

State Space Modeling of Multi-Machine Power System installed with PSS, for small signal stability analysis

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Abstract— This paper presents the detailed study and procedure for modeling Multi-machine power system installed with Power System Stabilizer (PSS). In this paper, a linearized State Space Model of Multi-machine power system, installed with PSS, has been realized. The procedure has been implemented on a standard IEEE 3 machine system for deriving its corresponding linearized state space model. Importance of modeling the Power System network, for carrying out the small signal stability analysis and damping out the low frequency oscillations, has also been discussed in this paper

Keywords: IEEE 3 bus system, Low frequency Oscillations, PSS, Small Signal Stability .

I. INTRODUCTION

Power system is continuously subjected to perturbations. It results Electromechanical oscillations to take place in power system [1]. Out of these oscillations, low frequency oscillations are basis for the study of small signal stability which lies in the range of 0.1 to 3 Hz [2]. The presence of oscillations make it necessary to introduce small signal stability studies and the damping criteria at the planning level [3]. If there is an inadequate amount of damping in the power system network then it leads the system to instability. The amount of damping and the frequency of oscillation depends upon the system operating conditions [1]. Thus, it is very necessary to perform the proper modeling of the system as it is the basis for all the dynamic stability studies[4].

The linearized Phillips-Hephron model has been used for ,many years for performing the small signal stability studies of a single-machine infinite-bus (SMIB) power system. Although the said model is linear it provides reliable results for low frequency oscillations [5]. Mathematically, we can describe small perturbation as a small deviation in the system state. Hence, all the equations described by linearized state space Phililps-Hephron model can be linearized around the equilibrium point. Consequently, the system can be analysed by utilizing all the properties of linear models [6]. If the modeling of the components of Power System is done adequately then it provides the close agreement between the simulations and the practical test results of the system [7-8]. Thus, electrical power system can be studied with the help of the mathematically developed models. These

models when subjected to the perturbations can be suitably analyzed by performing simulations. If the modeling of the power system network is done properly then the system can behave identical to the practical power system network. Power system operate in a very wide operating range. Hence it requires controllers to effectively damp out the oscillations. Power System Stabilizers (PSS), FACTS based controllers have been designed using various robust control techniques for damping the oscillations effectively [9-18]. In this paper the detailed procedure for modeling the Multi-Machine Power System has been discussed. This paper is organized as: Section II introduces the detailed procedure of Multi -Machine Power System Network, installed with PSS, modeling; Section III presents implementation of the said procedure in standard IEEE 3 Machine system; and Section IV discuss the importance of modeling for small signal stability studies

II. PROCEDURE FOR MODELING MULTI MACHINE POWER SYSTEM NETWORK INSTALLED WITH PSS

Consider, N machine power system network, installed with PSS. Here, for realizing the Philips-Hephron model of the power system network following fourth order model of synchronous generator has been considered:

$$\delta_i = \omega_0(\omega_i) \quad (1)$$

$$\dot{\omega}_i = \frac{1}{M_i} (P_{m_i} - P_i - D_i(\omega_i - 1)) \quad (2)$$

$$E'_{q_i} = \frac{1}{T'_{d0_i}} (-E_{q_i} + E_{fd0_i} + E'_{fd_i}) \quad (3)$$

$$\dot{E}'_{fd_i} = -\frac{1}{T_{A_i}} (E'_{fd_i}) + \frac{K_{A_i}}{T_{A_i}} (V_{ref_{g_i}} - V_{g_i} + u_{PSS_i}) \quad (4)$$

$$\left. \begin{aligned} \text{Here, } |V_{g_i}| &= \sqrt{v_{d_i}^2 + v_{q_i}^2} \\ V_{g_i} &= v_{d_i} + jv_{q_i} \\ v_{d_i} &= x_{q_i} i_{q_i} \\ v_{q_i} &= E'_{q_i} - x'_{d_i} i_{d_i} \\ E_{q_i} &= E'_{q_i} + (x_{d_i} - x'_{d_i}) i_{d_i} \\ P_i &= v_{d_i} i_{d_i} + v_{q_i} i_{q_i} \end{aligned} \right\} \quad (5)$$

