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XFEM Simulation of Semi Permeable Crack in Piezoelectric Materials

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Abstract: **-- Wide research is being carried on to investigate effect of electromechanical coupling behavior on fracture and crack growth of piezoelectric materials. In the present paper quasi-static crack growth using semi-permeable crack boundary conditions in a piezoelectric material has been analyzed using the extended finite element method (XFEM). Combined Mechanical and Electrical loading has been applied on a pre-cracked rectangular plate made of piezoelectric material with crack at its edge and centre. Stress intensity factors have been evaluated by interaction integral approach using the asymptotic crack tip fields.**

Index Terms: **— crack growth, piezoelectric material, quasi-static, semi-permeable, XFEM**

I. INTRODUCTION

Piezoelectric materials due to their electro-mechanical coupling characteristics have found their application in actuators, transducers sensors etc. Considering their wide potential and application, piezoelectric materials have got lot of interests among researchers to investigate their behavior under different circumstances. Fracture of piezoelectric materials have been critical in their application as these materials are brittle in nature and can lead to sudden fracture without any prior sign of failure. Presence of any discontinuity in the form of crack, hole or void in these materials further complicated their behavior analysis under applied conditions.

Researchers have been keen on investigating fracture of piezoelectric materials and effect of mechanical, electrical and combined (mechanical and electrical) loading under the presence of discontinuity. Parton [12] was first to investigate fracture of piezoelectric materials and analyzed deformation of piezoelectric medium under mechanical stress. Pak [13] Derived path-independent integral of fracture mechanics with the governing equations and boundary conditions for linear piezoelectric materials. Freiman and White [6] addressed issues on brittle fracture of piezoelectric materials for smart ceramics. Mechanical-electric coupling behavior of penny shaped crack under axis symmetric loading was analyzed by Zikun [17]. Kuna [9] implemented Finite Element Method to analyze crack in piezoelectric materials and evaluated fracture parameters. Shang et al. [14] Extended electromechanical fracture mechanics and finite element techniques to three-dimensional crack configurations. Penny-shaped cracks and elliptical cracks are analyzed, subjected to combined mechanical tension and electric fields. Ou and Chen[11] explored problems on near tip stress field and stress intensity factors for piezoelectric materials.

Daining Fang et al [5] proposed two fracture criteria for piezoelectric materials. One is based on a generalized stress intensity factor related to the stress field at the tip of the crack an another is based on the crack opening displacement (COD) related to the relative displacement of crack faces.

Most of the work discussed above on piezoelectric material has been carried out using numerical methods such as Boundary Element Method, Finite Difference Method or Finite Element Method. In the recent time Belytschko and Black [2] presented a minimal re-meshing finite element method for crack growth by adding discontinuous enrichment functions to the finite element approximation to account for the presence of the crack, Later on this method was known as the Extended Finite Element Method (XFEM). This numerical method has got wider acceptability for solving problems of fracture with discontinuity such as crack, hole, and inclusions in materials. Bhattacharya et al. [3] carried out fatigue crack growth simulation of alloy/ceramic functionally graded materials (FGMs) using extended finite element method (XFEM).

This method has found its applicability to piezoelectric materials also, Bechet et al [1] presented

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an application of the extended finite element method (X-FEM) to the analysis of fracture in piezoelectric materials with impermeable crack, and new enrichment functions were developed for cracks in piezoelectric structures. Sharma et al [13] analyzed impermeable subinterface crack problem in piezoelectric by the extended finite element method (XFEM). Effects of the crack distance to the interface, the crack inclination, the poling direction, the loading conditions, the domain of the J-integral computation, etc. on the field intensity factors. Further Bhattacharya et al [4] worked on the crack propagation in piezoelectric material considering impermeable conditions stress intensity factors were evaluated for mechanical and combined (mechanical and electrical) loading.

