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Elastic Stress Analysis of Hyperbolic Rotating Disks via Optimal Homotopy Asymptotic Method

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Abstract— ***This study, it is aimed to use an effective approximation method, called Optimal Homotopy Asymptotic Method (OHAM) for elastic stress analysis. This study is carried out with the idea that utilizable of the considered method in many areas and having advantages will add a new perspective to the rotating disc problems that can provide convenience and practicality. The considered disk is an annular disk with both free ends and is subjected to centrifugal force. The thickness variation is assumed as hyperbolic. Deformations occurred on the disk, and radial and tangential stresses are calculated with the proposed method. The approximate solutions are in very good agreement with the exact solution. It is observed that the results converge to exact solutions faster than other approximate methods, like the Improved Adomian Decomposition method, in the literature. The results of this study show that OHAM is a very practical method with fast results for rotating disk problems. The study also shows the advantages of OHAM being directly applicable to differential equations of rotating disks without any transform function. Results by the comparison of approximate results and exact solutions indicated that OHAM can effectively be used in the analysis of rotating variable thickness disks.***

Index Terms—angular limit velocity, elastic analysis, rotating disk, OHAM.

I. INTRODUCTION

Rotating disks have a wide area of utilization in engineering applications. Due to its wide usage, it is essential to understand its behavior under different conditions. Especially, the damage that can occur on rotating disks with high speed can cause vital problems. This risk necessitates studies on this subject particularly. Studies on different conditions and geometries are of particular importance in determining the optimum disk profile while still in the design phase, as it allows instant interpretation of the structural behavior of disk profiles. Rotating disk problems can be described with differential equations like many other physical phenomena.

Scientific literature involves different methods to solve rotating disk problems, but researchers still pay great attention to the subject. Although some problems can be solved easily with classical methods ın some conditions, some complex problems caused by material properties, boundary conditions, or geometry can not be solved with known classical methods. The development of numerical methods and analytical approach methods emerged from the need to solve these problems. Perturbation techniques [1] and numerical solutions [2] were produced for solid disks. Also, some analytical approximation techniques was studied for elastic-plastic analysis [3],[4]. [5] and [6] used the Runge-Kutta algorithm for the numerical solution while using hyper-geometric and Kummer functions in their analytical solution studies of the variable-thickness hollow rotating disk problem. [7] developed a numerical computational model for elastic and partially plastic stress states in rotating solid discs of variable thickness, and they used a shooting technique using Newton iterations to solve the model. The method is based on solving the boundary value problem by transforming it into an initial value problem. [8] used an approximate method called variable material properties (VMP) to solve the situations that do not have exact solutions in variable thickness rotating discs. [9] used Homotopy Analysis Method (HAM), Adomian Decomposition Method (ADM) and Variational Iteration Method (VIM) under different boundary conditions to provide semi-analytical solutions to the elastic analysis of hollow, functionally graded discs with varying thickness from the center to the ends. [10] and [11] presented a finite element method for elastic-plastic stress solution for a functionally graded rotating disk problem and used Runge-Kutta iteration method as a numerical solution for elasto-plastic stress analysis of rotating disks and pressure vessels made of functionally graded materials. [12] evaluated the elastic deformation analysis of a functionally graded rotating disc using the finite difference method. [13] used the finite element method for the analysis of two-dimensional functionally graded rotating circular and solid discs. [14] presented semi-analytical solutions for stress analysis of functionally graded rotating annular disks with arbitrary thickness variations. [15] presented a solution with complementary functions, aiming to reach the solution directly without requiring any acceptance in functionally graded disks. Complementary functions method is one of the effective methods used in the direct solution of differential equations by reducing the boundary value problem to the initial value problem. In [15], the differential equation set was solved using the Runge Kutta 4/5 method using the real boundary conditions of the problem. [16] proposed Improved Adomian Decomposition Method (IADM) for elastic analysis of variable thickness rotating disks. Researchers'

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interest in the development of analytical approach methods is always in search of "better" solutions.

