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Spatial Instability Analysis of Axisymmetric Boundary Layer on Circular Cylinder

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Abstract: **-- This paper presents linear stability analysis of incompressible axisymmetric boundary layer on a circular cylinder. The base flow is parallel to the axis of a cylinder and hence the angle of attack is zero. The pressure gradient is zero in the stream wise direction. The mass deficit effect is smaller compare to the Blasius boundary layer. The parallel base flow assumption is considered. The stability equations are derived for the disturbance flow quantities in cylindrical polar coordinates. Chebyshev spectral collocation method is used to discretize the stability equations. The discretized equations along with boundary conditions form a general eigenvalues problem. QZ algorithm is used to compute all the eigenvalues. The spatial growth rate of the disturbances is computed for different Reynolds number and azimuthal wave numbers. It is found that for convective instability flow should be temporally unstable.**

Keywords: Spatial, stability analysis, circular cylinder.

I. INTRODUCTION

The laminar-turbulent transition of the boundary layer is important to study in fluid mechanics to understand flow proper-ties. The growth of small disturbances is the first step towards the transition process and it is studied through linear stability analysis. Parallel flow assumption is considered here, linear local stability analysis is performed. In the present work, we study the stability analysis of an in-compressible axisymmetric boundary layer developed on the circular cylinder. The flow is developing in the spatial directions. The literature on the spatial stability analysis of incompressible flow over a circular cylinder is very sparse. Objectives of the present work are to study the spatial growth of the small disturbances at different stream wise location at different Reynolds numbers. The transverse curvature of the cylinder reduces with the increased Reynolds number.

II. PROBLEM FORMULATION

Linearized Navier-Stokes equations are derived for the disturbance flow quantities in cylindrical polar coordinates. The equations are normalized by free stream velocity (U_∞) and cone radius (a). The stream wise variation of the base flow is neglected hence the flow is parallel. The Reynolds number is computed based on a body radius of a cylinder and free stream velocity.

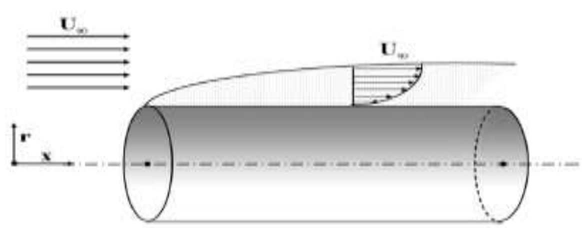

Fig 1.Boundary layer formation on circular cylinder

$$Re_a=\frac{U_\infty a}{\vartheta} \quad (1)$$

We follow the standard procedure for stability analysis with two dimensional perturbations to base flow. Disturbances are assumed to be in normal mode form with the amplitudes are varying in r direction.

$$U^*=U+u, \quad V^*=V+v \quad P^*=P+p \quad (2)$$

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$u(r,t) = \hat{u}(r)\, e^{i(\alpha x - n\theta + \omega t)}$ $v(r,t) = \hat{v}(r)\, e^{i(\alpha x - n\theta + \omega t)}$
$p(r,t) = \hat{p}(r)\, e^{i(\alpha x - n\theta + \omega t)}$ (3)

where,
r = radial coordinate,
x = stream wise coordinate,
ω= frequency of waves,
n = azimuthal wavenumber
a = cone radius at the inlet,
α = Stream wise wavenumber

The Linearized stability equations for disturbance flow quantities are obtained for axisymmetric case are

X component

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + v\frac{\partial U}{\partial r} + \frac{\partial p}{\partial x} - \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) = 0 \quad (4)$$

R component

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + \left(\frac{\partial p}{\partial r}\right) - \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2}\right) = 0 \quad (5)$$

here α is complex while ω is real
$\alpha = \alpha_r + i\alpha_i$
And continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \quad (6)$$

2.1. Boundary Conditions

At the solid surface of cylinder no slip and no penetration conditions are applied to all velocity disturbance amplitudes. At the free stream, far away from a solid wall, it is expected to vanish all velocity and pressure disturbances. The boundary conditions for pressure do not exist physically at the wall.

