

Static Response of Composite Induced by Piezoelectric Actuator

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Abstract—Piezoelectric materials are used a sensor as well as actuators. This has quick response property and extremely good coupling effects between the mechanical and electric properties. In this paper, an effort has been made to analyze the actuation behavior of various sizes of the piezoelectric patch which is surface bonded on the carbon fiber reinforced composite laminate and plate is simply supported. It was found that flexural deflection is depended upon the size of the piezoelectric patch.

Index Terms— Piezoelectric actuator, Simply Supported Plates, Composite laminate

NOMENCLATURE

| Symbol | Definition |
|---------------------------------|--|
| ϵ_{pe} | Piezoelectric strain |
| d_{31} | Piezoelectric constant |
| V | Voltage |
| t_{pe} | Thickness of actuator |
| $\sigma_x, \sigma_y, \tau_{xy}$ | Bending stress |
| t | Thickness of composite laminate |
| h | Half thickness of composite laminate |
| [D] | Bending matrix |
| [B] | Coupling matrix |
| k_x, k_y, k_{xy} | Plate curvature from the midplane |
| m_x, m_y | Bending moments in the plate |
| M_x, M_y | Internal moments of plate |
| w | Flexural deflection of the plate |
| Z | Position along with the thickness of the composite of the plate from the mid plane |

1. INTRODUCTION

Smart materials have one or more properties that can be appreciably changed in a well- ordered manner by external stimuli, such as stress, electric or magnetic field temperature. Application of smart materials are used in aerospace, civil engineering ,etc. In aerospace, which is called as ‘sensual’ devices that can sense their environment and developed for use in health and usage monitoring systems. In civil engineering, sensual structures are used to measures durability.

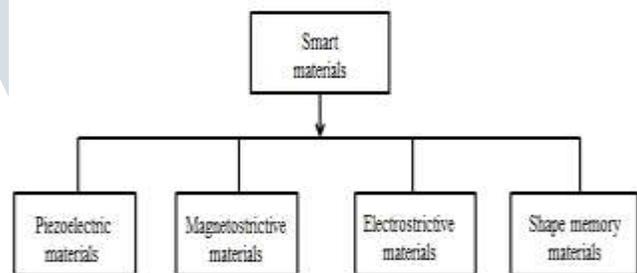


Figure 1: Classification of smart materials

One of the smart materials is piezoelectric materials, in which they undergo deformation when an electric field is applied across them and conversely produce voltage when strain is applied and therefore can be used as both actuators and sensors. Under an applied field, these materials generate a very low strain but cover a broad range of actuation frequency. With the ability to respond autonomously to changes in their environment, piezoelectric devices are widely used in structural applications such as noise reduction, shape, and vibration [3]. Piezoelectric materials are the surface bonded to the existing smart structures to form an online monitoring system, or embedded in composite structures without appreciably differ in size the strength of the structure. Crawley and de Luis [2] studied a beam and piezoelectric actuators that are surface bonded and embedded in a beam to examine the load transfer between host beam and actuator. S .C Her, C.S Lin [4] derived the deflections of a simply supported plate induced by piezoelectric actuators. The plate model is generated by a consisting of the two piezoelectric actuators that is

symmetrically bonded on the surface of a cross-ply composite plate subjected to an electric voltage. The present work investigated a MATLAB code is generated to find the deflections of simply supported composite plate bonded with the smart piezoelectric actuators symmetrically to the plate, with various sizes. In this paper, the aim is to analyze the actuation behavior of various sizes of the surface bonded piezoelectric patch for carbon fiber reinforced composite plate traverse deflection.

2. THEORETICAL MODEL

A model incorporating the classical laminate theory and plate theory is put forward to predict the deformation of the composite laminate plate. An analytical solution for bending moment of the laminated composite plate is excited by piezoelectric actuators was derived by using the composite laminate theory and piezoelectric effect. The achievability of controlling the deflected shape of the plate is determined by placing the actuators at various locations. To avoid the intricating the plate model, the following assumptions are:

- (1) The laminate of carbon fiber reinforced composite comprises of perfectly bonded layers and piezoelectric actuators are also assumed to be perfectly bonded to the surface of the composite. This suggests that strain is continuous across the thickness of composite as shown in Figure 4. There is no slide between the layers of composite.
- (2) For the properties of the composite are identified, the lamina is assumed to be homogenous. Each lamina is in a state of plane stress.
- (3) The laminate and actuators are in the state of plane stress.