In this study, the variable thickness rotating disk problem is solved with a very effective approximation method OHAM. OHAM was produced on the basis of Homotopi Analysis Method [17]. HAM, developed in [18] using the idea of homotopy in topology, later became a method used by many authors in solving various equations. HAM gave the authors an advantage in that it does not depend on small or large parameters, unlike perturbation methods. OHAM quite resembles HAM but more flexible. It provides an opportunity to control and adjustment of the convergence region and rate of convergence with an auxiliary parameter. This method is approached for variety flow problems, heat transfer problem, vibration problem, and many other equation types to prove and control its effectiveness [19]-[23].

This study is carried out with the idea that the utilizable of OHAM in many areas and having advantages will add a new practical and time-saving solution to the rotating disc problems.

II. VARIABLE THICKNESS ROTATING DISK

An annular rotating disk is shown in Fig. 1 The government equation of variable thickness rotating disk is based on (1).

$$\frac{d}{dr}(tr\sigma_r) - t\sigma_\theta + t\rho\omega^2 r^2 = 0 \qquad (1)$$

where t is the variable thickness, σ_r is the radial stress, σ_θ is the tangential stress, ρ is the density of disk material, ν is Poisson' ratio and ω is the constant angular speed of disk.

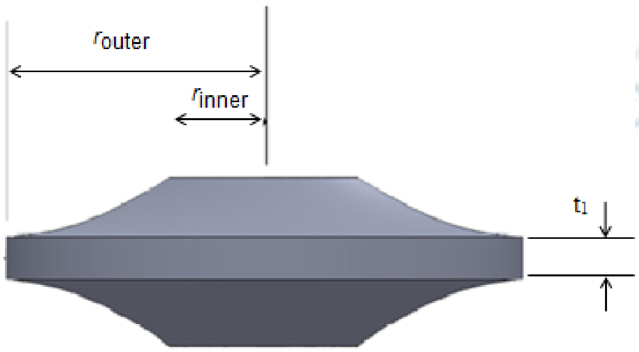

Fig. 1 Variable thickness annular disk profile

Because of the disk thickness is variable, t is a function of r as shown in (2).

$$t = t_1 r^{-s} \qquad (2)$$

t_1 is the constant thickness of disk at inner radius and s represents the geometry parameter of disk.

For elastic analysis, strain-displacement, stress-strain relations are given in (3).

$$\varepsilon_r = \frac{du}{dr} = \frac{1}{E}(\sigma_r - \nu\sigma_\theta)$$

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}(-\nu\sigma_r + \sigma_\theta) \qquad (3)$$

Elastic strains should satisfy the compatibility relation given in (4).

$$\frac{d}{dr}(r\varepsilon_\theta) - \varepsilon_r = 0 \qquad (4)$$

Defining a stress function as (5) lead to read tangential and radial stress in terms of Φ.

$$\Phi(r) = tr\sigma_r \qquad (5)$$

The radial and tangential stresses read as (6) and (7), respectively. Therefore, the strains are also rearranged according to Φ.

$$\sigma_r = \frac{\Phi}{tr} \qquad (6)$$

$$\sigma_\theta = \frac{1}{t}\frac{d\Phi}{dr} + \rho\omega^2 r^2 \qquad (7)$$

The governing equation is obtained in dimensionless form by using the non-dimensional variables as presented in (8).

$$\bar r = \frac{r}{r_{out}}, \quad \bar\sigma_{ij} = \frac{\sigma_{ij}}{\sigma_0}, \quad \bar\varepsilon_{ij} = \frac{\varepsilon_{ij}E}{\sigma_0}, \quad \bar\omega = \omega r_{out}\sqrt{\frac{\rho}{\sigma_0}} \qquad (8)$$

where σ_0 represent the yielding limit, Ɛ stands for the strains and E is the modulus of elasticity.

