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# Stable Reduced Order Models for Index-3 Second Order Systems

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***Abstract*-— A non-existing technique for stability preservation in reduced order models (ROMs) for index-3 second order systems (SOSs) in limited interval frequency is proposed in this paper. This achievement is based on making indefinite terms in algebraic Lyapunov equations definite for frequency limited reduction applications like filter design, signal reconstruction, controller design etc. Index-3 model is first transformed into index-0 generalized form and limited interval Gramians are computed from respective Lyapunov equations. Indefinite terms in Lyapunov equations have been made definite by assigning nearest positive eigenvalues. Obtained Gramians are balanced to achieve Hankel singular values on whom basis, stable ROMs is obtained by applying truncation. The proposed technique is widely useful for finite frequency interval applications of index-3 SOSs.**

***Index Terms*— Dynamic systems, Model reduction, Frequency limited Gramians, Hankel singular values**

## I. INTRODUCTION

In classical multibody dynamic systems, mathematical modeling is the key analysis and design parameter for the systems. Electrical networks constrained vibrational networks and multibody systems have second order structure [4-7]. The equation representing classical multibody dynamic system with second order structure can be represented as:

$$M\ddot{\rho}(t) + D\dot{\rho}(t) + K\rho(t) = B_2' u(t) \tag{1a}$$

$$C_2'\dot{\rho}(t) + C_1'\rho(t) = y(t) \tag{1b}$$

Where $M \epsilon R^{n\times n}$ , $D \epsilon R^{n\times n}$, $K \epsilon R^{n\times n}$ , $B_2' \epsilon R^{n\times m}$, $C_1'$, $C_2' \epsilon R^{p\times n}$ , $\rho(t) \epsilon R^n$ , $u(t) \epsilon R^m$ and $y(t) \epsilon R^p$, $n$ is the order of the system, $m$ is the number of input(s) and $p$ is the number of output(s) of the system. In a mechanical system , $D$ and $K$ are the mass, damping and stiffness matrices respectively. The system defined in (1) is called second order form system.

While modeling systems like (1), the system's order gets very large and becomes difficult to simulate, analyze or design controller for such system, moreover large storage and processing cost is the main constraint in implementing such systems [1]. To process such system, model order reduction (MOR) technique is advised which reduces the complexity by developing an approximate model of the system which have approximately similar response like the original system. The reduced system requires less storage and can easily be managed and is easier to analyze. The reduced model must preserve certain original system properties like stability, regularity, passivity, etc. The error in reduction must be small, and the technique must be converging and efficient [11],[21],[22]. The reduced order model for the system (1) is given in (2).

$$M_r\ddot{\rho}_r(t) + D_r\dot{\rho}_r(t) + K_r\rho_r(t) = B_{2r}' u(t) \tag{2a}$$

$$C_{1r}'\dot{\rho}_r(t) + C_{2r}'\rho_r(t) = y_r(t) \tag{2b}$$

Where, $M_r \epsilon R^{r\times r}$ , $D_r \epsilon R^{r\times r}$, $K_r \epsilon R^{r\times r}$ , $B_{2r}' \epsilon R^{r\times m}$, $C_{1r}'$, $C_{2r}' \epsilon R^{p\times r}$ and $r \ll n$.

Different reduction techniques for large scale system like balance truncation [8], and interpolatory method through iterative rational Krylov algorithm [9], [20], [21] are most used. Each method has pros and cons. The later technique is computationally efficient; however, it does not guarantee stability nor gives a prior error bound. The former technique (BT) gives a global bound on error stability preservation of the reduced system. The only drawback with this technique is to solve two Lyapunov equations for Gramian factors with computational efficiency.

MOR using BT is gaining wide acceptance these days. A lot of work that is being done is present in the literature [12],[13],[14]. Apart from the above discussion, in certain applications like filter design, feedback controllers, etc. needs minimum reduction error on finite frequency interval only. MOR using BT for the second order system in the limited interval frequency is presented in [15] and [16]. [17]

extended the work to time limited. For limited time/frequency interval ROMs are not guaranteed to be stable. Extending the discussion to second order index-3 systems, Gramian based MOR technique for a second order index-3 system using BT has been discussed in the literature [10], however, in case of limited interval frequency no such work exists in the literature to the best of authors knowledge which is discussed in this paper. Moreover, stability of ROM over finite frequency interval has been preserved in our proposed work.

