electronic  
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Vol 3, Issue 3, March 2017**

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of DER are modeled in economic terms. A set of indices are proposed in [6] for modeling and quantifying of the technical benefits of DERs.

The rest of this section introduces some of the previous works on DER placement and also presents the contributions of the present work that cover the weaknesses of the former studies.

#### **A. Literature Review**

A distributed generation (DG) capacity investment planning algorithm was proposed in [7] using a new heuristic approach from the perspective of a distribution company. Optimal solution for DG capacity placement was obtained through a cost-benefit analysis approach based on this new optimization model. The model aimed to minimize the disco's operating and investment costs as well as cost of power loss, but the other benefits gained by DG placement were not considered. The analytical optimization process for determining the optimal location and size of DER was aimed at minimizing power loss of distribution systems in [8]. Both radial and meshed distribution systems were considered.

Cost-benefit analysis is one of the other approaches used in the literature. For example, a new heuristic approach for DG capacity investment planning from the perspective of a distribution company was proposed in [9]. In order to solve the placement and sizing problem a multi-period AC optimal power flow (OPF) was proposed in [10]. Minimizing power loss was again the aim of the optimization algorithm. Loss reduction and reliability improvement were also handled in [9] as a cost/worth analysis.

Ahmadigorji et al. [11] proposed two multiobjective formulations based on the genetic algorithm (GA) and an  $\epsilon$ -constrained method as optimization techniques for the placement and sizing of DER in distribution networks. The optimization process was a compromise between reduction in power losses, reliability improvement, and reduction in power to be purchased from the power market and minimization of the cost of network upgrading.

Wang et al. [12] studied facility-location problems while taking into account a hybrid uncertain environment involving both randomness and fuzziness. Since the fuzzy parameters of the locating problem are represented in the form of continuous fuzzy variables, the determination of value-at-risk is inherently an infinite-dimensional optimization problem that is not possible to be solved analytically. Therefore a two-stage fuzzy facility location problem with value-at-risk,

was proposed in [13], which results in a two-stage fuzzy zero-one integer programming problem. Both the costs and demands are skillfully assumed to be fuzzy random variables in [14], a value-at-risk based fuzzy random facility location model is built and a hybrid modified PSO approach is proposed to solve such complicated problem.

A fuzzified multiobjective GA based algorithm was proposed in [15] for capacitor placement. Though the objective was finding the best location of capacitors in distribution networks, the model of objective function and the methodology can be used in DG placement problem. Das [16] proposed a method for reliability improvement and loss reduction by installing fixed capacitor in a distribution system. Though the problem was the capacitor placement, the reduction of power loss at peak load was skillfully modeled as one of the objective functions.

Two types of load uncertainties for planning studies can be identified in power systems' planning, uncertainties associated with load forecasting and short-term uncertainties related to time/weather factors. Both of them were considered in [17] and a simple GA-based optimization algorithm was used to extract the best location and size of DERs in a distribution system. A Monte Carlo simulation was used to model the stochastic nature of the system loads. The stochastic nature of system components and load growth forecasting was simulated and each possible system state was represented by a scenario in [18]. A scenario reduction technique was used to decrease the computational burden of large number of scenarios.

#### **B. Motivations and Contributions**

1) Developing a Long Term Stochastic Load Model for DER Placement Considering Stochastic Load Growth: A long term stochastic model for system uncertainties is presented in this paper that is suited for application along with PSO algorithm. The results of the case studies show the necessity of stochastic modeling of the problem. Some other studies in the literature have considered the stochastic nature of the load and system components, but uncertainties are modeled in just one hour or just one year. In planning problems, it is necessary to model the uncertainties in the entire planning horizon.

2) Considering System Operation in Planning Phase for Different System States: In this paper, the output power is scheduled for each load level to avoid the inconvenient rejection of more optimal solutions. In

**International Journal of Engineering Research in Electrical and Electronic  
Engineering (IJEREEE)  
Vol 3, Issue 3, March 2017**

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contrast, the previous works considered the output power of DERs to be fixed at the maximum rated value, while the load varies at each bus. This may render some optimal solutions infeasible due to violation of some constraints, such as voltage magnitude limits in some load levels, while in most of the other load levels there is no violation.

3) Application of Fuzzy Optimization Approach to Satisfy Different Objectives Simultaneously in DER Placement: So many studies have been conducted to reduce the cost of loss in distribution systems. Reduction of voltage deviation in order to reach a more flat voltage profile has also been the subject of many studies in distribution systems. In this paper, the reduction of loss and power purchased from the electricity market, loss reduction in peak load level, and reduction in voltage deviation are considered simultaneously as the objective functions. These objectives are first fuzzified and then integrated and introduced to a PSO Algorithm in order to obtain the solution which minimizes the value of integrated objective function.

4) Developing Adoptive Membership Functions: Fuzzy approach has been applied in previous works, such as [16] (for capacitor placement), but membership functions were pre-defined. This paper presents a method to find the appropriate membership functions in fuzzification process of objective functions. A method is also presented in order to make these objectives comparable with each other.

5) Improved Economic Modeling: Profit maximization is considered one of the objective functions, while in order to justify the investment on DER installation compared to the other investment opportunities, the benefit to cost ratio (BCR) is considered as a constraint whose value should be greater than a predefined value. This predefined value should be calculated based on the other investment opportunities.

The proposed method is tested on IEEE-30 bus radial distribution test system. The simulation results show the effectiveness of the proposed method in DER planning problems and the necessity of stochastic modeling.