Substituting stress-strain relations in compatibility equation in terms of dimensionless variables produces (9) for the considered problem in this study.

$$\frac{d^2\bar\Phi}{d\bar r^2} + \left(\frac{1}{\bar r} - \frac{1}{\bar t}\frac{d\bar t}{d\bar r}\right)\frac{d\bar\Phi}{d\bar r} + \left(-\frac{1}{\bar r^2} + \frac{\nu}{\bar r}\frac{1}{\bar t}\frac{d\bar t}{d\bar r}\right)\bar\Phi = -(3+\nu)\bar\omega^2\bar r\bar t \qquad (9)$$

This equation is solved according to annular disk boundary conditions as $\sigma_r(rin)=0$ and $\sigma_r(rout)=0$.

III. SOLUTION PROCEDURE OF OPTIMAL HOMOTOPI ASYMPTOTIC METHOD

The first step before applying the OHAM, the considered differantial equation is written in the form of (10).

$$L(u(x)) + g(x) + N(u(x)) = 0, \qquad B(u, \frac{du}{dx}) = 0 \qquad (10)$$

where L is a linear and N is a nonlinear operator, L(u(x)) and N(u(x)) are linear and nonlinear part of the equation. x denotes independent variable, u(x) is taken as an unknown function, g(x) is considered as a known function and B is a boundary operator. Having the freedom to choose the linear part is one of the most advantageous sides of the considered method.

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The second step is to construct a family of equation in accordance with the OHAM as in (11).

$$(1-p)[L(\varphi(x;p))+g(x)] = H(x,p)[L(\varphi(x;p))+g(x)+N(\varphi(x;p))] \quad (11)$$

p is embedding parameter that varies from 0 to 1. H(x,p) is an auxiliary function that is nonzero for p≠0 and it is equal to zero for p=0. $\varphi(x;p)$ is unknown function that its solution varies from $u_0(x)$ to $u(x)$ while p varies from 0 to 1, respectively.

$u_0(x)$ is the zeroth order problem, obtained from the constructed homotopy equation in (11) for p=0. The auxiliary function H(x,p) involves arbitrary constants showed c_n.

$\varphi(x;p)$ can be expanded in Taylor's series about p to get an approximate solution, as (12).

$$\varphi(x;p,c_i) = u_0(x) + \sum_{k=1}^{\infty} u_k(x;c_i)p^k, \quad \text{i=1,2,3..} \quad (12)$$

Substituting the value of $\varphi(x;p)$ into (11) the solutions for each approximation steps can be obtained as shown in (13).

$$L(u_0(x))\text{+g(x))=0}, \quad B\left(u_0, \frac{du_0}{dx}\right) = 0$$

$$L(u_1(x)) = c_1 N_0(u_0(x)), \quad B\left(u_1, \frac{du_1}{dx}\right) = 0 \quad (13)$$

$$L(u_k(x)) - L(u_{k-1}(x)) = c_k N_0(u_0(x)) + \sum_{i=1}^{k-1} c_i [L(u_{k-i}(x)) + N_{k-i}(u_0(x), u_1(x), ..., u_{k-i}(x))],$$

The convergence of the series $\varphi(x;p)$ depends upon the auxiliary constats c_n. If it is convergent at p=1, the solution is obtained as (14).

$$\tilde{u}(x;c_i) = u_0(x) + \sum_{k\geq 1} u_k(x;c_i), \quad \text{i=1,2,3..} \quad (14)$$

By substituting (14) into (10), it results the residual function in (15).

$$R(x,c_i) = L(\tilde{u}(x,c_i)) + g(x) + N(\tilde{u}(x,c_i)), \quad \text{i=1,2,3..} \quad (15)$$

If $R(x,c_i)=0$, then the solution is going to be exact. To determine the auxiliary constants the least square method can be used,

$$J(x,c_i) = \int_a^b R^2(x,c_i)dx, \quad (16)$$

where a and b are constant values depending on the considered problem. According to the least square method, for the optimization of equation (16), the unknown constants c_n's can be identified.

$$\frac{\partial J}{\partial c_1} = \frac{\partial J}{\partial c_2} = ... = \frac{\partial J}{\partial c_m} = 0 \quad (17)$$

OHAM requires solving a series of nonlinear algebraic equations for unknown convergence control parameters c_n.