These analyses were based on the assumptions of impermeable crack model for piezoelectric materials however there have been different approaches of semi-permeable and permeable crack of solving problems. In semi permeable crack electric lines of forces are partially allowed to pass through the crack faces, Hao and Shen et al [8] developed the model for semi permeable crack considering the opposite crack faces are as a set of parallel capacitors. Making a body cut, the occurring electric displacement component at the capacitor plates can be applied on the crack faces as distributed electrical charge density. This model has been named as Iterative Capacitor Analogy (ICA). Li S. [10] proposed permeable crack model to analyze crack growth in a piezoelectric ceramic. Both local and global energy release rates were calculated based on the permeable crack solution obtained. Wippler et al [16] investigated the influence of dielectric medium inside the crack. The accuracy of theoretical approach ‘‘capacitor analogy’’ for analyzing limited permeable cracks in piezoelectric ceramics is established by numerical methods and analytical estimations. Stress intensity factors were evaluated and it was observed that K_I is not influenced by permeability of crack medium however effect of permeability can be seen in K_{IV}

However XFEM has not been explored for analyzing semi-permeable crack. In the present work XFEM has been implemented to analyze semi-permeable crack boundary conditions. An iterative approach using has been applied for convergence of electric displacement component on crack faces and further stress intensity factors are evaluated using interaction integral, validation of results of K_{IV} for BaTio3 has been done with that of Iterative Capacitor Analogy (ICA), [16]. Further the methodology developed is applied for anlayzing quassi-static crack growth of PZT-4 pieozelectric materials

II. FORMULATION FOR SEMI PERMEABLE CRACK IN PIEZOELECTRIC MATERIALS:

Piezoelectric material constitutive relations are given by

$$\sigma_{ij} = C_{ijks}\varepsilon_{ks} - \xi_{sij}E_s \text{ and } D_i = \xi_{iks}\varepsilon_{ks} + z_{is}E_s \;\ldots.. \quad (1)$$

Where σ_{ij} is Cauchy stress tensor, D_i is the electric displacement vector. C_{ijkl}, ξ_{iks} and z_{is} are the elasticity constants, piezoelectric constants and dielectric permittivity respectively

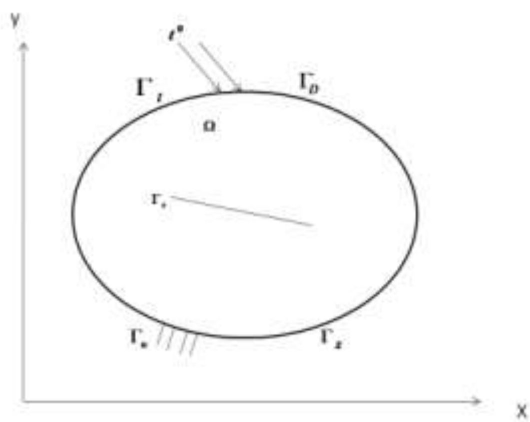

Fig.1 Physical piezoelectric domain with crack

If body forces are not present they are given as

$$\sigma_{ij,j} = 0,\; D_{i,j} = 0 \text{ on domain } \Omega \;\ldots\ldots\ldots\ldots.. \quad (2)$$

subjected to boundary condition on surface Γ

$$\sigma_{ij}n_j = t_j^0 \quad D_j n_j = \varpi^0 \quad u_j = u_j^0 \quad \chi = \chi^0 \;\ldots.. \quad (3)$$

Mechanical strain tensor ε_{ij} and electric field vector E_i are deduced as [9]

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i});\; E_i = -\chi_j \;\ldots\ldots \quad (4)$$

Where displacement vector u_i and electric potential χ_j are the field variables

Boundary conditions proposed by Hao and Shen, [8] for semi-permeable crack are as given by

$\sigma_{ij} n_j = 0 \; and \; \chi^+ = \chi^-, D_2^+ = D_2^- = D_2^c = \kappa_c \frac{\Delta\chi(x_1)}{\Delta u_2(x_1)},$
(5)

where "+" and "-" represent the upper and lower crack surfaces, $\Delta\chi(x_1)$ is the electrical potential jump, and $\Delta u_2(x_1)$ is the crack opening displacement; κ_c is the permittivity of medium between crack faces.
semi-permeable boundary conditions can change to impermeable one when $\kappa_C = 0$, and to permeable one when the jump in electric potential vanishes.