The rest of this paper is organized as follows. In Section II, an overview of the PSO algorithm is

presented. The long term scenario generation and reduction procedures are described in Section III. The proposed method is presented in Section IV. The simulation results are presented and discussed in Section IV. The conclusions are drawn in Section VI.

## II. PARTICLE SWARM OPTIMIZATION

Considering the fast growth in problem dimensions and great appeal to fast optimization algorithms in recent years, heuristic algorithms based on random search are widely used instead of the overall search in problem space [19]. Heuristic methods may be used to solve some combinatorial multi-objective optimization problems. These methods are called intelligent, because the move from one solution to another is done using rules based upon human reasoning. Heuristic algorithms may search for a solution only inside a subspace of the total search region. Although, they are able to give a good solution for certain type of problems in a reasonable computational time, they do not completely assure to reach the global optimum. The most important advantage of heuristic methods lies in the fact that they are not limited by restrictive assumptions about the search space like continuity, existence of derivative of the objective function, etc. Several heuristic methods can be addressed, such as tabu search (TS), simulated annealing (SA), genetic algorithms (GAs), and particle swarm optimization (PSO) [23]–[25]. Each one has its own pros and cons which make them possible to apply to the appropriate problems; in this paper PSO method is selected as an intelligent optimization method. Kennedy and Eberhart [26], [27] first introduced the PSO method, which is also an evolutionary computation technique. Similar to GA, PSO is a population-based optimization tool. The system is initialized with a population of random solutions and searches for the optimal solution by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation.

In PSO, the potential solutions, called particles, are flown through the problem space by following the current optimum particles. Compared to GA, the main advantage of PSO is its easy implementation and few numbers of parameters to be adjusted. It can be said that PSO has been successfully applied in many areas. Each individual in PSO flies in the search space with a velocity which is dynamically adjusted according to its own flying experience and its

companions' flying experience. Each individual keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) it has achieved so far. This value is called p-best, while another best value that is tracked by the global version of the particle swarm optimizer is the overall best value and its location, the so-called g-best, obtained so far by any particle in the population. At each time step, the PSO consists of velocity changes of each particle toward its p-best and g-best.

Acceleration is weighted by a random term, along with separate random numbers being generated for acceleration toward p-best and g-best. This new technique for nonlin-ear optimization involves simulating social behavior among individuals (particles) flying through a multidimensional search space, where each particle represents a single intersection of all search dimensions. The particles evaluate their positions relative to a goal (fitness) for any iteration, and particles in a local neighborhood share memories.

ith particle  $X_i$  is defined as a potential solution in D-dimensional space, where  $X_i = x_{i1}, x_{i2}, \dots, x_{iD}$ . Each particle also maintains a memory of its previous best position and a velocity along each dimension represented as  $P_i = v_{i1}, v_{i2}, \dots, v_{iD}$ . At each iteration, the  $P = [P_1, P_2, \dots, P_i, \dots, P_n]$  vector of the particle will be adjusted with the best fitness in the local neighborhood. This adjustment will be done using a factor g-best and with the best fitness of the population by a factor p-best. Velocity adjustment along each dimension, can be defined by (1), where it is used to compute a new position for the particle [23]–[24]

$$v_i = w \cdot v_{i-1} + c_1 \times \text{rand}(0, 1) \times (x_{igbest} - x_i) + c_2 \times \text{rand}(0, 1) \times (x_{ipbest} - x_i) \quad (1)$$

$$x_{i+1} = x_i + v_i \quad (2)$$

where

w: inertia weight factor, often decrease linearly from about 0.9 to 0.4 during a run [24].

$c_1, c_2$ : acceleration constants.

$\text{rand}(0,1)$ , random number between 0 and 1.

$x_{igbest}$ : The best particle among all individuals in the population.

$x_{ipbest}$ : The best history of position of particle  $x_i$ .

The constants  $c_1$  and  $c_2$  represent the weighting of the stochastic acceleration terms that pull each particle  $x_i$  toward

$x_{igbest}$  and  $x_{ipbest}$  positions. According to the literature  $c_1, c_2$  were often set to be 2.05. A suitable selection of inertia weight  $w$  in

(1) provides a balance between global and local explorations, thus requiring less iteration to find a sufficiently optimal solution. As it is originally developed,  $w$  often decreases linearly from about 0.9 to 0.4 during optimization process. The inertia weight  $w$  can be set according to (3) [23]

$$w = w_{max} - \frac{iter - iter_{min}}{iter_{max} - iter_{min}} (w_{max} - w_{min}) \quad (3)$$

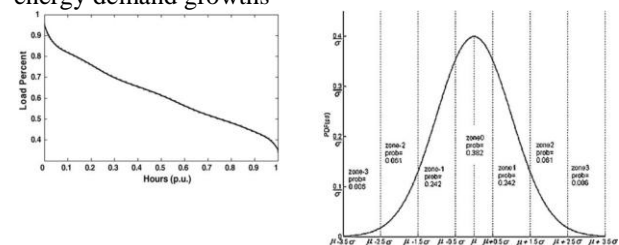
where  $iter_{max}$  is the maximum number of iterations (generations), while  $iter$  is the current number of iterations. Like GA, PSO is initialized by a population of random solutions with some advantages. It has memory to support the knowledge of good solutions by all particles. PSO has constructive cooperation between particles in order to share their information.