IV. NUMERICAL RESULTS

An annular rotating disk having hyperbolic profile that inner radius is 0.05m and the outer radius is 0.25 m is considered. The material properties are taken as 200 Gpa of elasticity modulus and 0.3 of Poisson's ratio. In fact, the solutions are expressed nondimensional form by taking $\rho\omega^2$ as unity and all equations as non-dimensional.

By taking consideration (10), linear and nonlinear operators are choosen as (18).

$$L(\varphi;p) = \frac{d^2\varphi}{dr^2} + \frac{3-s}{r}\frac{d\varphi}{dr}$$

$$N(\varphi;p) = -\frac{s(1+\nu)}{r^2} \quad (18)$$

Approximation solutions are calculated according to (13).

Zeroth order and first order approximate equations are shown in (19) openly.

$$\frac{d^2u_0(r)}{dr^2} = \frac{3-s}{r}\frac{du_0(r)}{dr} + g(r)$$

$$\frac{d^2u_1(r)}{dr^2} = -\frac{3-s}{r}\frac{du_1(r)}{dr} + c_1\left(-\frac{s(1+\nu)}{r^2}u_0(r)\right) \quad (19)$$

Numerical results for s=1 with different order of approximate solutions obtained using OHAM are compared with exact solutions in following figures.

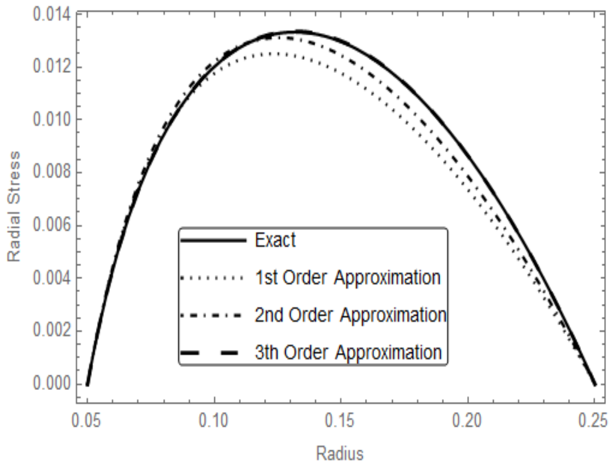

Fig 2. Comparison of radial stress results

Radial stress diagrams in Fig. 2 show that the results obtained with OHAM are getting closer to exact results when the order of approximation increases. An overlapping graph is obtained with the exact solution graph for the third order approximation solution without any deviations.

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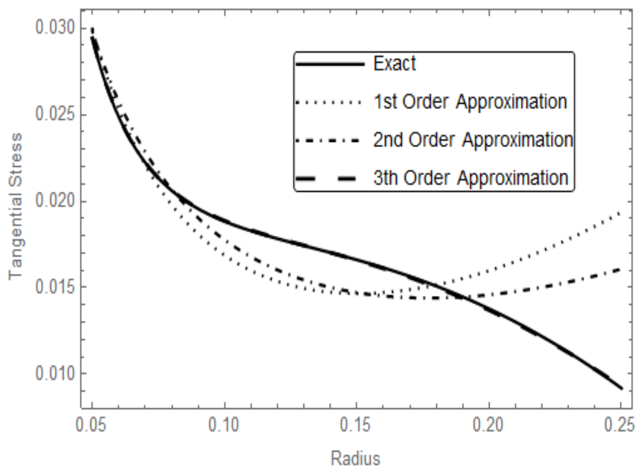

Fig 3. Comparison of tangential stress results

When the tangential graphs in Fig. 3 are taking into consideration, it is seen the third approximation result is in very good agreement with exact solution.