Enrichment of Crack in Piezoelectric Materials

Mechanical displacement and electric potential are derived using XFEM and given as

$$u^h(x) = \sum_{i \in t_n} S_i(x) u_i + \sum_{j \in s_n} S_j(x)(h(e^h(x)) - h(e_j))\lambda_j^c + \sum_{l \in tip_n} S_l(x) \sum_{k=1}^{6} (A_f(r,\theta,\omega_k{}^{re},\omega_k{}^{im}) - A_f(x,\omega_k{}^{re},\omega_k{}^{im}))\lambda_l^t$$
(6)

$$\chi^h(x) = \sum_{i \in t_n} S_i(x)\chi_i + \sum_{j \in s_n} S_j(x)(h(e^h(x)) - h(e_j))\gamma_j^c + \sum_{l \in tip_n} S_l(x) \sum_{k=1}^{6} (A_f^k(r,\theta,\omega_k{}^{re},\omega_k{}^{im}) - A_f^k(x,\omega_k{}^{re},\omega_k{}^{im}))\gamma_l^t$$
(7)

where S_i is the shape function associated with node i *and* A_f^k The asymptotic crack tip enrichment functions. $\lambda^c, \lambda^t, \gamma^c, \gamma^t$ are the enriched degree of freedom associated with the crack elements. $h(f)$ is heaviside function. Six fold enrichment functions are used as derived [1]

Interaction Integral

Interaction integral for two equilibrium states of cracked body is used. The first state is the actual state (1) while the second one is the auxiliary state (2). Superposition of the two gives

$J = J^{(1)} + J^{(2)} + I^{(1,2)}$ …………………(8)

$J^{(1)}$ and $J^{(2)}$ are the electromechanical J integrals for state 1 and 2 and

$$I^{(1,2)} = \int_A \left(\sigma_{ij}^{(1)} \frac{\partial u_i^{(2)}}{\partial x_1} + D_j^{(1)} \frac{\partial \chi^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)} \frac{\partial u_i^{(1)}}{\partial x_1} + D_j^{(2)} \frac{\partial \chi^{(1)}}{\partial x_1} - \tilde{Q}^{(1,2)}\delta_{1j} \right) \frac{\partial w}{\partial x_j} dA$$
(9)

in which

$$\tilde{Q}^{(1,2)} = \frac{1}{2}(\sigma_{ij}^{(1)}\varepsilon_{ij}^{(2)} - D_j^{(1)}E_j^{(2)} + \sigma_{ij}^{(2)}\varepsilon_{ij}^{(1)} - D_j^{(2)}E_j^{(1)}) \quad (10)$$

Interaction integral obtained above is used to find stress intensity factors as per [4]

Quasi-Static Crack Growth

The intensity factor obtained are used to find out equivalent stress intensity factor [5]

$$K_{eq} = A' K_I^{(1)} + B'' K_{IV}^{(1)} \quad (11)$$

Where and depends on material and can be find out as per [5]. Crack growth direction θ_c is determined on the basis on normal stress theory and is given by

$$\theta_c = 2\tan^{-1}\left(\frac{K_I^{(1)} - \sqrt{\left(K_I^{(1)}\right)^2 - 8\left(K_{II}^{(1)}\right)^2}}{4K_{II}^{(1)}} \right)$$
(12)

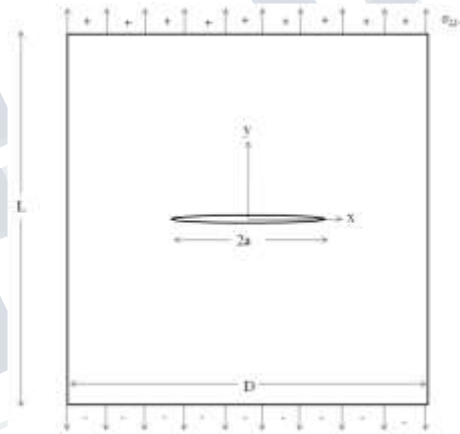

Fig.2 Griffith Crack Model for Piezoelectric Materials

III. PROBLEM DESCRIPTION, RESULT AND DISCUSSION

In the present work 2D problems of semi-permeable crack in piezoelectric materials have been analyzed. The problem is solved using the iterative procedure by initially considering the impermeable condition on the crack surfaces and then evaluating the electric displacement vector on enriched nodes D_2^c near to crack tip as per Eqn. (5).,

Electric displacement vector D_2^c determined above are then applied on enriched nodes this process is continued till the value of electric displacement vector converges D_2^c on enriched nodes. Once the value of D_2^c converges the same is applied to evaluate stress intensity factors for semi-permeable crack using interaction integral approach.

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Validation: Model developed is verified by solving Griffith model of as in Fig.2 (L=0.2m, D=0.2m, 2a=0.02m) and validating the results with those obtained by ICA (Iterative Capacitor Analogy) for limited permeable crack [16] for $BaTio_3$. Uniform mesh size of 160 nodes in X –direction and 160 nodes in Y-direction is applied so that the enriched nodes are very near to crack faces.

