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Reliability Analysis of Frames

[1] Karthik C B, [2] Amit Kumar Onkar, [3]Manjuprasad M, [4]Dinesh S V
[1]Post Graduate Student, [2] Senior Scientist, [3] Cheif Scientist, [4] Professor and Head
[2][3] Structural Technological Division, CSIR-National Aerospace Laboratories, Bengaluru.
[1][4] Dept of Civil Engg, Siddaganga Institute of Technology, Tumakuru.

Abstract: **In this paper, reliability analysis of framed structures are considered. Here, the uncertainties in geometry, loads and strength are considered with required distributions. The performance functions for bending stress, shear stress and deflection are derived from the finite element analysis. The performance functions are studied using Rackwitz-Fiessler algorithm and Hasofer-Lind reliability index is determined. A MATLAB program is developed for computing reliability index by using performance function and the statistical data.**

***Index terms*— Reliability Analysis; Rackwitz-Fiessler Algorithm; Reliability Index; Probability of failure.**

I. INTRODUCTION

Structural reliability analysis plays an important role in the analysis and design of structures. The main purpose of structural reliability analysis is due to many sources of uncertainty in the structural design. This may be due to uncertainties in material parameters or mathematical modeling. The parameters of the load and load carrying capacity of the structures are not deterministic quantities. Every engineering structure must satisfy the safety and serviceability requirement under the service load over it, which means it must be reliable against collapse and serviceability, such as excessive deflection and cracking. A typical structural reliability analysis deals with models that are mathematical idealizations of the physical processes. The idealization requires the definition of basic variables describing the geometry, loads and material properties. In the present work, reliability analysis is carried out by considering two example problems namely a simply supported beam and a portal frame. Reliability analysis is performed using Hasofer-Lind Reliability Index method in conjunction with Rackwitz-Fiessler Algorithm (AFOSM method) and the results are compared with those obtained using Monte-Carlo simulations.

II. RELIABILITY ANALYSIS

Reliability

Reliability is the probability of a structure performing its intended function adequately for the period intended under the given operation conditions. Reliability can be taken as the probability of endurance and is equal to one minus the probability of failure (1-P_f). Let the resistance of the structure be R and the load on the structure to be S. The structure is supposed to fail when R is less than S and its probability of failure is specified as

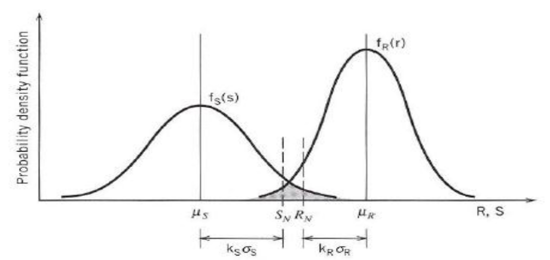

Fig 1. PDF of R and S[1]

$$\mu_Z = \mu_R - \mu_S \quad (1)$$

$$\sigma^2_Z = \sigma^2_R - \sigma^2_S \quad (2)$$

$$\beta = \mu_Z / \sigma_Z \quad (3)$$

Hasofer-Lind Reliability Index

Hasofer and Lind proposed a new approach which linearizes the LSF about a point which lies on the failure surface and corresponds to the maximum likelihood of failure occurrences. In this method the reliability is measured through the Hasofer–Lind safety index, which is defined as the minimum distance from the origin to the failure surface in the reduced (standard normal) co-ordinate system. The limitation of this method is that it is applicable only when the random variables are normal.

Hasofer-Lind Reliability Index and Rackwitz-Fiessler method

Rackwitz and Fiessler extended the Hasofer and Lind approach to include the distribution information of the random variables. Compared to other optimization algorithms, this algorithm requires the least amount of storage and a lower number of computations. For most

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situations this method not only converges, but also converges faster. However, under certain conditions the method may fail to converge and when the limit state function is highly complex and non-linear, it may converge slowly or even result in divergence. The algorithm is as follows

1. Formulate the limit state function. Determine the probability distributions and appropriate parameters for all random variables X_i (i=1, 2,…, n) involved.
2. Obtain an initial design point $\{x_i^*\}$ by assuming values for n-1 of the random variables X_i. (Mean values are often a reasonable choice.) solve the limit state function g=0 for the remaining random variable. This ensures that the design point is on the failure boundary.
3. For each of the design point values x_i^* corresponding to a non-normal distribution, determine the equivalent normal mean $\mu_{X_i}^C$ and standard deviation $\sigma_{X_i}^C$ using the below equations

$$\mu_X^C = x^* - \sigma_X^C[\varphi^{-1}(F_X(x^*))] \quad (4)$$

$$\sigma_X^C = \frac{1}{f_X(x^*)}\varphi[\varphi^{-1}(F_X(x^*))]$$

4. Determine the reduced variables $\{z_i^*\}$ corresponding to the design point.

$$z_i^* = \frac{x_i^* - \mu_{X_i}^C}{\sigma_{X_i}^C} \quad (5)$$

5. Determine the partial derivatives of the limit state function which respect to the reduced variables. For convenience define the column vector {G} as the vector whose elements are these partial derivatives multiplied by -1; i.e.,

$$\{G\} = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{bmatrix}, \text{ where } G_i = -\frac{\partial g}{\partial z_i} \quad (6)$$