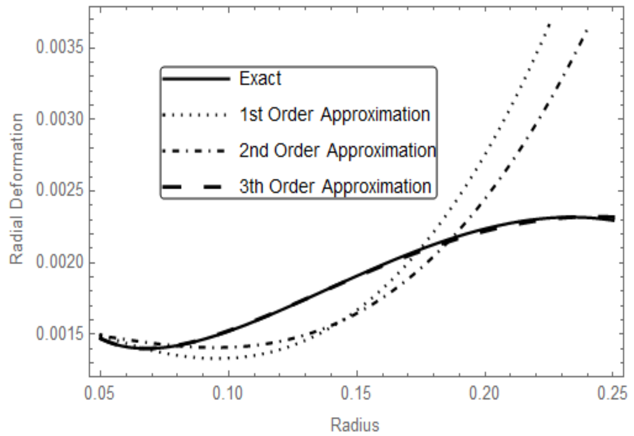

Fig 4. Comparison of radial displacement results

The radial displacement results seen in Fig. 4 are satisfactory enough, like radial and tangential stresses. Although the first two results do not behave the exact solution, the third approximation is in excellent agreement with exact.

V. CONCLUSION

Elastic stress and displacement analysis for rotating hyperbolic disks with constant angular velocity using OHAM is conducted in this work. This paper reveals that OHAM is a very strong method for solving variable thickness rotating annular disk problems and gives us a satisfactory precision solution as compared to other approximatin methods that avaliable in literature. In this study, third order solution was obtained by calculating c1, c2, and c3. The presence of c_n control parameters in OHAM simply controls and adjusts the convergence of the serial solution. Also, the OHAM can be applied to governing differential equation without any transform function. This method is quite simple in applicability, as it does not require discretization like numerical methods. OHAM has been developed to guarantee precision at all times. Flexibility in the auxiliary function facilitates this. Due to being suitable for leading equation of rotating disks, proposed method can be successfully used in the analysis of rotating variable thickness disks. The development of rotating disk equipment is possible by mathematically modeling and solving how it will behave under the conditions it will be exposed to. Most of the methods proposed in the literature for the solution of such problems refer to some assumptions or transformations in the mathematical model of the physical problem. The proposed method for the solution of rotating discs within the scope of this study can be applied directly without requiring any assumptions or linearization, thus allowing the closest solutions to the real behavior to be obtained. This study also shows that it is possible to offer solutions to disk problems in the closest and broadest framework with fewer steps. This study also contribute to the usage of the advanced mathematics applications in engineering fields.

REFERENCES

[1] L. H. You and J. J. Zhang, “Elastic-plastic stresses in a rotating solid disk,” Int. J. Mech. Sci., vol. 41, no. 3, pp. 269–282, Mar. 1999.

[2] L. H. You, S. Y. Long, and J. J. Zhang, “Perturbation solution of rotating solid disks with nonlinear strain-hardening,” Mech. Res. Commun., vol. 24, no. 6, 1997.

[3] M. H. Hojjati and S. Jafari, “Semi-exact solution of elastic non-uniform thickness and density rotating disks by homotopy perturbation and Adomian’s decomposition methods. Part I: Elastic solution,” Int. J. Press. Vessel. Pip., vol. 85, no. 12, 2008.

[4] M. H. Hojjati and S. Jafari, “Semi-exact solution of non-uniform thickness and density rotating disks. Part II: Elastic strain hardening solution,” Int. J. Press. Vessel. Pip., vol. 86, no. 5, pp. 307–318, May 2009.

[5] A. M. Zenkour, K. A. Elsibai, and D. S. Mashat, “Elastic and viscoelastic solutions to rotating functionally graded hollow and solid cylinders,” Appl. Math. Mech. (English Ed., vol. 29, no. 12, pp. 1601–1616, December 2008.

[6] A. M. Zenkour and D. S. Mashat, “Stress Function of a Rotating Variable-Thickness Annular Disk Using Exact and Numerical Methods,” Engineering, vol. 03, no. 04, pp. 422–430, 2011.

[7] A. N. Eraslan, T. Apatay and M.Gülgeç, “Homojen olmayan malzemeden yapılmış içi dolu dönen disklerin elastik-plastik gerilme analizi ”, J.Fac.Eng. Arch. Gazi Univ., vol. 23, no. 03, pp.627-635, 2008.