$u(x,0) = 0 \qquad v(x,0) = 0$

$u(x, r_{max}) = 0 \qquad v(x, r_{max}) = 0 \qquad p(x, r_{max}) = 0$ (7)

Eigen value problem solution

Stability equations are discretized using Chebyshev spectral collocation method (m points in the wall normal direction). The clustering of collocation points is employed in wall normal direction. The governing equations together with the boundary conditions forms quadratic eigenvalue problem of the form, And we know that for spatial case eigenvalue problem formed is,

$$A\begin{Bmatrix} u \\ v \\ p \end{Bmatrix} + \alpha \mathbf{B}\begin{Bmatrix} u \\ v \\ p \end{Bmatrix} + \alpha^2 C\begin{Bmatrix} u \\ v \\ p \end{Bmatrix} = 0 \quad (8)$$

So again equating above equations and previous one we will get following matrices

A=

$$\begin{bmatrix} -i\omega - \frac{1}{Re}\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}\right) & \frac{\partial U}{\partial r} & \frac{\partial}{\partial x} \\ 0 & -i\omega - \frac{1}{Re}\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} - \frac{1}{r^2}\right) & \frac{\partial}{\partial r} \\ 0 & \frac{\partial}{\partial r} + \frac{1}{r} & 0 \end{bmatrix}$$

$$B = \begin{Bmatrix} iU & 0 & 0 \\ 0 & iU & 0 \\ i & 0 & 0 \end{Bmatrix}$$ and

$$C = \begin{bmatrix} \frac{1}{Re} & 0 & 0 \\ 0 & \frac{1}{Re} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9)$$

Where the matrices A, B and C are real and of size 3×m×1, iω are eigenvalues and α is streamwise wavenumber. The QZ algorithm is used to solve the general eigenvalue problem. Here, QZ algorithm computes the full spectrum of eigenvalues. However shear flow become unstable due to very few leading eigen modes.

Base flow solution

Before going for stability analysis one should be fully aware of base flow. The base flow velocity profile is computed using finite difference method. The steady boundary layer equation is solved using second order finite difference.

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial r} = \frac{1}{Re}\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) \quad (10)$$

The following boundary conditions are applied to close the above problem.

$U(0, r) = U_\infty$; $U(x, 0) = 0$; $U(x, \infty) = U_\infty$

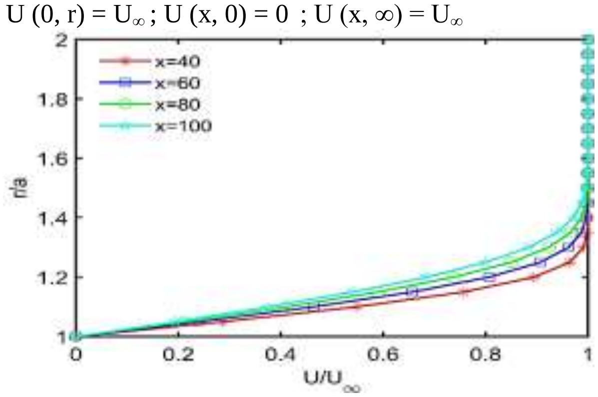

Figure 2 Variation of Base flow velocity in stream wise Direction (U) vs. r coordinate at Re=2000

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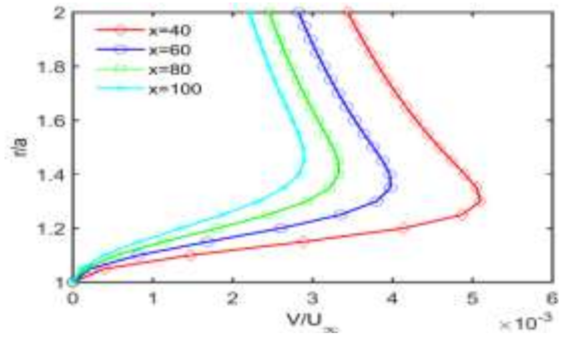

Figure 3 Variation of Base flow velocity in wall normal direction (V) vs. r coordinate at Re=2000

Figure 2 and 3 shows the variation of streamwise (U) and wall normal (V) base component velocity in the radial direction. Figure 4 and 5 shows the first derivative and second derivative of U velocity at different stramwise location. It shows that the magnitudes of radial derivatives reduce in the stream wise direction towards downstream.