6. Calculate an estimate of β using the following formula.

$$\beta = \frac{\{G\}^T\{z^*\}}{\sqrt{\{G\}^T\{G\}}}, \text{ where } \{z^*\} = \begin{bmatrix} z_1^* \\ z_2^* \\ \vdots \\ z_n^* \end{bmatrix} \quad (7)$$

7. Calculate a column vector containing the sensitivity factors using

$$\alpha = \frac{\{G\}}{\sqrt{\{G\}^T\{G\}}}$$

8. Determine a new design point in reduced variables for n-1 of the variables using

$$z_i^* = \alpha_i \beta \quad (8)$$

9. Determine the corresponding design point values in original coordinates for the n-1 values in step 7 using

$$x_i^* = \mu_{X_i}^C + z_i^* \sigma_{X_i}^C \quad (9)$$

10. Determine the value of the remaining random variables (i.e., the one not found in step 8 and 9) by solving the limit state function g=0.

11. Repeat step 3-10 until β and the design point $\{x_i^*\}$ converge.

III. PERFORMANCE FUNCTION

Assumptions

To define the limit state function, the following assumptions are made

1. Applied load to structure consists of live and dead load.
2. Applied steel is st_{37}.
3. Failure probability is investigated in elastic region.
4. Structure with compact section is I-shape.

Model for analysis

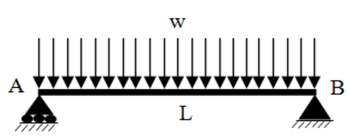

Fig 2. Beam Model

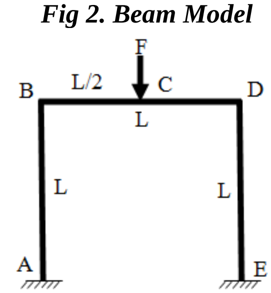

Fig 3. Frame Model

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Performance function for beam

1. For bending stress mode

$$g(M) = F_y - \frac{wL^2}{8.S_x} \quad (10)$$

F_y: Yield stress , L:Length of the member, w:Load, S_x: Section Modulus

2. For shear stress mode

$$g(v) = \frac{F_y}{\sqrt{3}} - \frac{wL}{2ht_w} \quad (11)$$

h: Height of the beam, t_w: Thickness of the web

3. For deflection mode

$$g(v) = \frac{L}{240} - \frac{5wL^4}{384EI} \quad (12)$$

E: Modulus of Elasticity,

Performance function for frame.

From fig 3, the structure and the loading conditions are symmetrical hence limit state equation for element AB is same for element DE and limit state equation for element BC is same for element CD.

1. For bending stress mode

For element AB

$$g(M) = F_y - \frac{FL(3I+2AL^2)}{24(3I+AL^2)S_x} \quad (13)$$

For element BC

$$g(M) = F_y - \frac{FL(15I+4AL^2)}{24(3I+AL^2)S_x} \quad (14)$$

2. For shear stress mode

For element AB

$$g(v) = \frac{F_y}{\sqrt{3}} - \frac{FAL^2}{8(3EI+AL^2)ht_w} \quad (15)$$

For element BC

$$g(v) = \frac{F_y}{\sqrt{3}} - \frac{F}{2ht_w} \quad (16)$$

3. For Deflection mode

For element AB

$$g(v) = \frac{L}{240} - \frac{FL^3(3I-(1/3)AL^2)}{108(3EI+AL^2)EI} \quad (17)$$

For element BC

$$g(v) = \frac{L}{240} - \frac{FL(105AIL^2+288I^2+2A^2L^4)}{192(3I+AL^2)AEI} \quad (18)$$

IV. RELIABILITY ANALYSIS.

The reliability analysis is carried out based on the random variables shown in table below.

Table 1 Statistical data of variables.

Variable	Unit	PDF	Mean	COV
W	kg/cm	Gumble	20	0.12
L	cm	Log-Normal	550	0.07
S_x	cm^3	Log-Normal	904	0.05
E	kg/cm^2	Gumble	$2.03*10^6$	0.1
f_y	kg/cm^2	Log-Normal	2400	0.1
I_x	cm^4	Gumble	16270	0.05
h	cm	Log-Normal	36	0.05
t_w	cm	Normal	0.8	0.05
A	cm^2	Deterministic	33.11	

Reliability Analysis of beam

In the below table the reliability indices of beam under various failure modes are shown.

