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A Mathematical Model of Fluid Flows in Open Rectangular and Triangular Channels

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Abstract: -- **Flow in a closed conduit is regarded as open channel flow, if it has a free surface. This study considers the flows of an incompressible Newtonian fluid through open rectangular and triangular channels. The effects of channel slope, energy coefficient, channel top-width and roughness coefficient on velocity distribution in the open rectangular and triangular channels are investigated. The governing equations of the flows are continuity and momentum equations. The finite difference approximation method is used to solve the governing equations because of its accuracy, stability and convergence and the results are represented graphically. It is found out that the velocity of flow increases as depth increases and an increase in the channel slope, energy coefficient and top-width leads to an increase in flow velocity whereas increase in roughness coefficient leads to a decrease in flow velocity for both rectangular and triangular channels. This study goes a long way in controlling floods, construction of channels and in irrigation.**

Key words: -- **Open channel, newtonian fluid, velocity, depth, finite difference**

I. INTRODUCTION

Flood routing in rivers with wide floodplains has received much attention by the researchers during the last few decades. Rivers, the arteries of nature, are one of the world's most valuable natural resources and are important to our lives in many ways. Unexpected precipitation, along with other causes like drainage modifications of the catchment, dam failures etc., increases the quantity of water flowing in a river which often leads to flooding. During heavy rainfall water would flow from high places towards lowlands resulting in excess water stagnation due to soil saturation. Such a rainfall becomes a disaster as it is unpredicted and the right measures have not been taken. This is because handling such unexpected amounts of water is a challenge even to engineers. Therefore, designing channels that would control such an environmental disaster and divert the same water to agricultural land is very much important. The fact that the flood problem still persists, there is need to come up with a hydraulically efficient channel, that is, a channel that would carry maximum discharge at a given slope, area of flow and roughness coefficient. Such a channel would be used to drive out excess water (floods) from the affected areas. In order to forecast, control and make efficient use of rivers and open channels, measurements of depth, discharge, and velocity of the hydrodynamic flow are essential. The measurements are usually done in two ways: 1) Directly by using an instrument where the flow properties are measured. 2) Indirectly by using numerical models to predict the behaviour and properties of the flow. Use of instruments is not feasible in open channels and rivers during flood events. In numerical modelling, solutions are obtained by performing successive iterations at each step until the numerical answer satisfies all the equations. Currently, the most popular numerical modelling techniques are finite difference and finite element methods.

The channel cross-section may be closed or open at the top. The channels that have an open top are referred to as open channels while those with a closed top are referred to as closed conduits. The forces at work in open channels are the inertia, viscosity and gravity forces. [1] Earliest study was carried out by French engineer Chezy. He revealed the Chezy's formula and Chezy's constant. Chezy's formula did not provide agreeable results to the engineers. [2] However, the most generally used formula in open channel is Manning formula and this formula is highly considered and required because it takes into account the coefficient of roughness called the Manning Coefficient. [3] Chow studiedand developed velocity formulas for open channel flows. This study focuses on a water flow in open rectangular and triangular channels and also studying efficient channels thus solving the problem of floods and also meeting the irrigation demands. [4] David C. Froehlich explained the governing equations for the width and depth constrained of trapezoidal channel. Also, his study provides reference for engineers of open channels and guiding standard for the hydraulic analysis. [5] Kwanza analysed the effects of channel width, slope of the channel and lateral discharge on fluid velocity

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and channel discharge for both trapezoidal and rectangular channels. They noted that the discharge increases as the specified parameters are varied upwards and also observed that trapezoidal channels are more hydraulically efficient channels than rectangular channels. [6] Kaushal patel worked on mathematical model of open channel flow for estimating velocity distribution through different surface roughness and discharge. This helps to predict the velocity profile of open channel flow, this would be usable for a wide range of Reynolds number by varying the discharge flow rate.

The present study is on analysing the effects of parameters such as channel slope, energy coefficient, channel top width and roughness coefficient on velocity distribution in the open rectangular and triangular channelsusing MATLAB program. Investigation of variation of velocity with depth is also carried out and to investigate the more hydraulically efficient channel between open rectangular and triangular channels. This would solve the problem of flooding during heavy precipitation as well as the shortage of water for irrigation.

MATLAB is one of the latest programs used in scheming window based software. It is a high-performance language for technical computing. It incorporates computation, visualization, and programming environment. Additionally, it is a modern programming language environment which as sophisticated data structures, contains built-in editing and debugging tools, and support object-oriented programming. These reasons make it atremendous tool for teaching and research. It has powerful built-in routines that facilitateanextensive variety of computations and easy to use graphics commands that make the visualization of results instantaneously available.