Table I: $BaTio_3$ material Properties [16]

Material Properties	$BaTio_3$
c_{11} (N/m^2)	16.6e+10
c_{12} (N/m^2)	7.66e+10
c_{13} (N/m^2)	7.75e+10
c_{33} (N/m^2)	11.3e+10
c_{44} (N/m^2)	4.29e+10
c_{66} (N/m^2)	3.06e+10
e_{31} (C/m^2)	-4.4
e_{33} (C/m^2)	18.6
e_{15} (C/m^2)	11.6
z_{11} C/Vm	14.343e-09
z_{33} C/Vm	16.283-09

Loading: Electrical load is kept constant ***D_2 =0.02 C/m^2*** while mechanical load ***σ_{22}*** is varied from ***20 MPa*** to ***80 MPa.*** Permittivity of free space inside the crack considered is κ_c =8.854e-12 F/m

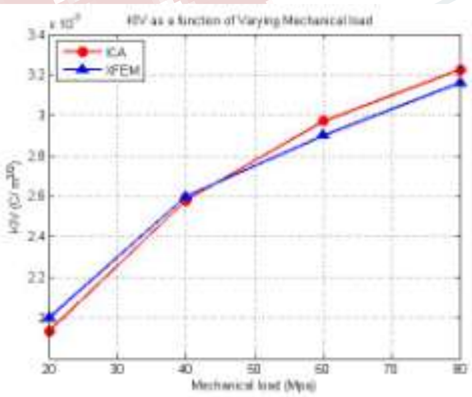

Fig.3 K_{IV} as a function of variation in mechanical load σ_{22} with constant electrical load of D_2 =0.02 C/m^2 for semi permeable crack

As shown in **Fig.3** for Griffith crack, it is observed that K_{IV} obtained using XFEM and interactional integral approach by applying D_2^c iteratively on enriched nodes is in agreement with those obtained by ICA (within ± 5% variation).

The approach validated above is further implemented to solve problem of edge cracked body and centre cracked body for semi-permeable crack growth.

Edge Crack body: quasi static crack growth for semi permeable condition

A rectangular plate of length, $L = 0.1\, m$ and height$D = 0.2\, m$, with initial crack length of $a = 0.02$ m has been considered for the analysis, XFEM together with level set method has been applied for modeling of crack without remeshing and finding out the enriched nodes. Six basis enrichment functions are used for enriched nodes. A uniform mesh of 160 nodes in X-direction and 160 nodes in Y-direction is applied. Remote loads of D_2 =0.02 C/m^2 and σ_{22}=20MPa have been applied on edge crack body. At each step crack extension of 0.01 m length has been considered for evaluating stress intensity factor. PZT-4 piezoelectric material properties [5] as in Table.II have been considered for the analysis. Permittivity of free space inside the crack considered is κ_c =8.854e-12 F/m.

Table II: Pzt-4 material Properties [5]

Material Properties	PZT4
c_{11} (N/m^2)	13.9e+10
c_{12} (N/m^2)	7.78e+10
c_{13} (N/m^2)	7.43e+10
c_{33} (N/m^2)	11.3e+10
c_{44} (N/m^2)	2.56e+10
c_{66} (N/m^2)	3.06e+10
e_{31} (C/m^2)	-6.98
e_{33} (C/m^2)	13.8
e_{15} (C/m^2)	13.4
z_{11} C/Vm	6e-09
z_{33} C/Vm	5.47e-09

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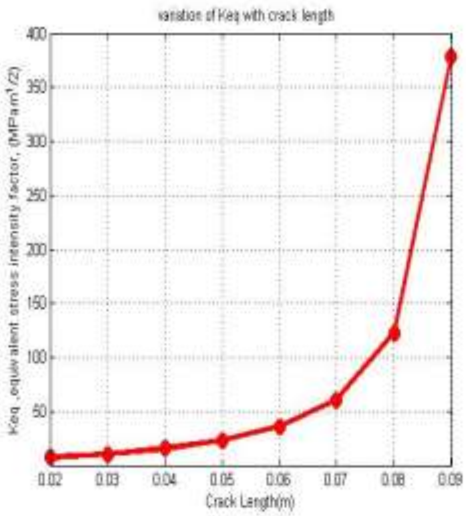

Fig. 4(a)