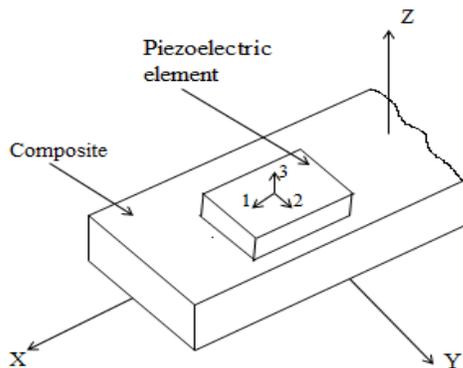


Figure 2: Represent of direction of Plate and PZT

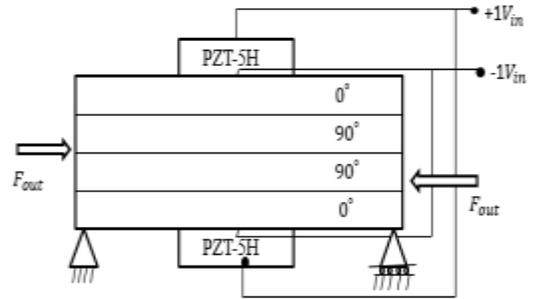


Figure 3: Schematic diagram of PZT-5H actuators are surface bonded on composite

a). Calculation of Bending moment

In this paper, consider two piezoelectric actuators that are symmetrically surface bonded on a cross-ply composite laminate are induced by applied voltage V along the direction of z-axis that is called polarization direction. For an unrestrained piezoelectric actuator, when functioned by a voltage along z-axis, there are equal strains in both x and y directions. The strains value of the piezoelectric actuator in terms of the applied voltage V, piezoelectric constant d_{31} and actuator thickness t_{pe} , as follows,

$$(\epsilon_x)_{pe} = (\epsilon_y)_{pe} = \epsilon_{pe} = \frac{d_{31}}{t_{pe}} V \tag{1}$$

d_{31} is defined as electric field applied in polarization direction (z-axis) and stress is applied in x direction. Moments and forces are generated in the bonded area of the composite laminated plate, as shown in figure 1 due to the coupling of the piezoelectric actuator to the composite. In view of this paper, attention on the deformation of the composite plate excited by the bending moment.

Bending moment is produced by the strains between the surface of the actuator and host plate and due to this bending moment, the strains across the thickness of composite plate can be expressed as,

$$\epsilon_x = Zk_x, \quad \epsilon_y = Zk_y, \quad \epsilon_{xy} = Zk_{xy} \tag{2}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = Z \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}^k \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \tag{3}$$

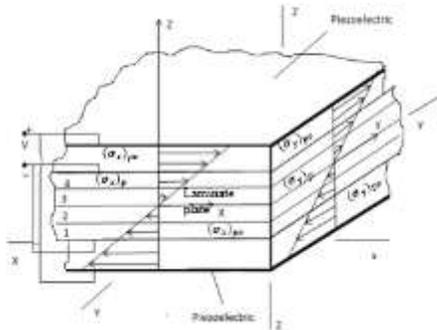


Figure 4: Strain distribution of laminated composite plate across the thickness

The bending stress in the piezoelectric actuator are

$$(\sigma_x)_{pe} = \frac{E_{pe}}{1-\nu_{pe}^2} [Zk_x + \nu_{pe}Zk_y - (1 + \nu_{pe})\epsilon_{pe}] \quad (4)$$

$$(\sigma_y)_{pe} = \frac{E_{pe}}{1-\nu_{pe}^2} [Zk_y + \nu_{pe}Zk_x - (1 + \nu_{pe})\epsilon_{pe}] \quad (5)$$