### III. STOCHASTIC LONG-TERM MODEL

In the proposed stochastic planning model, each possible system state is called scenario. These scenarios are created by the Monte Carlo simulation method to model long-term stochastic characteristics of the system components and bus loadings. Scenario reduction technique is used to decrease the number of created scenarios.

#### A. Monte Carlo Simulation Method

Fig. 1 depicts the annual load duration curve (LDC) that is modeled as multiple load blocks. This model is used as an infrastructure in consideration of forced outages of lines and load forecasting inaccuracies. Hours with similar loads are shown in each load block in Fig. 1. Future annual peak load and energy demand growths



**Fig. 1. Load duration curve (LDC) of IEEE Reliability Test System (RTS).**

**International Journal of Engineering Research in Electrical and Electronic  
Engineering (IJEREEE)  
Vol 3, Issue 3, March 2017**

are equal to the base year values times the regarding growth rates [15]. In this paper, the growth rate is expressed as an average growth rate, denoted by AGRP and AGRE, and a random component, RCPt and RCEt for annual peak loads and total energy demands, respectively. Normally distributed random components with certain standard deviations are aggregated with the average growth rates to reflect the uncertainties in economic growth and/or weather changes. Random trajectories in sth scenario and yrth year, represented by  $P_{yr,s}$  for peak load, and represented by  $E_{yr,s}$  for total energy demand, are expressed in the Monte Carlo simulation based on [18] as follows:

$$P_{yr,s} = P_{(yr-1),s} \times (1 + AGRP + RCP_{yr,s}) \quad (4)$$

$$E_{yr,s} = E_{(yr-1),s} \times (1 + AGRE + RCE_{yr,s}) \quad (5)$$

$$b_{yr,s} = a_s \times b_{l0} + b_s \quad (6)$$

where

$$a_s = \frac{E_{yr,s} - 8760 \times P_{yr,s}}{E_0 - 8760 \times P_0} \quad (7)$$

$$b_s = \frac{P_{yr,s} \times 0 - 0 \times P_{yr,s}}{E_0 - 8760 \times P_0} \quad (8)$$

The bus load of bus  $z$  at each load block is calculated by multiplying load distribution factor of that bus and load at each block

$$PD_{z,b,yr,s} = D_{z,b,yr} \times b_{l, yr,s} \quad (9)$$

Transmission line availability of line  $k$  at load block  $b$  in year  $yr$  denoted by  $UY_{k,b,yr}$  is used in the Monte Carlo simulation in which  $UY_{k,b,yr} = 1$  indicates that the transmission line  $k$  is available at load block  $b$  in year  $yr$  while  $UY_{k,b,yr} = 0$  indicates otherwise. Consequently, a scenario consists of  $RCP_{yr,s}$ ,  $RCE_{yr,s}$  and  $UY_{k,b,yr,s}$ .

Modeling the load uncertainties using a normal distribution causes some errors and difficulties. In practical power systems, historical data on bus loads is available, so one can categorize the load historical samples at each bus in several groups based on their differences as a measure. The mean value of each group can be considered as different states. The occurrence probability of each group can be defined as the number of historical samples in this group divided by the total number of samples. However, this method of modeling

cannot be used in our case studies as it lacks in historical data. So, a normal distribution is applied to model the continuous uncertainties.

The bus load of bus  $z$  at each load block is calculated by There are different approaches for modeling the uncer- multiplying load distribution factor of that bus and load at tainties associated with the unavailability of equipments. As- each block suming that each component has two states of failure and success, and components failures are independent of each

other, if the random value is less than the failure rate, the

component will be unavailable. This simple way of modeling considers multiple-outage modes, and is almost sufficient in a nonsequential simulation. When a nonsever failure occurs, some components can still be operated in different derated states. These states are not modeled in this paper indicates otherwise. Consequently, a scenario consists of Probability of each sample can be calculated using (11). Finally, the output of this step is a matrix, each column of which represents a sample of uncertain variable states

A. Objective Fuzzification

R

Each objective in fuzzy domain is associated with a mem-

prob(sm) =

rand(sm, r).

(11)

bership function. The membership function specifies the de-

r=1

gree of satisfaction of the objective. In the crisp domain,

the objective is either satisfied or violated, indicating mem-

C. Scenario Reduction

bership values of unity and zero, respectively. On the con-

The computational requirements for solving scenario-based

trary, fuzzy sets consider

varying

degrees

of membership

function

values from zero

to unity

[16]. The

present work

**International Journal of Engineering Research in Electrical and Electronic  
Engineering (IJEREEE)  
Vol 3, Issue 3, March 2017**

optimization models are directly affected by the number of considers the following objectives for the DER placement scenarios. Therefore an effective scenario reduction technique problem. could be very essential and useful in solving large-scale prob-  
 ✓ Maximization of the saving by minimization of the lems. Roh et al. [18] defines reduction technique as a scenario-based approximation with a smaller number of scenarios, and energy loss, power purchased and loss at the peak load a reasonably good approximation of the original system. The level due to the application of DERs. scenario reduction technique that is applied in this paper,  
 ✓ Minimization of the voltage deviation at network buses. controls the goodness-of-fit of approximation by measuring Before continuing further in this section let us return to the a distance of probability distributions as a probability metric. stochastic long term load model and find out how one can use After performing scenario reduction, a subset of scenarios with it for DER placement. regarding probabilities is selected that models the initial prob- In each scenario we have 12 load levels representing four ability distribution in terms of probability metrics. Efficient different load blocks of the three years of study horizon, algorithms based on backward and fast forward methods are each with a chance of occurrence. Each scenario itself