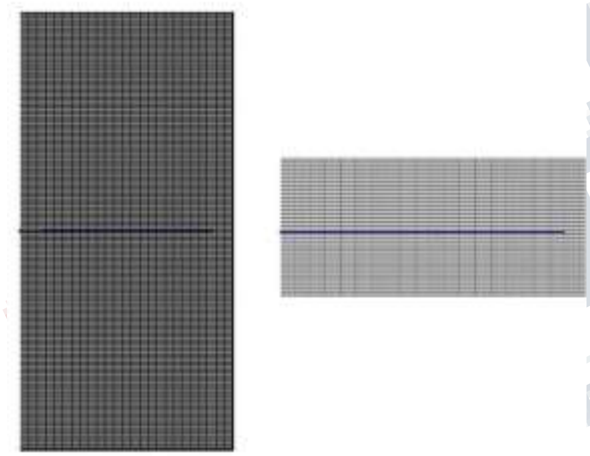

Fig. 4(b)

Fig. 4 (a) variation of Keq stress intensity factor for edge crack with crack length (b) crack path for edge crack

Fig.4 (a) shows variation of Keq with crack length for quasi static crack growth from 0.02 to 0.09 m of crack length for edge crack body for applied remote mechanical load of $\boldsymbol{\sigma_{22}=20MPa}$ and electric displacement vector $\boldsymbol{D_2 = 0.02\ C/m^2}$. Electric displacement D_2^c on enriched node is iterated for convergence and then Keq is obtained as per Eqn (5). Crack path is as shown in **Fig.4 (b)**

Centre Crack body: quasi static crack growth for semi permeable condition

A centre semi permeable crack in rectangular plate of length, $L = 0.2$m and height $D = 0.2$ m, with initial crack length of $2a = 0.04\,m$ has been considered for the analysis .A uniform mesh of 160 nodes in X-direction and 160 nodes in Y-direction is applied. Remote loads of $\boldsymbol{D_2 = 0.02\ C/m^2}$ and $\boldsymbol{\sigma_{22}=20MPa}$ have been applied on edge crack body. At each step crack extension of 0.01 m length on both side has been considered for evaluating stress intensity factor. PZT-4 material properties **Table.2** has been considered for the analysis. Permittivity of free space as mentioned earlier has been considered.

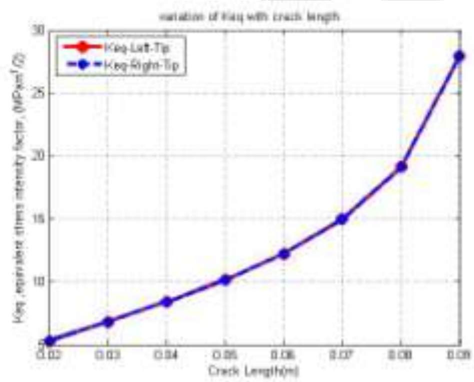

Fig. 5(a)

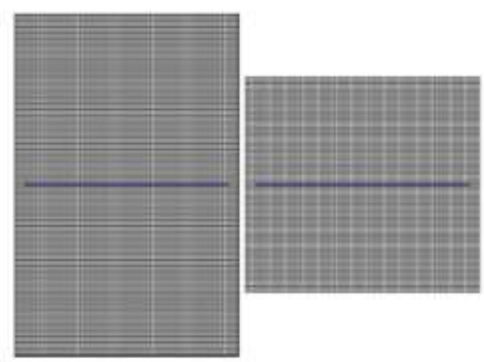

Fig. 5(b)

Fig. 5 (a) variation of Keq stress intensity factor for centre crack with crack length (b) crack path for centre crack

Variation of Keq with crack length for quasi static crack growth from 0.02 to 0.09 m of centre crack body is as shown in **Fig.5 (a)** for remote mechanical load of $\boldsymbol{\sigma_{22}=20MPa}$ and electric displacement vector $\boldsymbol{D_2 = 0.02\ C/m^2}$. Equivalent intensity factor obtained on both crack tip for centre crack has been identical. Crack path is as shown in **Fig.5 (b)**

IV. CONCLUSION

In the present work XFEM has been explored to analyze semi permeable crack for piezoelectric material. D_2^c Convergence has been obtained by iterative procedure and the validation of the results for K_{IV} has been carried out by comparing it with ICA method. Further the approach has been applied to analyze PZT-4 piezoelectric material quasi-static crack growth for edge crack and centre crack body with semi permeable crack conditions.

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