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Buckling Analysis of Non-Prismatic Column

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Abstract: **-- The constraints on the construction of various irregular shape are reduced remarkably with the advancement in manufacturing processes. The precise variation in the shape of the structure, in the form of any mathematical function, along the length is possible and can be achieved with help of three-dimensional printing technique. If 3-D printing technique is implemented to produce structural member, higher specific strength and stiffness can be achieved, with the same volume of material, by varying material distribution along the length of the member. In view of this, an attempt is made herein to investigate the critical load of a solid column due to the variation of shape in terms of a mathematical function. The present investigation is limited to the computation and comparison of critical buckling load of the solid clamped-pinned column with a linear, trigonometric and exponential variation of shape along the length by employing commercially available FE package ABAQUS®. Detailed FE analysis is carried out and results are compared and discussed for assumed variation.**

Keywords: **- Buckling of columns, FE analysis, ABAQUS, Irregular shape.**

I. INTRODUCTION

There exists a remarkable gap between the stress value of long compression member capable to withstand and stress up to which it can be loaded till it loses its stability [1-2]. The failure of a long compression member is governed by Euler's formula and on the basis of this theory one can say that Euler's load can be increased either by increasing moment of inertia or by employing the material with larger value of modulus of elasticity [2-3]. This research paper aims to investigate the possibility in increase of Euler's load by variation of moment of inertia i.e. removing material from over stiff location and distributing it to the weakest location such that total volume remains same. In attempt to do so, critical stress on the basis of Euler's formula vs. slenderness ratio for different cross-section for a given volume of 7600 cm3 and length equals to 400 cm along with the yield stress for different grade of steel used in India are reported in Fig. 1. It is evident from the Fig.1, that circular section is least effective as gap between the Euler's critical stress and the yield stress for different grade of steel is largest. So, in this investigation, Euler's load for different non-prismatic circular steel column with fixed volume of 7600 cm3 and length equals to 400 cm are computed, for clamped-pin boundary condition, using ABAQUS® and compared to find out which shape will give maximum increment in buckling load. This investigation explores the possibility of increment in Euler's load for biaxial symmetric section only, excluding the possibility of torsional or flexural- torsional buckling.

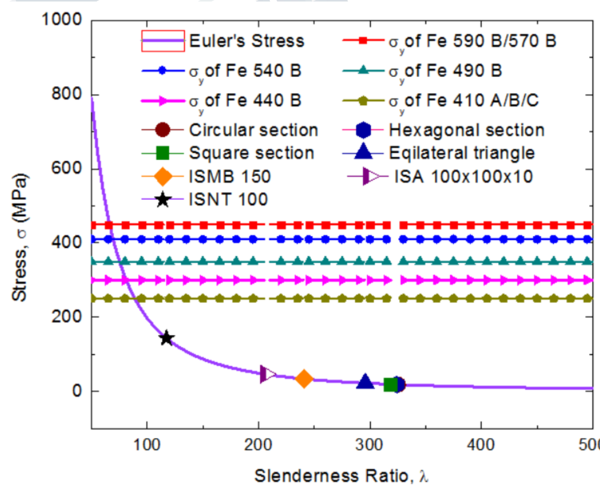

Fig. 1: Variation of Euler's critical stress with slenderness ratio for different grades of steel

II. COLUMN DESIGN

For present analysis, six different configurations of steel columns with circular cross-section are used. Fig. 2 shows different shapes of circular cross-section used in the present investigation. The dimension of these shapes is varied while maintaining the constant volume of 7600 cm3 with length equals to 400 cm, to find out the column which will result in

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maximum increment in Euler's load. It is to be noted that size of the column is chosen based on their application in automotive and aerospace industries. However, similar analysis can be carried out for columns used in buildings.