has derived that determine optimal reduced measures. An overview a probability. Combining the load levels of the scenarios, of the simultaneous backward reduction method, based on the total load probability density function (PDF) is ob- [25], is given in the following. tained. The load PDF is divided into equal sections each Let  $\xi_s (s = 1, \dots, Sc)$  denote  $Sc$  different scenarios, each with an occurrence probability. The centers of these sections with a probability of Probs, and Dists,  $s$  be the distance are introduced to the optimization algorithm as the load of scenario pair  $(s, s)$ . The simultaneous backward and fast levels. forward is given in the following steps: The membership function consists of lower and up- Step 1) Set  $S$  as the initial set of scenarios;  $DS$  is the set per bound values along with a strictly monotonically de- of scenarios to be omitted. The initial  $DS$  is null. creasing and continuous function are described in the Compute the distances of all scenario pairs following. Dists,  $s = Dist(\xi_s, \xi_s)$ ,  $s, s = 1, \dots, Sc$ ;

**B. Membership Function for the Net Saving**

Step 2)

for each scenario k,  $Distk(r) = \min Distk,s$ , while

The net saving at kth load level due to application of DER

$s, k$

$\in$

S and  $s = k, r$  is the index of scenario that

in a distribution system is given as the following (it should

has the minimum distance with scenario k;

be noted that load levels are in fact the stochastic load levels

Step 3)

compute  $DDk(r) = Probk$

$\times$

$Distk(r),$

$k \in S.$

that along with line outages reflect the system states in each

Step 4)

Choose d so that  $DDd = \min (DDk), k \in S;$  scenario)

$S = S - \{d\}, DS = DS + \{d\};$

$Probr = Probr + Probd;$

$Nyr$

$NDER$

Step 5) repeat steps 2-4 until the number to be deleted meets the predefined number of scenarios.

$NS =$

$KpT \text{ Peak } LRyrPeak -$

$KinvDERPiDER,Max$

$+$

$yr$

$i=1$

$KE \rho k LRk + KE \rho k$

$Pi,kDER - KkDERPi,kDER$

$Nk$

$NDER$

**IV. PROPOSED METHOD**

The aim of operation and planning in deregulated power systems is to maximize the social welfare through minimization of costs of the network, while the electric power is delivered to the customers with sufficient quality and reliability. Because of the high investment cost of DERs, there is considerable risk in their application. Therefore the optimal placement and sizing of DERs are the most important steps to be performed considering various aspects of distribution networks. The objectives of this study are loss

minimization, reduction of power which should be purchased from electricity market, loss reduction at the peak load level and improvement of voltage profile of the power system through proper application of DERs.

$k=1 \quad i=1$

(12) Where,  $Kp$  is a factor to convert peak power loss reduction

to dollar (\$/ kw); is the duration of peak load hours);

is the power loss reduction at peak load level at year

yr due to application of DERs (kw);  $KE$  is a factor to

convert energy losses to dollar (\$/ kwh);  $\rho k$  is the

probability of kth load level;  $LRk$  is the reduction in

power loss at kth load level

due to application of DERs (kwh);  $NDER$  is the number of

DERs;  $Pi,kDER$  is the power output of ith DER at kth load level (kwh);  $PkDER$  is the cost of operation and maintenance of DER

at kth load level (\$/ kwh);  $KinvDER$  is the cost of investment of DER (\$/ kw); and  $PiDER,max$  is the maximum capacity of ith

DER (kw).

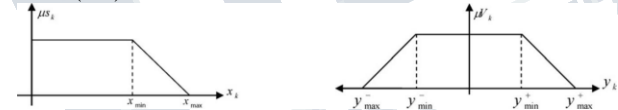


Fig. 3. Membership function of saving. Fig. 4. Membership function of node voltage deviation.

Considering a positive profit for application of DERs, for net saving in (13), we have  $Ns > 0$  that means

$Nk$

$NDER$

$Nyr$

$[KE \rho k LRk + KE \rho k$

$Pi,kDER] +$

$KP T \text{ Peak } LRyrPeak$

$k=1$

$i=1$

$yr$

$Nk$

$NDER$

$- KkDERPi,kDER \geq 0$

$k=1$

$i=1$

(13)

$Nk$

$NDER$

1.

$KDERP \text{ DER}$

$k$

$i,k$

$k=1$

**International Journal of Engineering Research in Electrical and Electronic  
Engineering (IJEREEE)  
Vol 3, Issue 3, March 2017**

$i=1$   
 $KE \rho_k LR_k + KE \rho_k$   
 $P_{i,kDER} +$   
 $K_{pT} Peak LR_{yrPeak}$   
 $N_k$

NDER

N<sub>yr</sub>

≤

$k=1$

$i=1$

$y_r$

(14)

Let us define

$x_k =$

$N_k$

NDER

.