Second order index-3 system is first transformed into index-0 generalized form, then using Lyapunov equation limited interval Gramians are calculated. The indefinite terms in algebraic Lyapunov equations are made definite for frequency limited reduction applications like filter design, signal reconstruction, controller design etc. This is done by assigning the indefinite terms in the Lyapunov equation nearest positive eigen values. Hankel singular values are obtained by balancing the Gramians. The proposed technique is useful for finite frequency interval applications of index-3 SOSs.

## II. PRELIMINARIES

BT for second order system using different approaches has been explained in [18]. This section focuses on the square root method. Consider a Linear time invariant system (LTI):

$$\tilde{E}\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) \quad (3a)$$

$$\tilde{y}(t) = \tilde{C}x(t) \quad (3b)$$

Where, $\tilde{E}, \tilde{A}, \epsilon\, R^{n\times n}$ , $\tilde{B}\ \epsilon\ R^{n\times m}$ and $\tilde{C}\ \epsilon\ R^{p\times n}$. $x(t)\ \epsilon\ R^n$ is state vector, and $u(t)\ \epsilon\ R^m$ is the input and $\tilde{y}(t)\ \epsilon\ R^p$ is the output at time $t$. The above is the generalized state space form as $\tilde{E}\ \neq I$ and is asymptotically stable. If the system is stable, then all the eigen values of the system must have negative real part, then the controllability gramian and observability gramian of the system are given as:

$$G_c = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(i\omega\tilde{E} - \tilde{A}\right)^{-1}\tilde{B}\tilde{B}^T\left(-i\omega\tilde{E} - \tilde{A}\right)^{-T}d\omega \quad (4)$$

$$G_o = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(-i\omega\tilde{E} - \tilde{A}\right)^{-T}\tilde{C}^T\tilde{C}\left(i\omega\tilde{E} - \tilde{A}\right)^{-1}d\omega \quad (5)$$

are the positive semidefinite, symmetric, and unique solutions of the generalized Lyapunov equations:

$$\tilde{E}G_c\tilde{A}^T + \tilde{A}G_c\tilde{A}_T = -\tilde{B}\tilde{B}^T \quad (6)$$

$$\tilde{E}^TG_o\tilde{A} + \tilde{A}^TG_o\tilde{E} = -\tilde{C}^T\tilde{C} \quad (7)$$

The main idea of BT is to diagonalize both Gramians with decreasing magnitude of their diagonal entries also called Hankel singular values (HSV). The state which is least involve in the system response have small HSV and the state with higher HSV have large involvement in the system response. The states having small HSV are truncated with the idea that system response is not affected much.

The controllability Gramian ($G_c$) and the observability Gramian ($G_o$ ) of the system can also be represented as:

$$G_c = \bar{R}\bar{R}^T \qquad G_o = \bar{L}\bar{L}^T$$

They are called the Cholesky factor of the Gramians. Performing singular value decomposition (SVD) of the Cholesky factor as:

$$\bar{L}^TE\bar{R} = [\bar{U}_{o1} \quad \bar{U}_{o2}]\begin{bmatrix}\hat{\Sigma}_{o1} & 0\\ 0 & \hat{\Sigma}_{o2}\end{bmatrix}\begin{bmatrix}\bar{V}_{o1}^T\\ \bar{V}_{o2}^T\end{bmatrix}$$

Where, $[\bar{U}_{o1} \quad \bar{U}_{o2}]$ and $\begin{bmatrix}\bar{V}_{o1}^T\\ \bar{V}_{o2}^T\end{bmatrix}$ are orthogonal and

$$\hat{\Sigma}_{o1} = diag(\varsigma_1, \varsigma_2, \dots\dots.., \varsigma_r)$$

$$\hat{\Sigma}_{o2} = diag(\varsigma_{r+1}, \varsigma_{r+2}, \dots\dots.., \varsigma_n)$$