and, i varies from 1 to N

In x-y coordinate system, Generator Bus voltage of N machines can be expressed as [19]

$$V_g = e^{j\delta} E'_q - jx'_d I_g + (x_q - x'_d) e^{j(\delta-90^\circ)} i_q \quad (6)$$

$$\begin{aligned} \text{Here, } V_g &= [V_{g1} \ V_{g2} \ \dots \ V_{gN}]^T \\ I_g &= [I_{g1} \ I_{g2} \ \dots \ I_{gN}]^T \\ E'_q &= [E'_{q1} \ E'_{q2} \ \dots \ E'_{qN}]^T \\ i_q &= [i_{q1} \ i_{q2} \ \dots \ i_{qN}]^T \\ e^{j\delta} &= \text{diag}(e^{j\delta_i}) \end{aligned}$$

$$\text{and, } (x_q - x'_d) e^{j(\delta-90^\circ)} = \text{diag}((x_{q_i} - x'_{d_i}) e^{j(\delta_i-90^\circ)})$$

Introducing the Internal Voltage of the generator which can be understood as addition to extra node behind the generator terminal. Let this voltage be denoted as " E_g "

Thus, from equation (5) we have,

$$V_g = e^{j\delta} E'_q + (x_q - x'_d) e^{j(\delta-90^\circ)} i_q - jx'_d I_g \quad (7)$$

$$V_g = E_g - jx'_d I_g \quad (8)$$

Let " Y_N " be the network admittance matrix with only nodes of generator terminal left. Hence, following Network equation can be obtained with " Y_N "

$$I_g = Y_N V_g \quad (9)$$

From equation (7) and (8), we have

$$\begin{aligned} I_g &= Y E_g \\ &= Y (e^{j\delta} E'_q + (x_q - x'_d) e^{j(\delta-90^\circ)} i_q) \end{aligned}$$

$$\text{Here, } Y = (Y_N^{-1} + jx'_d)^{-1}$$

$$\text{and, } |I_{g_i}| = \sqrt{i_{d_i}^2 + i_{q_i}^2}$$

$$I_{g_i} = i_{d_i} + j i_{q_i}$$

Linearization of I_g can be expressed in the matrix form as:

$$\left. \begin{aligned} \Delta I_d &= F_{dd} \Delta \delta + G_{dd} \Delta E'_q + H_{dd} \Delta I_q \\ \Delta I_q &= F_{qq} \Delta \delta + G_{qq} \Delta E'_q + H_{qq} \Delta I_q \\ \text{Further above equations can be reduced as:} \\ \Delta I_d &= F_d \Delta \delta + G_d \Delta E'_q \\ \Delta I_q &= F_q \Delta \delta + G_q \Delta E'_q \end{aligned} \right\} \quad (10)$$

$$\text{Here, } F_q = (I - H_{qq})^{-1} F_{qq}$$

$$G_q = (I - H_{qq})^{-1} G_{qq}$$

$$F_d = F_{dd} + H_{dd} F_q$$

$$G_d = G_{dd} + H_{dd} G_q$$

After obtaining the values of Parameters " F_d " & " G_d " the Philips-Hephron Model can be derived as:

$$\left. \begin{aligned} \Delta P &= K_1 \Delta \delta + K_2 \Delta E'_q \\ \Delta E_q &= K_4 \Delta \delta + K_3 \Delta E'_q \\ \Delta V_g &= K_5 \Delta \delta + K_6 \Delta E'_q \end{aligned} \right\} \quad (11)$$

System Coefficients K_1 to K_6 can be obtained by using equations (5), (10) and (11).