II. MATHEMATICAL ANALYSIS

Governing equations

Open channel flows are analyzed using continuity and momentum equations. In this study, the flow is unsteady, non-uniform and incompressible.

2.1 For rectangular channel

The continuity equation is given by

$$\frac{\partial y}{\partial t}+\frac{A}{b}\frac{\partial V}{\partial x}+V\frac{\partial y}{\partial x}-\frac{q}{b}=0 \quad (1)$$

The momentum equation is given by

$$\frac{\partial V}{\partial t}+\alpha V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}-g\left(S_o-S_f\right)=0 \quad (2)$$

Area of cross section, $A=b\times y$

Top-width, $T=b$

Wetted perimeter, $P=b+2\times y$

Energy coefficient, α=1

2.2 For triangular channel

The continuity equation is given by

$$\frac{\partial y}{\partial t}+\frac{A}{T}\frac{\partial V}{\partial x}+V\frac{\partial y}{\partial x}-\frac{q}{T}=0 \quad (3)$$

The momentum equation is given by

$$\frac{\partial V}{\partial t}+\alpha V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}-g\left(S_o-S_f\right)+\frac{qV}{A}=0 \quad (4)$$

Where S_f and $S_o$are the friction slope and slope of the channel bottom for both the channels.

Area of cross section, $A=0.5\times T\times y$

Wetted perimeter, $P=(4\times y^2+T^2)^{1/2}$

Energy coefficient, α=1

Conditions of flow for both rectangular and triangular channels

Same initial and boundary conditions are given for both the channels.

The initial conditions are

$$V(x,0)=14,\quad y(x,0)=1.8$$

And the boundary conditions are

$$V(x_0,t)=14,\quad y(x_0,t)=1.8$$
$$V(x_n,t)=14,\quad y(x_n,t)=1.8$$

III. METHOD OF SOLUTION

The governing equations (1), (2), (3) and (4) are non-linear first order partial differential equations of hyperbolic type. It is not possible to solve these equations analytically. The finite difference method will be used to obtain approximate solutions. In this technique, the partial derivatives in the equations are replaced by their corresponding finite difference approximations which results in a series of algebraic equations that can be solved on a computer.

Finite difference equations for rectangular channel are as follows:

$$y(i,j+1)=0.5\big(y(i-1,j)+y(i+1,j)\big)-\Delta t\left\{\frac{A}{b}\frac{V(i+1,j)-V(i-1,j)}{2\Delta x}+V(i,j)\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}-\frac{q}{b}\right\} \quad (5)$$

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$$V(i,j+1) = 0.5[V(i-1,j) + V(i+1,j)] - \Delta t\left\{\propto V(i,j)\frac{V(i+1,j) - V(i-1,j)}{2\Delta x} + g\frac{y(i+1,j) - y(i-1,j)}{2\Delta x} - g\left[S_0 - \frac{n^2}{2R^{\frac{4}{3}}}(V^2(i-1,j) + (V^2(i+1,j)\right]\right\} \quad (6)$$

Equations (5) and (6) are the continuity and momentum equations of an open rectangular channel in finite difference form.

Finite difference equations for triangular channel are as follows:

$$y(i,j+1) = 0.5\big(y(i-1,j) + y(i+1,j)\big) - \Delta t\left\{\frac{A}{T}\frac{V(i+1,j) - V(i-1,j)}{2\Delta x} + V(i,j)\frac{y(i+1,j) - y(i-1,j)}{2\Delta x} - \frac{q}{T}\right\} (7)$$

$$V(i,j+1) = 0.5[V(i-1,j) + V(i+1,j)] - \Delta t\left\{\propto V(i,j)\frac{V(i+1,j) - V(i-1,j)}{2\Delta x} + g\frac{y(i+1,j) - y(i-1,j)}{2\Delta x} - g\left[S_0 - \frac{n^2}{2R^{\frac{4}{3}}}(V^2(i-1,j) + (V^2(i+1,j)\right] + \frac{qV(i,j)}{A}\right\} (8)$$

Equations (7) and (8) are the continuity and momentum equations of an open triangular channel in finite difference form.

