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Study of Premiums Based on Fractional Ages Assumptions in Education Insurance

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Abstract— Insurance Company helps parents to plan their children's education funding by offering one of the insurance products named Education Insurance (EI). Benefits offered by EI are the funding stages of education for children from primary school to college level, parents'life protection due to the death within n-year term insurance approach, as well as rider benefits (additional insurance) in the form of standard premium returned when the child dies and waiver of premium if the insured (the parent) has permanent total disability. The net single premium of this product is the sum of the base premiums, including the insured life protections premium and the standard premium for the education stage fund, as well as the rider premium which includes the waiver of premium and the return of premium. The premium calculation is based on the discrete calculation model where the payment of the insurance benefit is paid immediately after the claim by using a fractional age assumption. Age group of children (0 to 18) years using Balducci approach and age group (20 to 50) using UDD. The model obtained from this continuous discrete transformation enable the fair benefit at the point of age of the fraction when decrement occurs.

I. INTRODUCTION

(Soleh and Noviyanti 2011) has modeled insurance premiums using endowment insurance involving two insureds. In that study discrete tables were still used in calculating the life chances of both the insured. The discrete table does not describe the age of the claim accordingly. Therefore, based on (Bowers 1997; Jones 2000; Jordan 1991; Li 2016; London 1997) a fractional age assumptions approach was developed in calculating the probability of occurrence.

Determination of insurance premiums will depend on the type of product offered. In this study, education insurance products will be developed. Insurance Company helps parents to plan their children's education funding by offering one of the insurance products. This product is named a Education Insurance (EI). A child education policy is a life insurance product specially designed as a savings tool to provide an amount of money when your child reaches the age for entry into college (18 years and above).

The four benefits that will be received from this product are the funding stages of education for children from primary school to college level, parents' life protection due to the death within n-year term insurance approach, rider benefits in the form of standard premium returned when the child dies and waiver of premium if the insured (the parent) has permanent total disability. all this time, rider premiums are only additional premiums offered by the insurance company. The net single premium of this product is the sum of the base premiums, including the insured life protections premium and the standard premium for the education stage fund, as well as the rider premium which includes the waiver of premium and the return of premium. The basic theory of premium calculation is developed from (Bowers 1997; Jordan 1991; London 1997; Soleh 2011).

In this research, the premium calculation is based on the discrete calculation model where the payment of the insurance benefit is paid immediately after the claim by using fractional age assumptions (FAAs). Several studies on FAAs have been carried out by (Jones 2000; Li 2016). They only study survival function and have not applied them to insurance products.

II. FRACTIONAL AGES ASSUMPTIONS

Fractional age assumptions (FAAs) are used frequently in actuarial calculations. When combined with life table probabilities, they allow one to fully specify the age at death distribution. FAAs are necessary in the calculation of net single premiums for insurance benefits payable at the moment of death or actuarial present values (APVs) of annuities with payments more frequent than annual (Bowers 1997; Jordan 1991; London 1997). FAAs are also needed when policy issue ages are not integers. In estimating mortality and other decrement rates, FAAs are often required (Jones 2000; Jordan 1991; Li 2016).

In determining the probability of remaining life (_t p_x), it is necessary to use a mortality table to obtain an S(x) value, which is an opportunity precisely at the valuation point that can be determined, while in the calculation of premiums, the opportunity value is needed between the valuation points. (Bowers 1997) developed fractional age assumption to overcome this problem by using the value of l_(x+s)where $0 \leq s \leq 1$, namely by assuming that there is a mathematical function between valuation points (x) and (x + 1) so that premium payments done discretely but benefits can be given continuously.

According to (Bowers 1997) there are three assumptions used to determine the probability of life and the probability of death for fraction age, namely the assumption of uniform distribution of death (UDD abbreviated), Exponential, and

Hyperbolic (Balducci). The selection of assumptions for refining the model for calculating the size of education insurance premiums will be based on Figure 1 which informs the opportunity for the death of someone aged 0 to 60 years based on TMI III 2011.

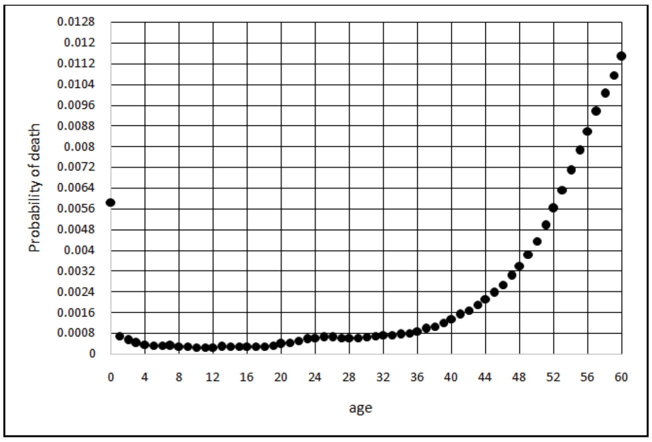


Figure 1. Plotting Indonesian Mortality Table (TMI III 2011)

According to Figure 1 it can be seen that the probability of death at the age of 0 is relatively high and tends to decline at later ages. Meanwhile after 20 years of age, the probability of death again rises to the age of 60. Hyperbolic assumption (Balducci) is a survival function that forms a hyperbolic function. This hyperbolic assumption causes a force of mortality (the chance that someone at a certain age will die a moment later) decreases with time. So this assumption is suitable for use in calculating actuarial quantities of insurance products to calculate the probability of death of children aged 0 to 18 years. Or in other words, at the time of birth humans are so susceptible to the risk of death, but the more time this risk decreases and will rise again when humans enter old age. Furthermore, for each age group of 24 to 60 years will use the UDD assumption in calculating the value of a survival probability from a person.