The bending moments per unit length m_x and m_y which is acting on the composite plate as shown in figure 5 are calculated as:

$$m_x = C_1\epsilon_{pe}; m_y = C_2\epsilon_{pe} \quad (6)$$

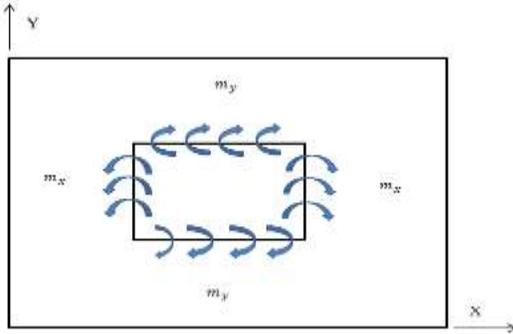


Figure 5: Bending moment acting on the laminated composite plate excited by the PZT actuator

Where,

$$C_1 = A_1(D_{11})_p + A_2(D_{12})_p \quad (7)$$

$$C_2 = A_1(D_{12})_p + A_2(D_{22})_p \quad (8)$$

$$(D_{11})_{pe} = (D_{22})_{pe} = \frac{1}{3} \frac{E_{pe}}{1+\nu_{pe}^2} ((t+h)^3 - t^3) \quad (9)$$

$$(D_{12})_{pe} = \frac{1}{3} \frac{\nu_{pe}E_{pe}}{1+\nu_{pe}^2} ((t+h)^3 - t^3) \quad (10)$$

$$(B_{11})_{pe} = \frac{1}{2} \frac{E_{pe}}{1-\nu_{pe}^2} ((t+h)^2 - t^2) \quad (11)$$

$$(D_{11})_p = \frac{1}{3} \sum_{k=1}^N \bar{Q}_{11}^k (Z_k^3 - Z_{k-1}^3) \quad (12)$$

$$(D_{22})_p = \frac{1}{3} \sum_{k=1}^N \bar{Q}_{22}^k (Z_k^3 - Z_{k-1}^3) \quad (13)$$

$$(D_{12})_p = \frac{1}{3} \sum_{k=1}^N \bar{Q}_{12}^k (Z_k^3 - Z_{k-1}^3) \quad (14)$$

$$(D_{66})_p = \frac{1}{3} \sum_{k=1}^N \bar{Q}_{66}^k (Z_k^3 - Z_{k-1}^3) \quad (15)$$

b). Deflection of plate:

In this work, applying from the classical lamination plate theory the equilibrium equations of plate internal moments M_x, M_y, M_{xy} and actuator induced moments m_x, m_y as,

$$\frac{\partial^2(M_x - m_x)}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2(M_y - m_y)}{\partial y^2} = 0 \quad (16)$$

The inner moments of composite plate M_x, M_y, M_{xy} are expressed in terms of the flexural displacement w , and the following governing equation of the composite laminate plate:

$$(D_{11})_p \frac{\partial^2 w}{\partial x^4} + 2H_1 \frac{\partial^4 w}{\partial x^2 \partial y^2} + (D_{22})_p \frac{\partial^2 w}{\partial y^4} = P \quad (17)$$

Where,

$$H_1 = (D_{12})_p + 2(D_{66})_p \quad (18)$$

$$P = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} \quad (19)$$

For a simply supported rectangular laminate, the flexural displacement w are expressed in terms of Fourier series as:

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (20)$$

$$W_{mn} = \frac{P_{mn}}{(D_{11})_p \alpha^4 + [2D_{12} + 4D_{66}] \alpha^2 \beta^2 + D_{22} \beta^4} \quad (21)$$

$$P_{mn} = \frac{4}{\alpha \beta} \left[-\frac{m_y \alpha^2 + m_x \beta^2}{\alpha \beta} (\cos \alpha x_1 - \cos \alpha x_2) (\cos \beta y_1 - \cos \beta y_2) \right] \quad (22)$$

Where $\alpha = \frac{m\pi}{a}$; $\beta = \frac{n\pi}{b}$

3. NUMERICAL SIMULATION FOR STATIC RESPONSE

The flexural deflections are calculated for a simply supported composite plate which is surface bonded furnished with the PZT-5H actuators. The composite material is carbon/epoxy with the stacking sequence of $[0/90]_s$ [4]. The dimensions of laminated composite plates are length $a=380\text{mm}$, width $b=300\text{mm}$, thickness $t_p = 1.5876$ [4] as shown in figure 4.