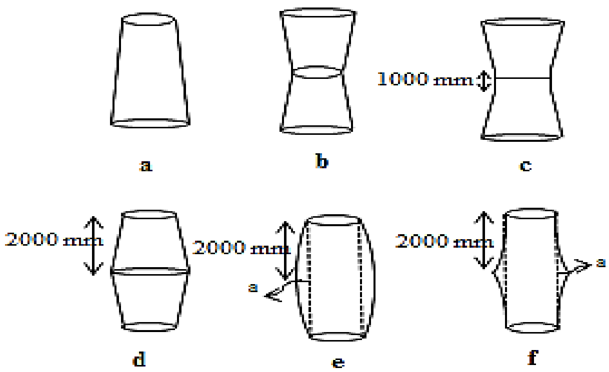

Fig. 2: Column shape (a) uniform taper throughout the length (b) taper with maximum diameter at ends (c) taper with maximum diameter at ends and uniform diameter on either side of centre (d) taper with maximum diameter at centre (e) sinusoidal variation, and (f) with exponential variation

III. FINITE ELEMENT MODELLING

The columns with different shapes, having one end clamped and other end pinned, is modelled using deformable, revolution feature available in ABAQUS® as shown in Fig. 3. Steel is used as material for column having density 7800 kg/m3, modulus of elasticity as 210 GPa, and Poisson's ratio as 0.3. Linear perturbation buckling analysis is carried out to compute the Euler's load for column by discretising column with C3D20 continuum elements available in ABAQUS®. Further, to avoid the effect of mesh on the final results, mesh size is varied by varying seeding i.e. 0.1, 0.05, 0.02, 0.015, 0.01, 0.008, 0.0075 as reported in Fig. 4 and converged mesh i.e. 0.0075 is used for further analysis. The concentrated load is applied at the centroid of the pinned end of the column.

IV. RESULTS AND DISCUSSIONS

In the present investigation, columns are analysed under three major diameter variations i.e. linear, sinusoidal and exponential. Further, under linear variation, four different combinations are considered and the same are discussed in next section.

(i) Linear variation:

(a) Column is uniformly tapered such that maximum diameter occurred at the clamped end and minimum diameter at pinned end. The Euler's load for different combination of diameter are computed maintaining the constraint of constant volume and length using ABAQUS®. A plot of ratio of Euler's load of tapered column to Euler's load for uniform circular column versus ratio of diameter of tapered column at clamped end to diameter of uniform circular column is reported in Fig. 5. It is evident from Fig. 5 that, when diameter of clamped end approaches to diameter of uniform circular column, Euler's load comes closer to that of uniform circular column but no variation in diameter for this shape results in higher value of Euler's load.

(b) Column is tapered such that diameter is maximum at both ends while minimum at centre. Euler's load for different combination of diameter is computed while maintaining the constant volume and length constraint. Fig. 6 shows that the maximum value of Euler's load is obtained when diameter at centre is 0.73 times the diameter of uniform circular, but no combination of diameter in this configuration is going to yield higher value of Euler's load.

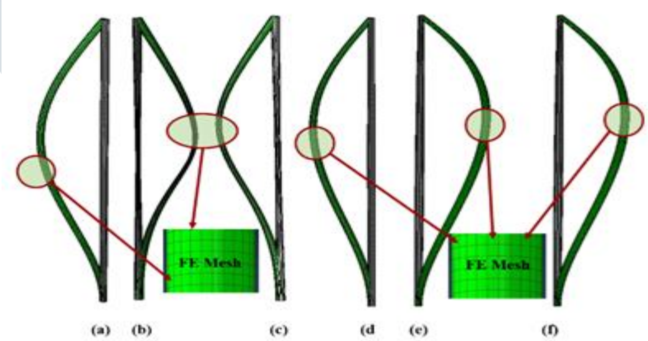

Fig. 3: Finite element model in base state and first eigen vector (a) uniform taper throughout the length (b) taper with maximum diameter at ends (c) taper with maximum diameter at ends and uniform diameter on either side of centre (d) taper with maximum diameter at centre (e) sinusoidal variation, and (f) with exponential variation