Now reduced order model (ROM) is calculated as:

$$\grave{E} = \tilde{L}^T\tilde{E}\tilde{R}\ ,\ \grave{A} = \tilde{L}^T\tilde{A}\tilde{R}\ ,\ \grave{B} = \tilde{L}^T\tilde{B}\ ,\ \grave{C} = \tilde{C}\tilde{R} \quad (8)$$

where,

$$\tilde{L} = \bar{L}\bar{U}_{o1}\hat{\Sigma}_{o1}^{(-\frac{1}{2})} \qquad \tilde{R} = \bar{R}\bar{V}_{o1}\hat{\Sigma}_{o1}^{(-\frac{1}{2})}$$

In this paper quadrature rule is used to solve the Lyapunov equations to find both controllability and observability Gramians. This paper focus on the stability preservation in the limited frequency interval of the ROM using BT by making the indefinite terms in the Lyapunov equation definite. The below section discusses the MOR LR-ADI of second order index-3 system in detail.

## III. PROPOSED METHOD

A linearized holonomically constrained equation of motion [2] with index-3 structure is represented as:

$$M\ddot{\rho}(t) + D\dot{\rho}(t) + K\rho\,(t) + G^T\sigma(t) = B_2\,u(t) \quad (9a)$$

$$G\sigma(t) = 0 \quad (9b)$$

$$y(t) = C_2\rho(t) \quad (9c)$$

Where $\rho\ \epsilon\ R^{n_\rho}$ is a position vector with coordinates related to the degree of freedom of the individual masses and $\sigma\ \epsilon\ R^{n_\sigma}$ is a vector with $n_\sigma$ ( $n_\sigma < n_\rho$) the unknown parameters, called Lagrange multiplier. The matrices $M, D, K\ \epsilon\ R^{n_\rho\times n_\rho}$ are mass, stiffness, and damping respectively and $G\ \epsilon\ R^{n_\sigma\times n_\rho}$ is known as constraint matrix. $B_2\ \epsilon\ R^{n_\rho\times m}$ is the input matrix related to the input vector $u(t)\epsilon\ R^m$ and $C_2\ \epsilon\ R^{p\times n}$ is the output matrix related with output vector $y(t)\epsilon\ R^p$,$n$ is the order of the system,

$m$ is the number of input(s) and $p$ is the number of output(s) of the system.

Before applying BT on the system in (9), first it is converted into generalized form. The first step is the conversion from index-3 to index-0 which is explained in the coming section. Then second step is to transform the index-0 system into first order generalized form so that BT can be applied. All these steps are explained in the coming sections.

### A. Second Order Index-3 to Index-0 Conversion

This section describes methods to transform second order index-3 system into an index-0 system. Multiplying (9a) on both sides by $GM^{-1}$:

$$GM^{-1}M\ddot{\rho}(t) + GM^{-1}D\dot{\rho}(t) + GM^{-1}K\rho(t) = GM^{-1}\varphi B_2 u(t)$$

$$GM^{-1}D\rho^{\cdot}(t) + GM^{-1}K\rho(t) = GM^{-1}\varphi B_2 u(t)$$

$$(GM^{-1}G^T)^{-1}GM^{-1}D\rho^{\cdot}(t) + (GM^{-1}G^T)^{-1}GM^{-1}K\rho(t) - (GM^{-1}G^T)^{-1}GM^{-1}B_2 u(t) = \varphi(t)$$

Now putting $\varphi(t)$ in (9a),

$$M\ddot{\rho}(t) + \Theta D\dot{\rho}(t) + \Theta K\rho(t) = \Theta B_2 u(t) \quad (10)$$

where $\Theta = I_{n_\rho} - G^T(G M^{-1}G^T)^{-1}GM^{-1}$ (11)

Where $I_{n_\rho}$ is an identity matrix of size $n_\rho$. $\Theta$ is a projection which meets some properties defined and proved in [10]. After applying the Theorem:

$$\Theta^T \rho(t) = \rho(t) \quad (12)$$

Proof for (12) is given in [10], the system in (10) becomes:

$$\Theta M\Theta^T\ddot{\rho}(t) + \Theta D\Theta^T\dot{\rho}(t) + \Theta K\Theta^T\rho(t) = \Theta B_2 u(t) \quad (13a)$$

Applying (12) on the output equation in (9c), the system becomes:

$$y(t) = C\Theta^T\rho(t) \quad (13b)$$

System in (13) is now ind ex-0 second order system.