KDERP DER

$k_{i,k}$

$k=1$

$i=1$

$N_k$

NDER

N<sub>yr</sub>

$P_{i,kDER} +$

$[KE \rho_k LR_k + KE \rho_k$

$K_{pT} Peak LR_{yrPeak}$

$k=1$

$i=1$

$y_r$

C. Membership Function for the Node Voltage Deviation

Basic purpose of this membership function is that the deviation of nodes voltage should be minimized. At kth load level of the load duration curve, let us define

$y_k = \max(\text{abs}(V_n - V_{i,k}))$  for  $i = 2, 3, \dots, NB$

(19)

where,  $V_{i,k}$  is the voltage magnitude of node  $i$  at  $k$ th load level in per unit and  $V_n$  is the nominal voltage magnitude that is equal to one in per unit. It should be noted that it is for the case that the substation is located at bus 1. The less the maximum value of nodes voltage deviation, the high the assigned membership value and vice versa. Fig. 4 shows the membership function for maximum node voltage deviation defined in [16]. Based on Fig. 3 and taking into account  $y_{max-} \leq y_k \leq y_{max+}$  we have

$(y_{max+}$

$- y_k)$

$y_k$

≥

0

$(y_{max+}$

$y_{min+})$

$(y_{max-}$

$y_k)$

–

$y_k$

0

(20)

$\mu_{V_k} =$

–

≤

$(y_{max-}$

$- y_{min-})$

$k$

$\min \leq$

$k \leq$

+

$\min$

$\mu_{V_k}$

1

for  $y_{-}$

$y$

$y$

(21)

=

$\mu_{V_k}$

$k$

= 0

for  $y$

$k$

≥

$y_{+}$

or  $y$

$k$

≤

$y_{-}$ .

(22)

max

max

$\mu_{sk} = (x_{max} - x_k)$  for  $x_{min} \leq x_k \leq x_{max}$

$x_{max} - x_{min}$

$\mu_{sk} = 1$  for  $x_k \leq x_{min}$

$\mu_{sk} = 0$  for  $x_k \geq x_{max}$ .

(15) In this paper,  $y_{min+}$  and  $y_{max+}$  are considered to be 0.05 and 0.10, respectively.  $y_{min-}$  and  $y_{max-}$  are assumed  $-0.05$  and  $-0.10$ , respectively. Considering  $V_n = 1$ ,  $y_{min+} = 0.05$  means the minimum system voltage will be 0.95 p.u. and it means that



if the minimum system voltage is greater than or equal to 0.95 p.u., the membership value is one. Similarly,  $\mu_{\max} = 0.10$

(16) means the minimum system voltage allowed will be 0.90 p.u. and if the minimum system voltage is less than or equal to 0.90 p.u., the assigned membership value will be zero. For more

(17) information about practical issues about how the membership functions and weighting factors should be chosen please refer to [12]–[14].

(18)

In this paper,  $x_{\max}$  is assumed to be 1.0; in order to achieve  $x_{\min}$  the proposed method is once performed without consideration of voltage improvement as one of the objectives. The value of  $x_{\min}$  is determined based on the maximum profit to cost ratio. It means if the maximum profit to cost ratio achieved is 0.6,  $x_{\min}$  will be 0.375. The  $x_{\min}$  of 0.375 means unity membership value is assigned if the savings is 37.5% or more, and the  $x_{\max}$  of 1.0 means zero membership value is assigned if the profit is zero percent of the cost or has a negative value.

#### D. Fuzzy Formulation for Several Objectives

The two fuzzified objectives described in the previous section are dealt with by integrating them into a fuzzy satisfaction objective function  $F$ , through appropriate weighting factors ( $K$ ) as (23).

$$\text{Max } F = K_1 \times \mu_{sk} + K_2 \times \mu_{V_k} \quad (23)$$

$K_1$  and  $K_2$  are weighting factor considered in this paper for investigation of the impact of each one of objectives in

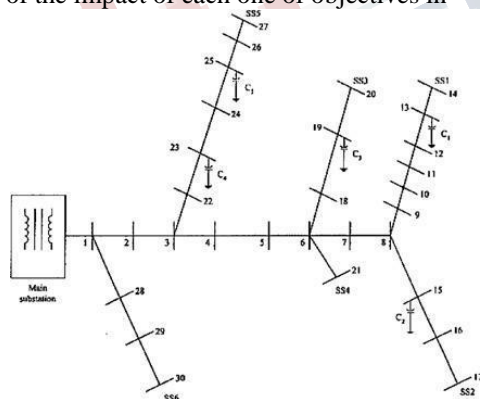


Fig. 5. IEEE 30-bus distribution system.

planning of the DERs.  $K_1 = 0$  and  $K_2 = 1$  means that only improving voltage profile is considered as the

objective of optimization and vice versa; while  $K_1 = 0.5$  and  $K_2 = 0.5$  means that these two objectives are assumed to be equally important. The weighting factors can be determined according to the preferences of the operators [16].

#### V. Simulation Results

In order to test the effectiveness of the proposed method, IEEE 30-bus distribution system including 22 load points, six auxiliary substations and a main feeder [26] is chosen. This system is shown in Fig. 5.

The load pattern in the peak load level of the present year is shown in Table I. Both real (kW) and reactive (kVAR) loads are specified. The base value of voltage and power are 23 (kV) and 100 (MVA), respectively. Table I also shows the resistance and reactance of the lines. After scenario aggregation ten load levels are considered, each with an occurrence probability. These load levels are presented in Table II as the percentage of the peak load level of the present year. One can consider more load levels for the sake of accuracy. The PSO parameters are presented in Table III.

It is assumed that the size of DERs varies in 100 (kW) steps. The investment and operation costs of DERs are borrowed from [27].