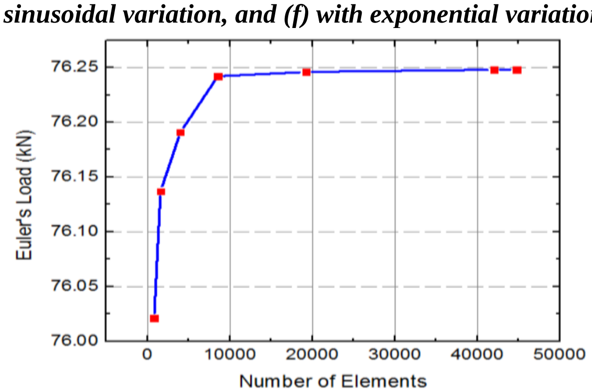

Fig. 4: FE mesh convergence for column

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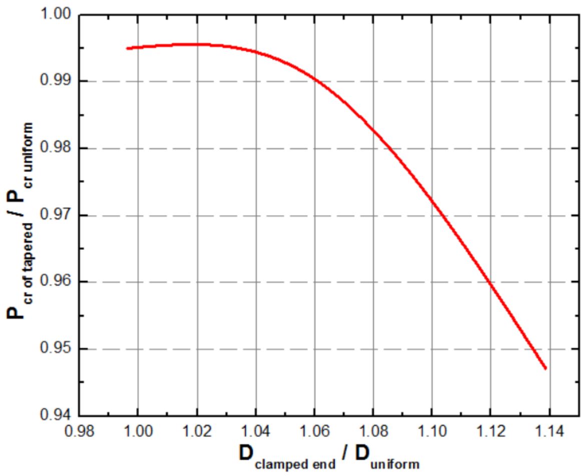

Fig. 5: Normalized Euler's load for uniform tapered column

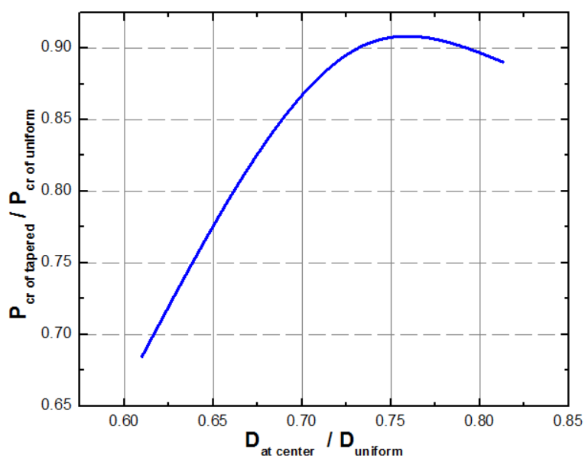

Fig. 6: Normalized Euler's load for tapered column with minimum diameter at centre

(c) Column is tapered such that diameter is maximum at both the ends with uniform diameter at middle for 500 mm on either side of centre. The Euler's load for a combination of diameter i.e diameter of centre is 20 mm for which diameter at the ends is 31.46 mm comes out to be 61.94 kN which is substantially lower than the Euler's load for uniform circular column. This shape supports the conclusion obtained from previous one that there is no benefit in increasing the diameter at the end by reducing diameter at centre below the diameter of uniform circular column.

(d) Column is tapered such that diameter is minimum at the ends while maximum at the centre. The normalized curve of Fig. 7, for Euler's load of such variation for different diameter combination, shows that when diameter at centre is in the range of 1.05 to 1.10 times the diameter of uniform circular section, maximum increment of Euler's load can be obtained.

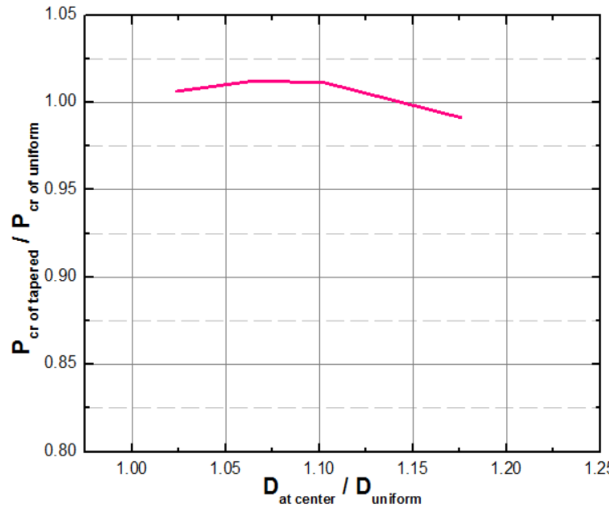

Fig. 7: Normalized Euler's for tapered column with maximum diameter at centre

(ii) Sinusoidal variation:

The diameter of cylinder is varied in accordance with sinusoidal function such that diameter is maximum at the centre and minimum at the clamped and pinned ends in accordance with Eq. 1.

$$4000 \times 1900 = \int dv = \int_0^l \pi (r_0 + a\sin(\frac{\pi y}{l}))^2 dy \quad (1)$$

The normalized Euler's load for sinusoidal variation for different diameter combination is reported in Fig. 8 and it is evident from this figure that sinusoidal variation gives maximum increment in Euler's load when diameter at center for sinusoidal variation is 1.05 to 1.06 times of diameter of uniform circular section.