Conditions of flow for rectangular and triangular channels in finite difference form

The initial conditions in finite difference form are

$$V(i,0) = 14, \quad y(i,0) = 1.8$$

The boundary conditions in finite difference form are

$$V(x_0,j) = 14, \quad y(x_0,j) = 1.8$$
$$V(x_n,j) = 14, \quad y(x_n,j) = 1.8$$

In these equations, i denotes the distance along the channel while j denotes time. x_0 and x_n denotes the entry point and exit point respectively of the section of the channels. A uniform mesh in which $\Delta x = 0.1$ and $\Delta t = 0.0012$ have been considered. The convergence is achieved by the condition, $\frac{\Delta y}{(\Delta t)^2} < \frac{1}{2}$.

IV. RESULTS AND DISCUSSION

The equations (5), (6), (7) and (8) were solved using MATLAB program. The effects of the various flow parameters on the flow velocity are shown graphically below from Fig. 4.1 to Fig 4.5.

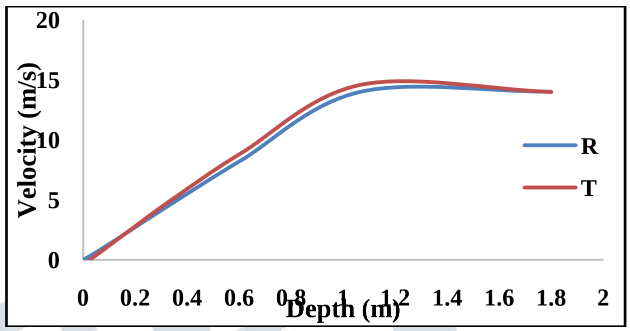

Figure 4.1: Velocity distribution for open rectangular (R) and triangular (T) channels

T=3, n=0.012, S_0=0.02, α=1, y=1

From figure 4.1, for a fixed flow depth of the channel, the velocity distribution for a triangular channel are higher than for a rectangular channel. A large wetted perimeter in rectangular channel causes high resistance to the fluid motion from the walls of the channel. Thus velocity reduces due to high resistance. A triangular channel has a less wetted perimeter compared to a rectangular channel which causes less resistance hence high flow velocities.

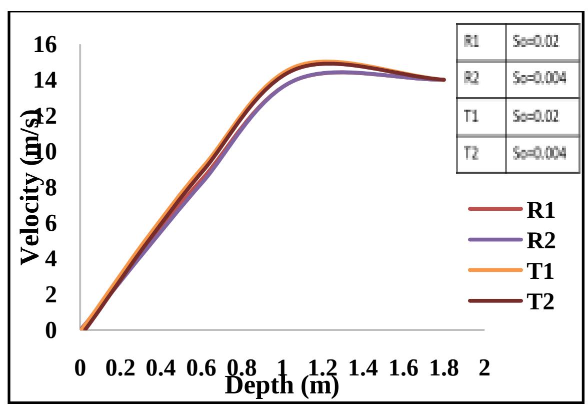

Figure 4.2: Effects of varying channel slope (S_0) on velocity

From figure 4.2, for the same depth increasing the channel slope from 0.004 to 0.02 increases the velocity of the

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flow in both channels. In other words, the velocity values when the channel slope is 0.004 are lesser than when the channel slope is 0.02.

According to Manning's velocity formula, velocity is directly proportional to the slope and therefore as the slope increases velocity also increases.

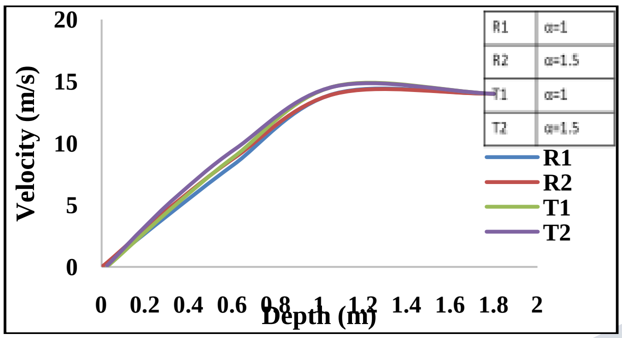

Figure 4.3: Effects of varying energy coefficient (α) on velocity

From figure 4.3, as the energy coefficient increases from 1 to 1.5 velocity increases for both the channels. According to kinetic theory of matter concept, fluid molecules possess kinetic energy. If flow energy increases, kinetic energy of the particles also increases making the particles to travel faster and hence velocity increases.