#### A. Fuzzy Optimization Problem Considering Several Objectives, Deterministic Case

Before testing the proposed stochastic method, a deterministic version of the proposed method is tested on IEEE 30-bus distribution system in this subsection. The solution of this deterministic problem can be compared with the solution of stochastic problem to show the necessity of the stochastic modeling of the problem. The values of the load and energy growth rates are considered to be 0.08. The load model with nine levels is used for the three-year time horizon. Table IV shows these load levels and the regarding time durations. The proposed stochastic approach can be simply modified for

**TABLE I**  
**IEEE 30-Bus Distribution System Data**

**International Journal of Engineering Research in Electrical and Electronic  
Engineering (IJEREEE)  
Vol 3, Issue 3, March 2017**

| From Bus i  | To Bus j | Active load at j (MW) | Reactive load at j (MVAR) | $r_{ij}$ (pu) | $x_{ij}$ (pu) |
|-------------|----------|-----------------------|---------------------------|---------------|---------------|
| Main Feeder | 1        | 0                     | 0                         | 0.0963        | 0.3219        |
| 1           | 2        | 0.5220                | 0.1740                    | 0.0414        | 0.0022        |
| 2           | 3        | 0                     | 0                         | 0.0659        | 0.0651        |
| 3           | 4        | 0.9360                | 0.3120                    | 0.2221        | 0.1931        |
| 4           | 5        | 0                     | 0                         | 0.1045        | 0.0909        |
| 5           | 6        | 0                     | 0                         | 0.3143        | 0.1770        |
| 6           | 7        | 0                     | 0                         | 0.2553        | 0.1438        |
| 7           | 8        | 0                     | 0                         | 0.2553        | 0.1438        |
| 8           | 9        | 0.1890                | 0.0630                    | 0.2506        | 0.1412        |
| 9           | 10       | 0                     | 0                         | 0.2506        | 0.1412        |
| 10          | 11       | 0.3360                | 0.1120                    | 0.7506        | 0.4229        |
| 11          | 12       | 0.6570                | 0.2190                    | 0.3506        | 0.1975        |
| 12          | 13       | 0.7830                | 0.2610                    | 0.1429        | 0.0805        |
| 13          | 14       | 0.7290                | 0.2430                    | 0.2909        | 0.1639        |
| 8           | 15       | 0.4770                | 0.1590                    | 0.0898        | 0.0781        |
| 15          | 16       | 0.5490                | 0.1830                    | 0.1377        | 0.0775        |
| 16          | 17       | 0.4770                | 0.1590                    | 0.2467        | 0.1390        |
| 6           | 18       | 0.4320                | 0.1440                    | 0.0915        | 0.0795        |
| 18          | 19       | 0.6720                | 0.2240                    | 0.3005        | 0.2612        |
| 19          | 20       | 0.4950                | 0.1650                    | 0.2909        | 0.1639        |
| 6           | 21       | 0.2070                | 0.0690                    | 0.1143        | 0.0994        |
| 3           | 22       | 0.5220                | 0.1740                    | 0.1066        | 0.1054        |
| 22          | 23       | 1.9170                | 0.0630                    | 0.0649        | 0.0641        |
| 23          | 24       | 0                     | 0                         | 0.1083        | 0.0941        |
| 24          | 25       | 1.1160                | 0.3720                    | 0.2760        | 0.2399        |
| 25          | 26       | 0.5490                | 0.1830                    | 0.2009        | 0.1746        |
| 26          | 27       | 0.7920                | 0.2640                    | 0.2857        | 0.1609        |
| 1           | 28       | 0.8820                | 0.2940                    | 0.0881        | 0.0047        |
| 28          | 29       | 0.8820                | 0.2940                    | 0.3091        | 0.1741        |
| 29          | 30       | 0.8820                | 0.2940                    | 0.2106        | 0.1187        |

**TABLE II**  
*Quantized Load Levels and Their Respecting Probability*

|    | Load  | Probability |
|----|-------|-------------|
| 1  | 0.626 | 0.120       |
| 2  | 0.693 | 0.141       |
| 3  | 0.759 | 0.140       |
| 4  | 0.826 | 0.121       |
| 5  | 0.892 | 0.085       |
| 6  | 0.958 | 0.085       |
| 7  | 1.025 | 0.092       |
| 8  | 1.091 | 0.079       |
| 9  | 1.157 | 0.077       |
| 10 | 1.224 | 0.060       |

*deterministic problem by substituting the probability of load levels ( $p_k$ ) with load level durations of Table IV.*

*In order to find the maximum attainable profit, which as discussed earlier is an important factor in*

construction of membership functions, firstly the voltage deviation is omitted from the objective function and the maximum profit found is \$1389923.00. Now we can find the suitable membership function regarding to profits and solve the problem. Table V shows the optimal solution of the deterministic problem.

Fig. 6 shows the voltage profile for compensated and uncompensated systems for the peak load level of the present year. As can be seen in this figure, voltage deviation is

**TABLE III**  
*PSO Algorithm Parameters*

|                       | Swarm Size | C1  | C2  | W1  | W2  | Iter <sub>max</sub> |
|-----------------------|------------|-----|-----|-----|-----|---------------------|
| Deterministic Problem | 30         | 1.7 | 1.7 | 0.8 | 0.4 | 150                 |
| Probabilistic Problem | 50         | 1.7 | 1.7 | 0.8 | 0.4 | 200                 |