### B. Second Order to Generalized Form Conversion

Consider the index-0 second order system in (13). In the next section the system in (9) is converted to standard first order form which is represented as:

$$\Phi E_1\Phi^T\dot{x}(t) = \Phi A_1\Phi^T x(t) + \Phi B_1 u(t) \quad (14a)$$

$$y(t) = C_1\Phi^T x(t) \quad (14b)$$

Where,,

$$\Phi = \begin{bmatrix}\Theta & 0\\ 0 & \Theta\end{bmatrix},\quad E_1 = \begin{bmatrix}M & 0\\ 0 & M\end{bmatrix},\quad A_1 = \begin{bmatrix}0 & M\\ -K & -D\end{bmatrix},$$

$$B_1 = \begin{bmatrix}0\\ B_2\end{bmatrix},\ C_1 = [0 \quad C_2] \ \text{ and } \ x(t) = \begin{bmatrix}\rho(t)\\ \rho^{\cdot}(t)\end{bmatrix}$$

The equations in (14) can also be written as

$$E\dot{x}(t) = A\,x(t) + Bu(t) \quad (15a)$$

$$y(t) = C\,x(t) \quad (15b)$$

Where,

$$E = \Phi E_1\Phi^T,\ A = \Phi A_1\Phi^T,\ B = \Phi B_1 u(t),\ C = C_1\Phi^T$$

### C. Limited Frequency Interval Gramians

The limited frequency interval controllability and observability Gramians for the system in (15) are given as:

$$G_{c\delta} = \frac{1}{2\pi}\int_\delta (i\omega E - A)^{-1}BB^T(-i\omega E - A)^{-T}d\omega \quad (16)$$

$$G_{o\delta} = \frac{1}{2\pi}\int_\delta (-i\omega E - A)^{-T}C^TC(i\omega E - A)^{-1}d\omega \quad (17)$$

Where $\delta$ = [$-\omega_2, -\omega_1$] ∪ [$\omega_1, \omega_2$] is chosen such that the limited frequency interval Gramians are ensured to be real, symmetric, and positive definite.

The frequency limited interval Gramians defined in (16) and (17) are the solutions of continuous time algebraic Lyapunov equation:

$$EG_{c\delta}A^T + AG_{c\delta}E^T = -EHBB^T - BB^TH^TE^T \quad (18)$$

$$EG_{o\delta}A^T + AG_{o\delta}E^T = -E^TH^TC^TC - C^TCHE \quad (19)$$

where $E$, $A$, $B$ and $C$ are of second order structure and

$$H = \frac{1}{2\pi}\int_\delta (i\omega E - A)^{-1}d\omega$$

### D. Solution for Lyapunov Equation and Cholesky Factor

Solving Lyapunov equation for large scale systems is quite tedious due to the right-hand side's indefiniteness, as a result Cholesky factorization can't be done. To solve such systems quadrature rule is used that uses nodes $\omega_j$ and weights $\gamma_j$ at numerous points in the limited frequency interval.

The frequency limited Gramian in (12) and (13) becomes:

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Vol 7, Issue 10, October 2021

$$G_{c\delta} \approx \frac{1}{2\pi}\sum_{j=1}^{k}\gamma_j\left\{\left(i\omega_j E - A\right)^{-1} BB^T\left(i\omega_j E - A\right)^{-T} + \left(i\omega_j E - A\right)^{-1} BB^T\left(i\omega_j E - A\right)^{-T}\right\} \quad (20)$$

Assuming $\gamma_j$ to be positive then (20) becomes:

$$G_{c\delta} \approx \frac{1}{2}[B_{1\delta}, B_{1\delta}^{-}, \dots, B_{k\delta}, B_{k\delta}^{-}][B_{1\delta}, B_{1\delta}^{-}, \dots, B_{k\delta}, B_{k\delta}^{-}]^{*} \quad (21)$$