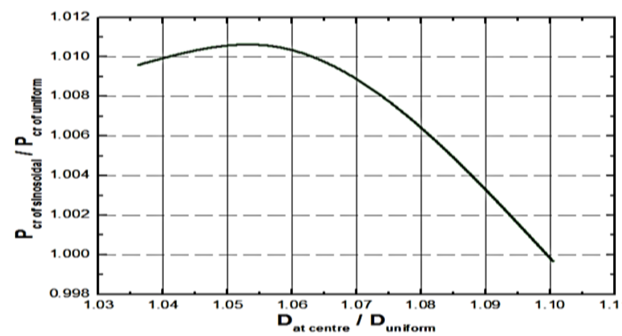

Fig. 8: Normalized Euler's load for sinusoidal variation

(iii) Exponential variation:

The shape of column is varied in the form of scaled exponential variation such that column have maximum

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diameter at the centre and minimum at clamped and pinned end. Herein, diameter at a section is governed by Eq. 2.

$$v = 4\times 1.9 = \int dv = \int_0^2 \pi (r_0 + ae^{1.095y-6.9})^2 dy \quad (2)$$

The normalized Euler's load for exponential variation for different combination is shown in Fig. 9 and it is evident that no combination of diameter is going to increase the Euler's load above Euler's load of uniform circular column.

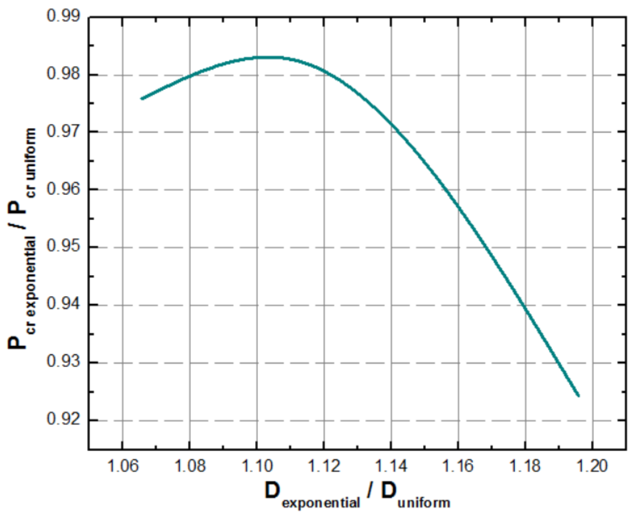

Fig. 9: Normalized Euler's load for exponential variation

V. CONCLUSIONS

This investigation explores the possibility of increment in Euler's load for biaxial symmetric section only using FE analysis. Based on this extensive FE analysis, following conclusions are drawn:

1. FE investigation of uniform tapered column throughout the length suggest that, inorder to increase the Euler's load, diameter should be equal or greater than diameter of uniform circular section at both the ends, there is no benefit in increasing the diameter at clamped end by reducing the diameter at pinned end.
2. Results of FE investigation on tapered column, with maximum diameter at the clamped and pinned ends while minimum diameter at the centre, suggest that there will not be any increment in Euler's load if diameter at centre becomes significantly smaller than diameter of uniform circular section i.e. there is no benefit in increasing the diameter at the ends by reducing diameter at the centre.
3. Results of FE investigation on tapered section with maximum diameter at the centre while minimum at the ends shows maximum increment in Euler's load when minimum value of diameter at the ends is close to the value of diameter of uniform circular section. If the value of diameter at the ends reduces significantly below the diameter of uniform circular section in order to obtain larger diameter at the centre the Eigen's value reduces drastically.
4. Sinusoidal variation of diameter throughout the length gives maximum increment in Euler's load when diameter at the centre is in between 1.05 to 1.06 times the diameter of uniform circular column.

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