Phillips-Hephron model as described from equations (11) can be written in State Space form as:

$$\begin{aligned} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} &= \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ -M^{-1}K_1 & -M^{-1}D & -M^{-1}K_2 & 0 \\ -T'_{d0}{}^{-1}K_4 & 0 & -T'_{d0}{}^{-1}K_3 & T'_{d0}{}^{-1} \\ -T_A^{-1}K_5K_A & 0 & -T_A^{-1}K_6K_A & -T_A^{-1} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_A^{-1}K_A \end{bmatrix} \Delta u_{PSS} \end{aligned} \quad (12)$$

Here, $M = \text{diag}(M_i)$

$D = \text{diag}(D_i)$

$T'_{d0i} = \text{diag}(T'_{d0i})$

$K_A = \text{diag}(K_A)$

$T_A = \text{diag}(T_A)$

State Vector ' ΔX ' = $[\Delta \delta \ \Delta \omega \ \Delta E'_q \ \Delta E'_{fd}]^T$

Output Vector ' ΔY ' = $[\Delta \omega]$

Output Signal from PSS:

$$' \Delta u' = [\Delta u_{PSS}]$$

Equation (12) represents the Linearized State Space model of N machine power system installed with PSS. Its

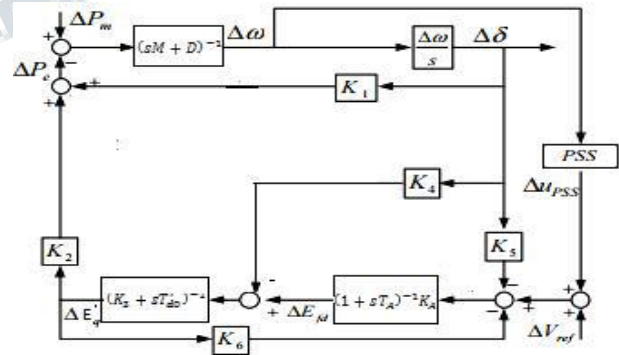


Fig. 1 Linearized State Space model of N machine power system installed with PSS.

Systematic procedure for realizing Linearized State Space Model of the Multi-Machine Power System is shown in Figure. 2.

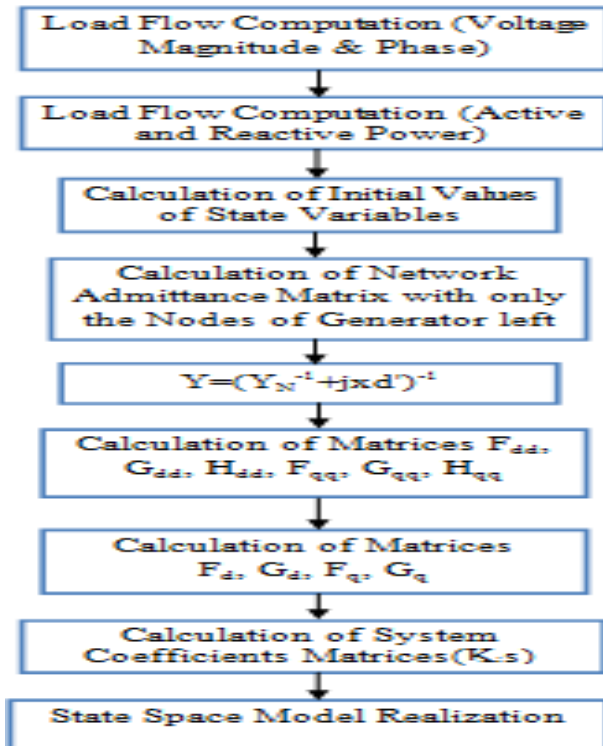


Fig.2 Systematic procedure for realizing Linearized State Space Model of the Multi-Machine Power System

III. PHILIPS HEPHRON MODEL OF IEEE 3 MACHINE SYSTEM

Fig.3 shows the standard IEEE 3 Machine System with their initial conditions. Table I & II shows the parameters of generator, transformer and transmission lines considered for modeling the considered system.