Where, $B_{j\delta} = \sqrt{\frac{\gamma_j}{\pi}}\left(i\omega_j E - A\right)^{-1} B.$

Equation (21) gives low rank approximation of $G_{c\delta}$ and $\frac{1}{\sqrt{2}}[B_{1\delta}, B_{1\delta}^{-}, \dots, B_{k\delta}, B_{k\delta}^{-}]$is rank Cholesky factor. Re-writing (21) as:

$$[B_{j\delta}, B_{j\delta}^{-}][B_{j\delta}, B_{j\delta}^{-}]^{*} = 2[Re(B_{j\delta})\ Im(B_{j\delta})] \times [Re(B_{j\delta}), \mathrm{Im}(B_{j\delta})]^{T} \quad (22)$$

The equation (22) results in Cholesky factor:

$$G_{c\delta} \approx \acute{R}_\delta \acute{R}_\delta^T \quad (23)$$

Where,

$$\acute{R}_\delta = [Re(B_{1\delta}), \dots, Re\ (B_{k\delta}), Im(B_{1\delta}), \dots Im\ (B_{k\delta})] \quad (24)$$

Frequency limited interval observability Gramian is approximated as:

$$G_{o\delta} \approx \grave{L}_\delta \grave{L}_\delta^T \quad (25)$$

Where,

$$\grave{L}_\delta = [Re(C_{1\delta}), \dots, Re\ (C_{k\delta}), Im(C_{1\delta}), \dots Im(C_{k\delta})] \quad (26)$$

Where,

$$C_{j\delta} = \sqrt{\frac{\gamma_j}{\pi}}\left(-i\omega_j E - A\right)^{-1} C^T.$$

### E. Balance Truncation for Frequency Limited Second Order System

Equations (23) and (25) gives Frequency limited controllability and observability Gramian factors also known as Cholesky factors. Applying singular value decomposition on the relation.

$$\grave{L}_\delta^T\ \Phi\ E\ \Phi\ \acute{R}_\delta = [U_{\delta 1}\quad U_{\delta 2}]\begin{bmatrix}\Sigma_{\delta 1} & 0\\ 0 & \Sigma_{\delta 2}\end{bmatrix}\begin{bmatrix}V_{\delta 1}^T\\ V_{\delta 2}^T\end{bmatrix}$$

Right and left balancing and truncation transformation can be obtained as:

$$\mathbb{R}_\delta = \acute{R}_\delta\, V_{\delta 1}\Sigma_{\delta 1}^{\left(-\frac{1}{2}\right)} \quad (27)$$

$$\mathbb{L}_\delta = \grave{L}_\delta\, U_{\delta 1}\Sigma_{\delta 1}^{\left(-\frac{1}{2}\right)} \quad (28)$$

By applying the left and right transformation on the system in (15), we get:

$$E_{r\delta} = \mathbb{L}_\delta^T E\ \mathbb{R}_\delta \qquad A_{r\delta} = \mathbb{L}_\delta^T A\ \mathbb{R}_\delta \quad (29)$$
$$B_{r\delta} = \mathbb{L}_\delta^T B \qquad C_{r\delta} = C\ \mathbb{R}_\delta$$

Equation (29) gives the reduced order model in limited frequency interval of second order index-3 system defined in (9).

### F. Remarks

For limited interval, when right hand side of (18) and (19) becomes indefinite, the stability in ROM is not guaranteed in this case. In order to make ROM stable, the proposed method makes right hand side positive definite by computing nearest positive definite eigen values, so that modified eigen values remains closest possible to the original eigen values.

## IV. NUMERICAL RESULTS

The method described above is tested on a stable second order index-3 system to check the efficiency and accuracy of the proposed scheme. The system has been reduced for different reduction orders.