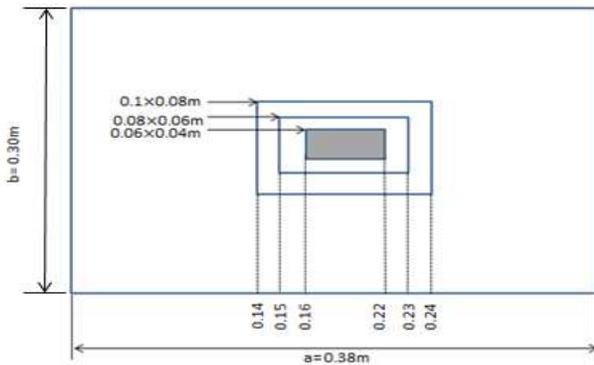


Figure 6: Three different sizes of PZT actuators

The properties of actuator PZT-5H are piezoelectric constant $d_{31} = -285 \times 10^{-12} \text{ V/m}$, thickness of piezoelectric actuator $t_{pe}=0.15876\text{mm}$ and the applied voltage is 1V. The flexural deflections of plates agitated by various dimensions of PZT-5H actuators were computed using the created MATLAB code.

Table I Properties of composite plate and actuator [4]

| Properties | Carbon/Epoxy | Actuator |
|------------|--------------|----------|
| E_1 | 108 GPa | 72.4GPa |
| E_2 | 10.3GPa | - |
| G_{12} | 7.13 GPa | - |
| N | 0.28 | 0.512 |

TABLE II MAXIMUM DEFLECTION OF THE LAMINATED COMPOSITE PLATE INDUCED BY THE PZT ACTUATORS AT DIFFERENT DIMENSIONS

| Size and position of PZT actuator | Deflection value |
|-----------------------------------|------------------|
| Small | 2.6564e-16 m |
| Medium | 5.3128e-16 m |
| Large | 8.8546e-16 m |

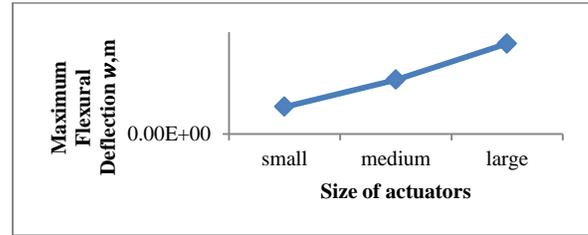


Figure 7: Flexural deflections of different sizes of PZT actuators

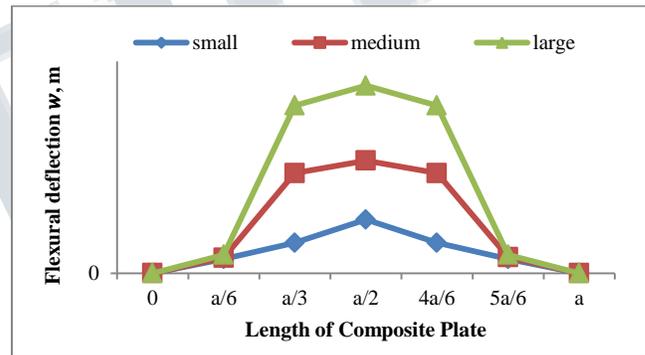


Figure 8: Flexural deflection of the composite plate along the length of the plate with different sizes of PZT actuator

CONCLUSIONS

In this study, piezoelectric actuators were symmetrically surface bonded on a composite laminate to obtain the flexural deflection. From the results, it was concluded that the transverse deflections increases with the size of the actuator by applying electric voltage in reverse polarity. Maximum deflections was achieved for the large size of the PZT actuator at the centre of the plate.

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