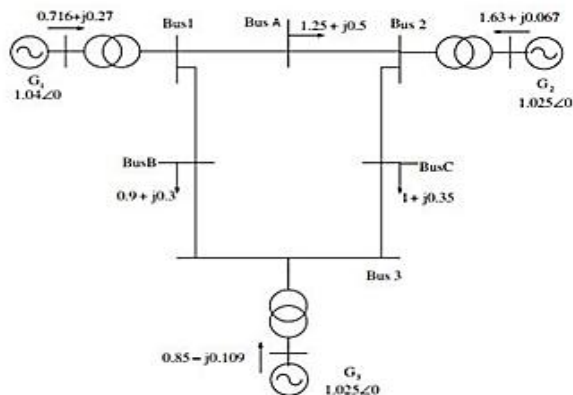


Fig.3 Systematic procedure for realizing Linearized State Space Model of the Multi-Machine Power System

	G1	G2	G3
x_d	0.146	0.8958	1.313
x'_d	0.0608	0.1189	0.1813
T_{d0}	8.96	6	5.89
x_q	0.0969	0.8645	1.258
x'_q	0.0969	0.1969	0.25
M	47.2	12.8	6.02
D	0	0	0
K_A	200	200	200
T_A	0.02	0.02	0.02

Table 1. Parameters of Generator

Node	Node	Resistance	Reactance	Susceptance
1	A	0.01	0.085	0.088 X 2
1	B	0.017	0.092	0.079 X 2
A	2	0.032	0.161	0.153 X 2
B	3	0.039	0.17	0.170 X 2
2	C	0.085	0.072	0.0745 X 2
C	3	0.0119	0.1008	0.1045 X 2
G1	1	0	0.0576	0
G2	2	0	0.0625	0
G3	3	0	0.0586	0

Table 2. Parameters of Transformer and Lines

For Modeling the three machine system following steps have been performed:

1. Calculation of Initial Values.
2. Calculation of Network Admittance Matrix only nodes of generator terminal
3. Calculation of System Coefficients
4. Realization of Linearized State S

A. Calculation of Initial Values

Load Flow Studies are carried out to calculate initial values of parameters " δ_{i_0} ", " ω_{i_0} ", " E'_{q_0} ", " E'_{fd_0} ". In order to carry out Load Flow Studies, Power World Simulator Software has been used.

B. Calculation of Network Admittance Matrix with only nodes of generator terminal left

When node "k" is eliminated, the modified elements of Network Admittance Matrix can be given by

$$Y'_{ij} = Y_{ij} - \frac{Y_{ik}Y_{kj}}{Y_{kk}} \quad (13)$$

Here, $i, j = 1, \dots, N \neq k$

With the help of equation (13) all the nodes except generator nodes can be eliminated from the Network Admittance matrix and the modified Admittance matrix will be left with only generator terminal nodes. MATLAB code has been made for calculating admittance matrix "Y". For the given system configuration it has found to be:

$$Y = \begin{bmatrix} 0.9258 - 2.4877i & 0.3005 + 1.8418i & 0.2798 + 1.4325i \\ 0.3005 + 1.8418i & 0.5588 - 2.4219i & 0.1081 + 1.2244i \\ 0.2798 + 1.4325i & 0.1081 + 1.2244i & 0.4569 - 2.2602i \end{bmatrix}$$