Consider a system with $n = 12$, $m = 1$ and $p = 1$ with system matrices as:

$$M = \begin{bmatrix}
7.3 & 5.1 & 3.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5.1 & 7.3 & 5.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3.3 & 5.1 & 7.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 7.3 & 5.1 & 3.3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 5.1 & 7.3 & 5.1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 3.3 & 5.1 & 7.3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 7.3 & 5.1 & 3.3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 5.1 & 7.3 & 5.1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 3.3 & 5.1 & 7.3 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.3 & 5.1 & 3.3\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.1 & 7.3 & 5.1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.3 & 5.1 & 7.3
\end{bmatrix}$$

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Vol 7, Issue 10, October 2021

$$D = \begin{bmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \end{bmatrix}$$

$$K = \begin{bmatrix} 2.4 & 0.5 & -0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.5 & 2.4 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.03 & -0.5 & 2.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4 & -0.5 & -0.03 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 2.4 & -0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.03 & -0.5 & 2.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.4 & -0.5 & -0.03 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.03 & -0.5 & 2.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.4 & -0.5 & -0.03 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.4 & -0.5 & -0.03 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 2.4 & -0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.03 & -0.5 & 2.4 \end{bmatrix}$$

$$G = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$$
$$B_2 = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]^T$$
$$C_2 = [0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1]$$

The matrices $M, D, K \in R^{n_\rho \times n_\rho}$ are the mass, stiffness, and damping matrices respectively and $G \in R^{1\times n_\rho}$ is known as constraint matrix. $B_2 \in R^{n_\rho \times m}$ is the input matrix related to the input vector $u(t) \in R^m$ and $C_2 \in R^{p\times n}$ is the output matrix The system is reduced to the order of r = 4, 6, and 7 in interval $\delta = [0.1,10]$ using the method described above. The bode plot for the original system, reduced system for infinite frequency interval and reduced system for finite frequency interval is shown in Fig. (1), Fig (2) and Fig (3) for reduction orders 4th ,6th, and 7th respectively.

The eigen values for the reduced orders in interval $\delta = [0.1,10]$ in unstable and stable condition are shown in (I) for 4th ,6th and 7th orders respectively. Stability of the system is preserved in the limited frequency interval by assigning nearest positive eigen values to the indefinite term in the Lyapunov equation and making them definite.