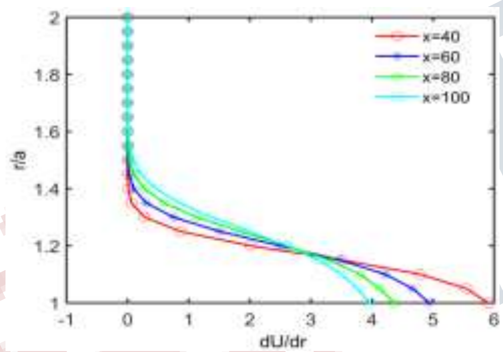

Figure 4 Variation of first derivative of Base flow velocity in wise direction ($\frac{\partial U}{\partial r}$) vs. r coordinate for Re=2000

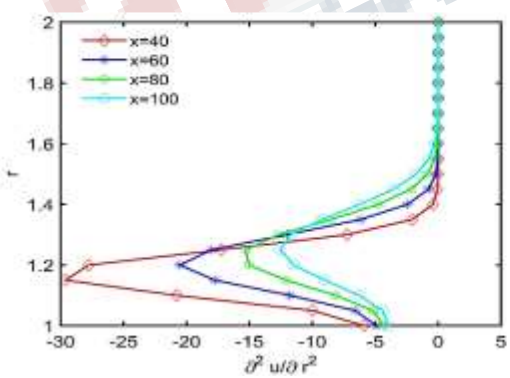

Figure 5 Variation of second derivative of Base flow velocity in streamwise direction ($\frac{\partial^2 U}{\partial r^2}$) vs r coordinate for Re=2000

III. CODE VALIDATION

The temporal eigenvalue problem is solved for for α=2.73 N=0 at Re=12439 and at x=47 for axisymmetric case.The obtained spectrum is in good agreement with the research paper of Tutty et. al. (2008) [4] .Hence for α=2.73 the least stable eigenmode is having C_r=0.317.Now in spatial stability case for C_r=0.317 the least stable eigenmode found with α_r=2.73 at the same streamwise location and Reynolds number.The temporal and spatial stability spectrums are shown in figure 6 and 7.

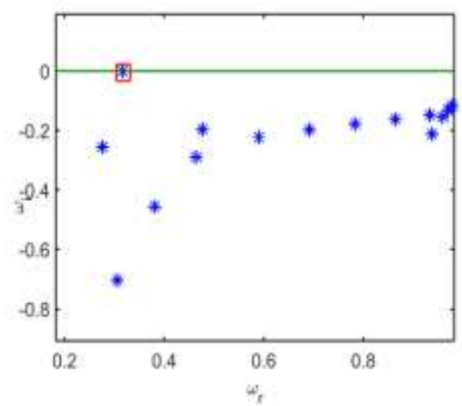

Figure 6 spectrum of eigenvalues at N=0 at Re=12439 and at x=47 and ω= 0.8654 for spatial case

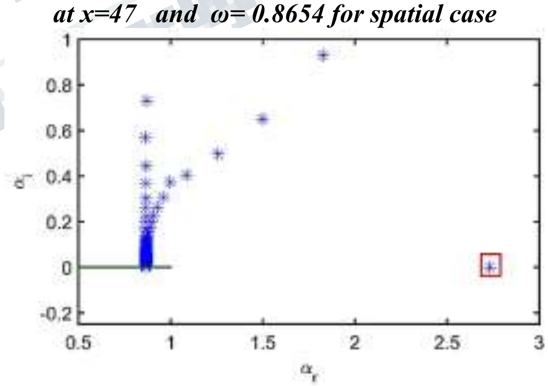

Figure 7 spectrum of eigenvalues at N=0 at Re=12439 at α=2.73 for temporal case

IV. RESULTS AND DISCUSSION

The spatial stability analysis is performed for axisymmetric mode (N=0).The Reynolds numbers Re=11000, 12439,15000 and frequency ω_r = 0.8000 are considered .The polyeig, a MATLAB function is used to solve the eigenvalue problem. At different streamwise locations x=20 to x=200 the stability analysis is performed. The effect of Reynolds number frequency, and effect of transverse curvature is discussed.