C. Calculation of System Coefficients

With the help of equations (5),(10),(11) System Coefficients for the considered network can be calculated. With the help of MATLAB code system coefficients for the considered system is found as:

$$K_1 = \begin{bmatrix} 2.6787 & -1.8822 & -0.7965 \\ -2.0458 & 2.6864 & -0.6406 \\ -1.2407 & -0.9213 & 2.1619 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 3.1940 & -1.3824 & -0.3280 \\ 1.6568 & 2.9970 & -0.263 \\ 0.5396 & -0.8503 & 2.22 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 1.0920 & -0.115 & -0.1437 \\ 0.1502 & 2.8556 & -0.7332 \\ -0.7308 & -1.4822 & 3.5967 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 0.0485 & -0.0023 & -0.0463 \\ -1.5692 & 2.2312 & -0.662 \\ -1.4179 & -1.0613 & 2.4792 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} -0.0187 & -0.0096 & 0.00283 \\ -0.1083 & 0.0595 & 0.0488 \\ -0.1077 & -0.0766 & 0.1843 \end{bmatrix}$$

$$K_6 = \begin{bmatrix} 0.9470 & 0.0738 & 0.1006 \\ 0.6312 & 0.5265 & 0.2438 \\ 0.5984 & 0.1351 & 0.5969 \end{bmatrix}$$

D. Realization of Linearised State Space Model

With the help of equation (11) following linearized State Space Model have been realized for considered system with the help of a MATLAB code:

$$\begin{bmatrix} \dot{\Delta\delta} \\ \dot{\Delta\omega} \\ \dot{\Delta E'_q} \\ \dot{\Delta E'_{fd}} \end{bmatrix} = A \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix} + B \Delta u_{PSS}$$

A =

$$\begin{bmatrix} 0 & 0 & 0 & 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0 & 0 & 0 & 0 & -0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & -0.2 & 0.1 & 0 & 0 & 0 & -0.1 & -0.2 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & -0.4 & 0 & 0 & 0 & -0.1 & 0.1 & -0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0.3 & -0.4 & 0.1 & 0 & 0 & 0 & 0 & -0.5 & 0.1 & 0 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & -0.4 & 0 & 0 & 0 & 0.1 & 0.3 & -0.6 & 0 & 0 & 0 & 0.2 \\ 186.6 & 96.1 & -282.7 & 0 & 0 & 0 & -9470.1 & -738.3 & -1006.1 & -50 & 0 & 0 & 0 \\ 1083.3 & -595.1 & -488.2 & 0 & 0 & 0 & -6311.6 & -5264.9 & -2438 & 0 & -50 & 0 & 0 \\ 1077.3 & 765.8 & -1843 & 0 & 0 & 0 & -5983.8 & -1350.6 & -5968.8 & 0 & 0 & -50 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

IV. IMPORTANCE OF STATE SPACE MATRIX FOR SMALL SIGNAL STABILITY ANALYSIS

Low frequency oscillations that occurs in the power system network may leads to instability if not properly damped. With the help of calculated State Space matrix "A" the small signal stability analysis can be done so that proper controller can be installed in the system to damp out low frequency oscillations.

Eigen value analysis of the matrix "A" helps in finding the oscillatory modes of the system. For the considered IEEE 3 machine system, oscillatory modes of the system comes out to be:

- 1) -24.9799 ±31.6073i
- 2) -24.9802 ±8.5695i
- 3) -0.3336 ±8.5357i
- 4) -1.8881 ±10.6960i

By analyzing suitably the electromechanical modes of the Multi-Machine network, controllers can be designed to damp out the oscillations effectively, when perturbation is injected into the system.

V. CONCLUSION

In this paper, the procedure for modeling the Multi-Machine Power System installed with PSS has been explained in detail. With the Help of a MATLAB Code the Linearized State Space model of standard IEEE three machine system has been derived and obtained for the specific operating conditions. Oscillatory modes have been successfully identified for the derived State Space model. Importance of modeling in the small signal stability studies have also been briefly discussed.

VI. NOMENCLATURE

- xq,xd,xd'- q-axis, d-axis, d-axis transient reactance of generator respectively,
- Vg- Generator Terminal Voltage,
- Eq'- Transient EMF in q-axis of generator, Efd- Field Voltage,
- Ka,Ta- Voltage Regulator Specifications (Gain and Time constant of excitation system),
- Td0'- Open Circuit Field Time constant,

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