Table 2 Failure probability of beam using AFOSM method

Failure Mode	Reliability Index	Failure Probability
Bending Stress	4.86	$2.89*10^{-6}$
Displacement	4.58	$1.11*10^{-5}$
Shear Stress	8.29	$4.49*10^{-16}$

Table 3 Failure probability of beam using MCS

Failure Mode	Reliability Index	Failure Probability
Bending Stress	4.95	$3.37*10^{-7}$
Displacement	5.21	$9.44*10^{-8}$
Shear Stress	8.40	$2.23*10^{-17}$

Reliability analysis of beam is also performed for different magnitudes of load under different failure modes using AFOSM method and shown in Tables 4 -6.

Table 4 Bending Stress failure mode

W	β	Pf	Direction Cosines			
			α_{Sx}	α_L	α_w	α_{Fy}
20	4.86	$2.89*10^{-6}$	-0.24	0.53	0.44	-0.67

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25	3.83	0.0002	-0.24	0.57	0.47	-0.62
30	2.96	0.0049	-0.24	0.59	0.49	-0.57
35	2.22	0.0334	-0.24	0.61	0.51	-0.54
40	1.58	0.1129	-0.24	0.62	0.52	-0.52
45	1.03	0.2345	-0.23	0.63	0.54	-0.49
50	0.53	0.3455	-0.23	0.63	0.55	-0.48

Table 5 Shear Stress failure mode

W	B	Pf	Direction Cosines				
			α_h	α_{tw}	α_w	α_{Fy}	α_L
20	8.29	$4.49*10^{-16}$	-0.09	-0.09	0.19	0.12	-0.96
25	7.81	$2.22*10^{-14}$	-0.13	-0.12	0.24	0.15	-0.93
30	7.31	$9.66*10^{-13}$	-0.16	-0.99	0.29	0.19	-0.90
35	6.81	$3.33*10^{-11}$	-0.18	-0.18	0.33	0.22	-0.87
40	6.31	$8.56*10^{-10}$	-0.20	-0.20	0.37	0.24	-0.84
45	5.83	$1.58*10^{-8}$	-0.22	-0.22	0.40	0.26	-0.81
50	5.37	$2.10*10^{-7}$	-0.24	-0.24	0.43	0.28	-0.78

Table 6 Deflection Mode of Failure

W	B	Pf	Direction Cosines			
			α_I	α_E	α_w	α_L
20	4.58	$1.11*10^{-5}$	-0.21	-0.56	0.40	0.68
25	3.69	$4.40*10^{-4}$	-0.21	-0.51	0.41	0.71
30	2.95	0.005	-0.21	-0.48	0.42	0.73
35	2.34	0.025	-0.21	-0.46	0.43	0.74
40	1.81	0.077	-0.20	-0.44	0.43	0.75
45	1.35	0.159	-0.20	-0.42	0.44	0.76
50	0.94	0.254	-0.20	-0.41	0.44	0.76

Reliability Analysis of Frame

The reliability indices of frame element AB under various failure modes are given in Tables 7 and 8.

Table 7 Failure Probability for element AB using AFOSM method

Failure Mode	Reliability Index	Failure Probability
Bending Stress	7.08	$7.20x10^{-13}$
Displacement	8.93	$2.13x10^{-19}$
Shear Stress	9.59	$4.40x10^{-22}$

Table 8 Failure Probability for element AB using MCS

Failure Mode	Reliability Index	Failure Probability
Bending Stress	7.17	$3.74x10^{-13}$
Displacement	8.97	$1.48x10^{-19}$
Shear Stress	9.61	$3.62x10^{-22}$

Similarly, reliability indices of frame element BC under various failure modes are given in Tables 9 and 10

Table 9 Failure Probability for element BC using AFOSM method

Failure Mode	Reliability Index	Failure Probability
Bending Stress	4.17	$1.5x10^{-5}$
Displacement	6.50	$4.01x10^{-11}$
Shear Stress	8.36	$3.13x10^{-17}$

Table 10 Failure Probability for element BC using MCS

Failure Mode	Reliability Index	Failure Probability
Bending Stress	4.22	$1.22x10^{-5}$
Displacement	6.58	$2.35x10^{-11}$
Shear Stress	8.41	$2.05x10^{-17}$

RESULTS.

Based on statistical features of random variables, failure probability for specified modes of beam is summarized as follows. Results of failure probability for assumed beam is negligible. According to Table 2, the highest failure probability corresponds to displacement and the lowest failure probability is achieved in shearing stress. In Tables 4-6, the probability of beam for the failure modes are compared. As shown in the table, the failure probability of displacement is more than bending stress for load less than 30 kg/cm. However, for load more than 30 kg/cm failure mode in bending stress is more critical. For the frame considered, the element AB is safer than the element BC. This is because load is directly acting on the member BC. Both elements are safer for shear compared to bending and deflection.

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