[8] M. H. Hojjati and A. Hassani, “Theoretical and numerical analyses of rotating discs of non-uniform thickness and density,” Int. J. Press. Vessel. Pip., vol. 85, no. 10, pp. 694–700, October 2008.

[9] A. Hassani, M. H. Hojjati, G. Farrahi, and R. A. Alashti, “Semi-exact elastic solutions for thermo-mechanical analysis of functionally graded rotating disks,” Compos. Struct., vol. 93, no. 12, 2011.

[10] A. T. Kalali, B. Hassani, and S. Hadidi-Moud, “Elastic-plastic analysis of pressure vessels and rotating disks made of functionally graded materials using the isogeometric approach,” J. Theor. Appl. Mech., vol. 54, no. 1, 2016.

[11] A. T. Kalali, S. H. Moud, and B. Hassan, “Elasto-plastic stress analysis in rotating disks and pressure vessels made of functionally graded materials,” Lat. Am. J. Solids Struct., vol. 13, no. 5, pp. 819–834, 2016.

[12] M. H. Jalali and B. Shahriari, “Elastic Stress Analysis of Rotating Functionally Graded Annular Disk of Variable Thickness Using Finite Difference Method,” Math. Probl. Eng., vol. 2018, pp. 1–11, 2018.

[13] H. Zafarmand and B. Hassani, “Analysis of two-dimensional functionally graded rotating thick disks with variable thickness,” Acta Mech., vol. 225, no. 2, pp. 453–464, February 2014.

[14] M. N. M. Allam, R. Tantawy, and A. M. Zenkour, “Thermoelastic stresses in functionally graded rotating annular disks with variable thickness,” J. Theor. Appl. Mech., vol. 56, no. 4, 2018.

[15] C. Boğa, V. Yıldırım, and M. Jeddinia, “Tamamlayıcı fonksiyonlar yöntemi ile fonksiyonel derecelendirilmiş malzemeden yapılmış değişken kesitli dönen disklerde elastik gerilme-şekil değiştirme analizi,” Internatıonal Partıcıpatıon III. Aegean Composıte Materıals Symposıum, November 2015.

[16] S. Mert Kutsal and S. B. Coşkun, “Deformation Analysis of Variable Thickness Rotating Disks Using an Improved Adomian Decomposition Technique,” Int. J. Appl. Mech., vol. 12, no. 1, 2020.

[17] V. Marinca and N. Herişanu, “Application of Optimal Homotopy Asymptotic Method for solving nonlinear equations arising in heat transfer,” Int. Commun. Heat Mass Transf., vol. 35, no. 6, pp. 710–715, July 2008.

[18] S.J. Liao, “The proposed homotopy analysis techniques for the solution of nonlinear problems,” Ph.D.dissertation, Shanghai Jiao Tong University, Shanghai, China, 1992.

[19] V. Marinca, N. Herişanu, C. Bota, and B. Marinca, “An optimal homotopy asymptotic method applied to the steady flow of a fourth-grade fluid past a porous plate,” Appl. Math. Lett., vol. 22, no. 2, 2009.

[20] V. Marinca, N. Herişanu, and I. Nemeş, “Optimal homotopy asymptotic method with application to thin film flow,” Cent. Eur. J. Phys., vol. 6, no. 3, 2008.

[21] V. Marinca, N. Herisanu, and I. Nemes, “A new analytic approach to nonlinear vibration of an electrical machine,” Proceedings of the RomanianAcademy, Vol. 9, pp. 229–236, 2008.

[22] V. Marinca, N. Herişanu, and T. Marinca, “Approximate Solutions to a Cantilever Beam Using Optimal Homotopy Asymptotic Method”, Applied Mechanics and Materials, Vol. 430, pp. 22-26, 2013

[23] M. R. Mufti, M. I. Qureshi, S. Alkhalaf, and S. Iqbal, “An Algorithm: Optimal Homotopy Asymptotic Method for Solutions of Systems of Second-Order Boundary Value Problems,” Math. Probl. Eng., vol. 2017, 2017.