4.1 Real and imaginary part of streamwise wave number VS x location at different reynolds' number

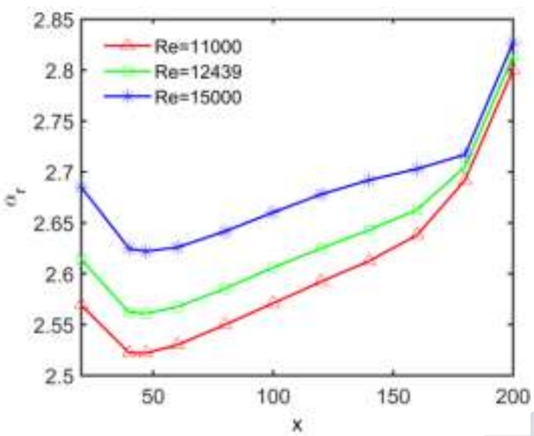

Figure 8 variation of α_r with x location at Diff. Re numbers

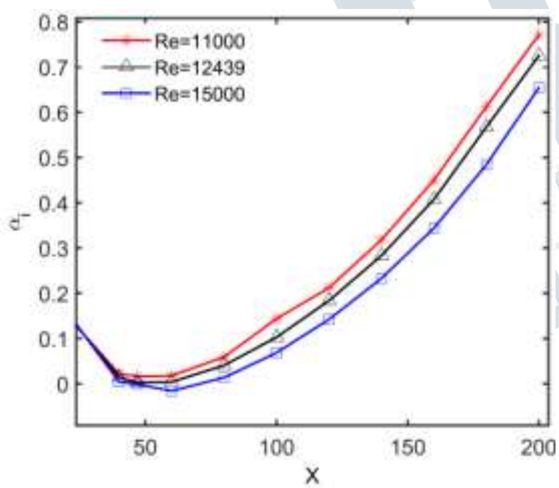

Figure 9 variation of α_i with x location at Diff. Re numbers

From the above figure it is evident that as the Reynolds's number increases flow is becoming more and more unstable as in spatial stability analysis imaginary part is zero when the flow is stable and as it increasing from zero, more and more it will become unstable from figure 9 it can be visualize that for Re=12439 at x=47 α_i=o which means flow is neutral there.

4.2 Spatial amplification rate (A_x) versus x location at different Reynolds' number

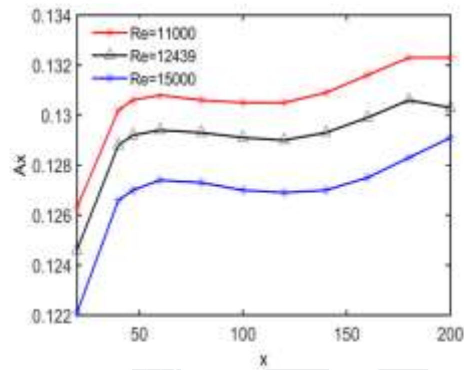

Figure 10.spatial amplification growth rate at diff. Re

4.3 Normalized Spatial amplification rate $(A_x)_n$ versus x location at different reynolds' number

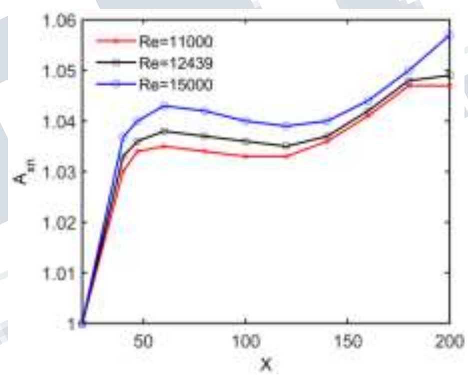

Figure 11 normalized.spatial amplification growth rate at diff. Re

In figure 10 x location VS spatial amplification rate (A_x) graph is plotted, and it gave the information that as the turbulent spot moving in x direction spatial amplification rare increases which means that turbulent spot increases rapidly as it approaches at downstream direction, and as the Reynolds' number increases for same x location Ax is more. And in figure 11 graph of x VS normalized spatial amplification rate $(A_x)_n$ is plotted ,here rates are normalized by the spatial amplification rate available at x=20 so to check whether it is increasing or decreasing in downstream and it can be observed from the graph that as we marching on x direction , normalized spatial amplification rate $(A_x)_n$ is above 1 which simply means that flow is becoming more and more unstable at further downstream and turbulent spot is rapidly getting bigger.

V. CONCLUSIONS

Local stability analysis of boundary layer on the circular cylinder at different stream wise location is performed. It is found that at critical Reynold's number temporal growth and spatial growth is zero. At Re= 12439 beyond x=47 the flow becomes laminar as shown in figure 9.At higher Reynold's number the spatial growth of the disturbances is found higher this proves that the transverse curvature has significant damping effect on the disturbances ,this proves that spatial instability requires temporal instability of the flow. It is perquisite for spatially stable flow to be temporally stable, means flow can be spatially stable if and only if flow is temporally stable.

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