**TABLE IV**  
*Load Levels and Durations for Deterministic Case*

|   | Load level | Duration in 3-year (hrs) | Duration in 3-year (%) |
|---|------------|--------------------------|------------------------|
| 1 | 0.500      | 2000                     | 0.076                  |
| 2 | 0.540      | 2000                     | 0.076                  |
| 3 | 0.583      | 2000                     | 0.076                  |
| 4 | 0.700      | 5260                     | 0.200                  |
| 5 | 0.756      | 5260                     | 0.200                  |
| 6 | 0.816      | 5260                     | 0.200                  |
| 7 | 1.000      | 1500                     | 0.057                  |
| 8 | 1.080      | 1500                     | 0.057                  |
| 9 | 1.166      | 1500                     | 0.057                  |

**TABLE V**  
*Optimal Solution for Case V.1 – Deterministic Optimization Problem Considering Several Objectives for BCR>1.3*

| Bus | Size of DER (KW) |
|-----|------------------|
| 6   | 600              |
| 11  | 1500             |
| 14  | 300              |
| 15  | 300              |
| 22  | 900              |
| 25  | 1700             |

lower for compensated system, which demonstrates that the algorithm can effectively mitigate the voltage deviation while the value of profit is still acceptable comparing with the maximum attainable profit gained in previous case study.

### B. Fuzzy Optimization Problem Considering Several Objectives, Stochastic Case

In this case study the proposed stochastic approach is used to find the best solution of the optimization problem considering several objectives. In order to find the shape of membership function of the first part of the objective function, initially the stochastic problem is solved considering the profit as the objective function to find the maximum attainable value of profit. Table VI show the results of placement problem for the single objective problem. As can be seen in this table the maximum attainable profit with the minimum acceptable BCR is \$6658313.10. At the next stage the stochastic problem is solved considering all the objectives and the results are shown in Table VII. Fig. 7 shows convergence characteristic for the stochastic problem by PSO. This figure depicts the change in the BCR of the best solution (g-best) versus iterations of the algorithm. As it can be seen, the PSO have rapid convergence characteristic.

**TABLE VI**  
**Optimal Solution For Case V.2 - Stochastic Single objective Problem, For BCR>1.3**

| Bus                | Size of DER (KW) |          |                         |           |
|--------------------|------------------|----------|-------------------------|-----------|
| 13                 | 1900             |          |                         |           |
| 17                 | 1700             |          |                         |           |
| 19                 | 1000             |          |                         |           |
| 21                 | 1000             |          |                         |           |
| 24                 | 2800             |          |                         |           |
| 3-year Profit (\$) | BCR              | LR (MWh) | LR <sup>Peak</sup> (KW) | PPR (MWh) |
| 6658313.1          | 1.3004           | 2585.36  | 1042.9                  | 220752    |

**TABLE VII**  
**Optimal Solution for Case V.2 - Stochastic Optimization Problem Considering Several Objectives, for BCR>1.3**

| Bus                | Size of DER (KW) |          |                         |           |
|--------------------|------------------|----------|-------------------------|-----------|
| 2                  | 1900             |          |                         |           |
| 14                 | 1400             |          |                         |           |
| 15                 | 1800             |          |                         |           |
| 17                 | 400              |          |                         |           |
| 24                 | 500              |          |                         |           |
| 26                 | 500              |          |                         |           |
| 27                 | 1400             |          |                         |           |
| 3-year Profit (\$) | BCR              | LR (MWh) | LR <sup>Peak</sup> (KW) | PPR (MWh) |
| 6255752.6          | 1.3001           | 3671.32  | 942.2                   | 197015    |

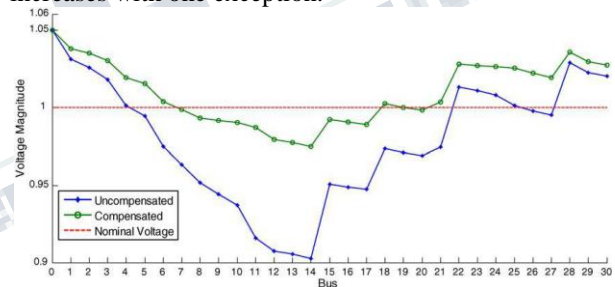
**TABLE VIII**  
**Statistical Analysis of the Results**

|                         | Best solution | Worst solution | Mean value | Standard deviation % |
|-------------------------|---------------|----------------|------------|----------------------|
| 3-year Profit (\$)      | 6255752.6     | 6256712.1      | 6256236.6  | 0.0076               |
| LR (MWh)                | 3671.32       | 3653.01        | 3660.3     | 0.249                |
| LR <sup>Peak</sup> (KW) | 942.2         | 938.0          | 941.1      | 0.235                |
| PPR (MWh)               | 197015        | 195101         | 196107     | 0.487                |

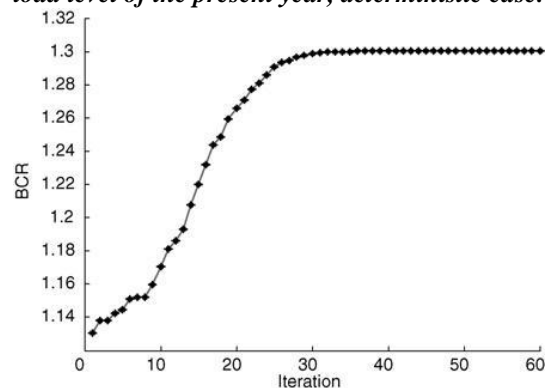
### C. Statistical Analysis of the Results

Since the algorithm used here is a heuristic optimization algorithm and the stochastic nature of the system has been taken into account, the results derived from the proposed method might vary in each run. In order to investigate the effect of these factors, statistical analysis of the results is discussed in this subsection. The proposed algorithm is run 50 times to find the best solution to the problem discussed in case study B (stochastic case with both objectives included).