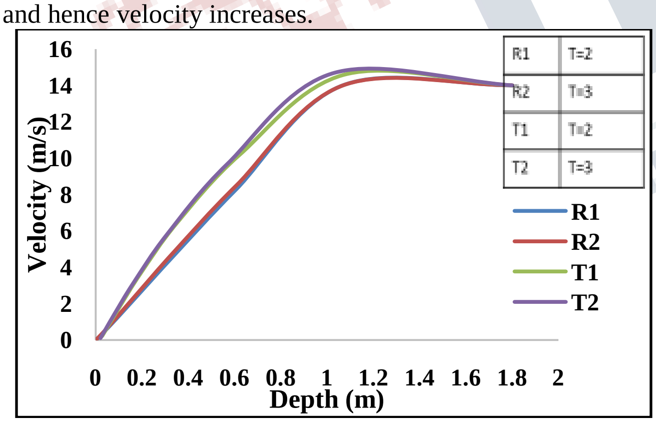

Figure 4.4: Effects of varying top-width (T) on velocity

From figure 4.4, for the same depth of flow the velocity distribution for triangular channels are higher compared to the rectangular channel when the top width is 3m and 2m and also the velocity distribution for the triangular channel when top width is 3m are greater than when top width is 2m. This is for the reason that when top width is 3m the hydraulic radius is higher than when top width is 2m. Higher hydraulic radius entails higher velocity as velocity is directly proportional to the hydraulically radius.

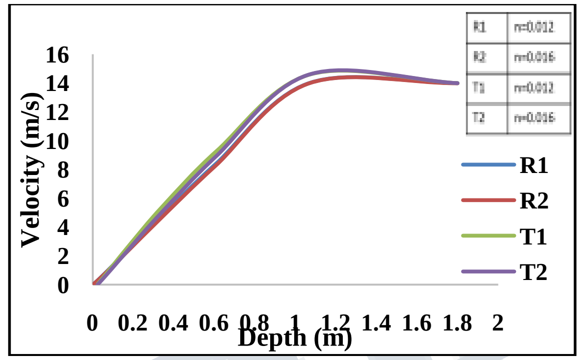

Figure 4.5: Effects of varying roughness coefficient (n) on velocity

From figure 4.5, for the same depth varying roughness coefficient the triangular channel shows higher velocities compared to rectangular channel. Furthermore for a triangular channel the velocity distribution is higher when roughness coefficient is 0.012 compared to the velocity distribution when the roughness coefficient is 0.016. An increase in roughness coefficient results to enormous shear stresses at the sides of the channel. Due to enormous shear stresses there is decrease in the flow velocity.

Likewise varying two and three parameters holding other parameters constant were studied. Results show the same trend as compared to varying one parameter.

V. VALIDATION

From this study when energy coefficient (∝) and roughness coefficient (n) are negligible, the results obtained are same as given by Kwanza et al., [5] and when all parameters compared with circular cross-sectional open channel flow the results obtained show the same trend from data given by Kinyanjui. M.N. [17].

VI. CONCLUSION

The objective of this work was to investigate the effects of the energy coefficient, top-width, roughness coefficient and channel slope on the flow velocity for rectangular and triangular channels. The various flow parameters were varied one at a time while holding the other parameters constant. The inference is that for both rectangular and triangular channels increasing energy coefficient, channel slope and top width leads to an increase in flow velocity whereas increasing roughness coefficient leads to decrease in velocity. Flow velocity increases with increase in depth and becomes maximum slightly below the free surface. For the same cross-sectional area rectangular channel results in least wetted perimeter than triangular channel. Therefore, rectangular channel is more hydraulically efficient channel compared to triangular channel. The results obtained by varying one, two and three parameters were similar. It ensures the efficiency of the program (MATLAB).

RECOMMENDATIONS

Further research should be carried out on

i) Effects of lateral inflow on discharge
ii) Effects of lateral outflow on discharge
iii) Fluid flows through elliptic channels
iv) Fluid flows in rectangular and triangular channels and solving the partial differential equations using finite element method or any other numerical technique and to compare the efficacy of different method.

NOMENCLATURE

q Lateral inflow(m^3/s/m)
L Length of the channel (m)
Q Discharge (m^3/s)
A Cross-section area of flow (m^2)
b Breadth of the channel floor (m)
So Slope of the channel bottom
S_f Friction slope
P Wetted perimeter (m)
T Top width of the free surface (m)
R Hydraulic mean depth or Hydraulic radius (m)
y Depth of the flow (m)
V Mean velocity of flow (m/s)
x Distance along the main flow direction (m)
t Time (s)
g Acceleration due to gravity (ms^{-2})
n Manning's coefficient of roughness
α Energy coefficient

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