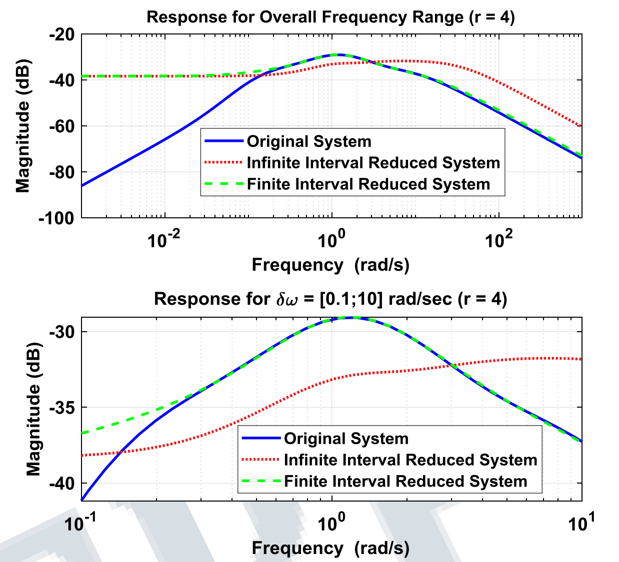


**Fig 1:** System Response for Overall and Limited Frequency interval in 4th Order

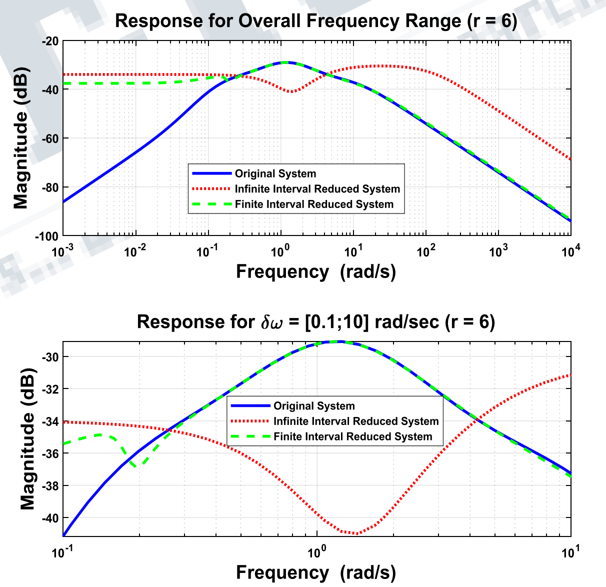


**Fig 2:** System Response for Overall and Limited Frequency interval in 6th Order

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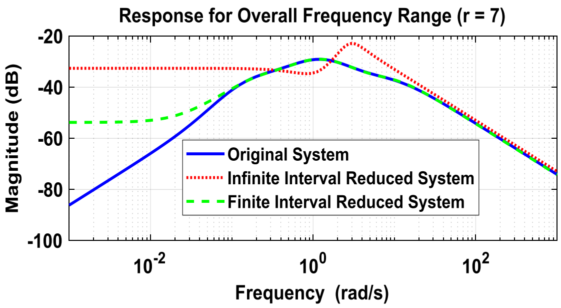
Vol 7, Issue 10, October 2021



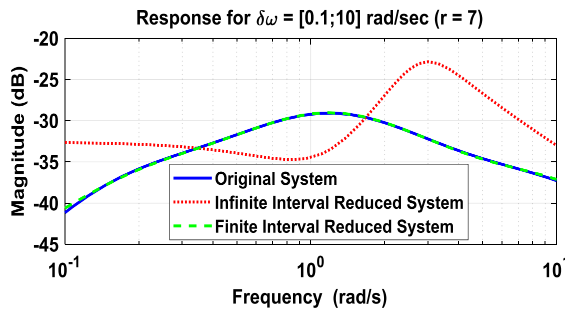




**Fig 3:** System Response for Overall and Limited Frequency interval in 7th Order

**Table i.** Eigen values of reduced system in interval $\delta = [0.1,10]$ for 4th ,6th and 7th order

| Reduction Order | | Eigen Values | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4th | Indefinite/unstable | 8.1052 | -1.8231 | -0.8951 | -0.2564 | | | |
| | Definite/Stable | -14.6433 + 0.0000i | -1.0199 + 0.6451i | -1.0199 - 0.6451i | -0.1161 + 0.0000i | | | |
| 6th | Indefinite/unstable | -14.1601 + 0.0000i | -1.0383 + 0.6254i | -1.0383 - 0.6254i | 0.0745 + 0.1726i | 0.0745 - 0.1726i | -0.1325 + 0.0000i | |
| | Definite/Stable | -13.6459 + 0.0000i | -1.0232 + 0.6456i | -1.0232 - 0.6456i | -0.0352 + 0.1794i | -0.0352 - 0.1794i | -0.0990 + 0.0000i | |
| 7th | Indefinite/unstable | -13.8321 + 0.0000i | -1.0606 + 0.6729i | -1.0606 - 0.6729i | -0.7487 + 0.0000i | 0.1029 + 0.2296i | 0.1029 - 0.2296i | -0.1712 + 0.0000i |
| | Definite/Stable | -7.7785 + 3.2557i | -7.7785 - 3.2557i | -0.9652 + 0.4483i | -0.9652 - 0.4483i | -1.0623 + 0.0000i | -0.1752 + 0.1014i | -0.1752 - 0.1014i |

## V. CONLUSION

Stability preserving method in ROMs for index-3 continuous time SOSs in limited frequency interval is proposed. Linearized systems with holonomic constrains exits in mechanical, multibody, and other fields of science. It has been shown that the Index-3 model is first transformed into index-0 generalized form and limited interval Gramians are computed from respective Lyapunov equations. Stability is preserved by allocating the Indefinite terms in Lyapunov equations by definite through nearest positive eigenvalues. The controllability and observability Gramians are balanced to achieve Hankel singular values based on which truncation is applied to obtain stable ROMs. The proposed technique is widely useful for finite frequency interval applications of index-3 SOSs like filter design, signal reconstruction, controller design etc.

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