Table VIII shows the results of the statistical analysis. The PSO parameters are considered to be fixed in all runs. As can be seen the standard deviation of the solutions is very low. This shows the robustness of the proposed algorithm against some factors such as initial population of PSO algorithm. Another study is also conducted to analyze the effects of an increase in the degree of uncertainty associated with system loads. The standard deviation of peak load and energy growth is changed from 2% to 5% and the results are presented in Table IX. As can be seen in this table the value of the profit decreases as the degree of uncertainty increases. It is also interesting that as the degree of uncertainty increases, the maximum value of voltage deviation (19) increases with one exception.



**Fig. 6. Voltage profile at peak load level for compensated and uncompensated systems at the peak load level of the present year, deterministic case.**



**Fig. 7. Optimization procedure by PSO for the stochastic problem.**

**Table IX**  
*Effects of Uncertainties on the Optimal Solution*

|                                | $\sigma = 2\%$ | $\sigma = 3\%$ | $\sigma = 4\%$ | $\sigma = 5\%$ |
|--------------------------------|----------------|----------------|----------------|----------------|
| 3-year Profit (\$)             | 6255752.6      | 6255402.5      | 6254610.1      | 6254222        |
| Maximum voltage deviation (pu) | 0.0412         | 0.0435         | 0.0431         | 0.0442         |

**TABLE X**  
*Voltage Deviation at Peak Load Level*

|             | First year | Second year | Third year |
|-------------|------------|-------------|------------|
| Without DER | 0.0350     | 0.0378      | 0.0381     |
| With DER    | 0.028502   | 0.026649    | 0.025138   |

reason may lay under this fact that with increase in these standard deviations the total objective function will definitely decrease, but each objective may show unexpected trend.

#### D. Discussion

In order to show the effectiveness of the proposed method this section provides a discussion about the results of case studies.

1) As said earlier the proposed algorithm schedules the output of DERs in each load levels individually to avoid the unwanted rejection of optimal solutions. To illustrate this, consider the result of Table VII. The reduction in power purchased (PPR) from the electricity market simply shows that in all load levels the maximum output of DERs is not scheduled. The maximum power which can be supplied by DERs is 7900 kW, so the maximum energy which can be supplied is  $3 \times 8760 \times 7.9 = 207612$  (MWh) while the energy served by DERs is 197015 (MWh). Again the minimum BCR of 1.3 is considered as a constraint.

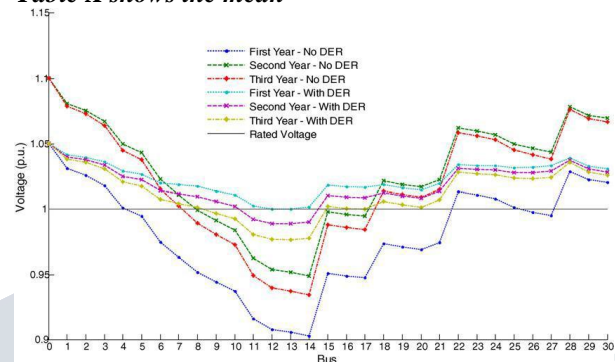
2) Comparing the results of Table VII with those presented in Table VI, the profit and the maximum voltage deviation are less for the problem in which several objectives are considered. This shows that the improvement of voltage profile causes a small reduction in profit.

3) Comparing the results of stochastic algorithm (Table VII) to those obtained in case study (Section V-I) for deterministic problem, one can understand the necessity of the stochastic modeling of the problem. In order to clarify this point the solution of deterministic problem is fed into the stochastic model to calculate the stochastic profit gained by applying best solution of deterministic problem. The profit value is 4412295.2 (\$) which is so much lower than the stochastic profit gained by best solution of stochastic problem reported in Table VII (6255752.6 (\$)).

4) The voltage profile in the peak load level of each year of the three-year time horizon of the study for

compensated and uncompensated cases are presented in Fig. 8. As can be seen in this figure the proposed method can effectively mitigate the voltage deviation. It should be noted that the optimal power flow algorithm for peak load level of years 2 and 3 in uncompensated case did not converged with the main feeder voltage of 1.05 (p.u.); so the voltage of main feeder for these states is considered to be 1.1 (p.u.).

*Table X shows the mean*



**Fig. 8. Voltage profile at peak load level of each year for compensated and uncompensated systems, stochastic case. deviation of voltage at different load points and in fact summarized the results presented in Fig. 8.**

## VI. CONCLUSION

A PSO based fuzzy stochastic long term optimization methodology considering several objectives was proposed in this paper for optimal placement and sizing of DERs. As the results of case studies showed, ignoring the uncertainties in DER placement problem renders the stochastic saving gained non-optimal.

The optimization algorithm simultaneously sought the re-duction in power loss; power purchased from the electricity market, power loss at the peak load level, and deviation of the voltage magnitude at the load points. A proper modeling of the economic aspects of the problem was also presented in this paper. The results of case studies showed that the proposed algorithm effectively guaranteed the justification of investment on DERs. The results also showed that the method scheduled the output power of DERs in each load level, and avoided the inconvenience rejection of more optimal solutions. In fuzzy-fying process the membership functions were obtained with an effective method, instead of the predefined membership functions. Some other aspects of the proposed method are discussed in case studies.

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