Linear Assumption (Uniform Distribution of Death (UDD)) is also called linear interpolation, where the chance of survival is a linear function that causes increased force of decrement and random variable S (Survival) with uniform distribution for (x, x + 1).

Hyperbolic Assumptions (Balducci Distribution) are also called harmonic interpolations or Balducci distributions, where survival is assumed to follow a hyperbolic function which causes the force of decrement to decrease for (x, x + 1). In education insurance, 0-year-old children can be included as scholarship beneficiaries. 0-year-old children are very vulnerable to the risk of death. The risk decreases as the child grows. So that Balducci's assumption is more appropriate to be used to calculate the chances of survival of children who are being educated.

The transformation of fractional age assumption in this study refers to Table 1 from (Jones 2000; Jordan 1991). The values of l_x, p_x, and q_x are obtained from the Indonesian Mortality Table (TMI) III 2011.

Table 1. The Transformation of Fractional Age Assumption

Notation	UDD	Balducci
l_{x+s}	$l_x - s.d_x$	$\frac{l_{x+1}}{q_x + s.q_x}$
${}_sp_x$	$1 - s.q_x$	$\frac{1-q_x}{1-(1-s)q_x}$
${}_{1-s}q_{x+s}$	$\frac{(1-s)q_x}{1-sq_x}$	$(1-s)q_x$

III. THE BASE PREMIUM

In this research, the base premium includes the insured life protections premium and the standard premium for the education stage fund. The base premium, P_p is the sum of the standard premium, P_S and the life protections annual premium, P_r as follows.

$$P_p = P_s + P_r \quad (1)$$

This base premium is the premium used in calculating the Waiver of Premium and Return of Premium.

A. The insured life protections premium

Life insurance is a contract between an insurer (x) and a policyholder in which the insurer guarantees payment of a death benefit to named beneficiaries upon the death of the insured. The insurance company promises a death benefit in consideration of the payment of premium by the insured.

This premium is used to protect parents from die not by accident (J=1) and accidental deaths (J=2). Both are mutually exclusive. The premiums model for this life insurance uses single life double decrement.

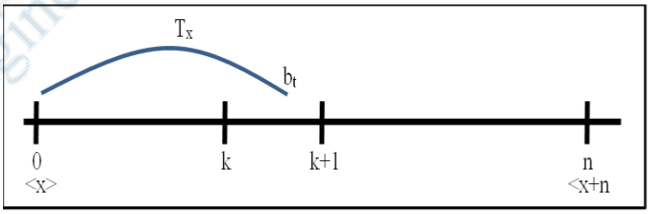


Figure 2. Present Value of Benefit based on the Lifetimes of a Person Aged *x* Years

Figure 2 explains the relationship between TX and KX in term insurance n. Let b_t shows the benefits someone will receive at (x + t). Let the insurance benefits that will be paid by the company if the parents die not by accident (*J=1*) are 100% of Insurance Money (UA) which rises 5% compounded annually. The insurance benefits can be stated as follows.

$$b_1 = \begin{cases} 100\,\%.\,UA.\,(1{,}05)^k, & k = 0,1,2,\ldots,n-1 \\ 0 \quad , & k = n, n+1, \ldots \end{cases}$$

The insurance benefits that will be paid by the company if the parents die due to an accident (*J=2*) are 200% of Insurance Money (UA) which rises 5% compounded annually. The insurance benefits can be stated as follows.

$$b_2 = \begin{cases} 200\,\%.UA.(1{,}05)^k, & k = 0,1,2,\dots,n-1 \\ 0 \quad , & k = n, n+1, \dots \end{cases}$$

The discount factor that will be used in calculating insurance premiums based on fractional age assumptions is formulated as follows.

$$v^{k+s} = \frac{1}{(1+i)^k} \cdot \frac{1}{\left(1+\frac{i}{12}\right)^s}, \qquad (2)$$

where $s = o/12$ is a fractional age value of $0 < s < 1$ and $o = 1,2,3,\cdots,12$ and i is the actuarial interest rate. The value of s is the monthly period conversion between points k and k+1. Figure 3 is Illustration of fractional value o.

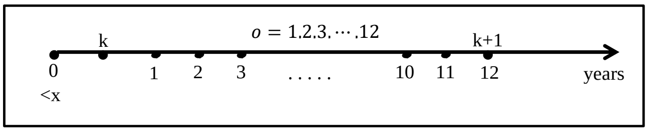


Figure 3. Illustration of Fractional Value o

Let Z_1 is the present value of insurance benefits b_J and Z_1 is expressed as

$$Z_1 = b_j . v^{k+s},$$

where K is random variable which curtate life times and j is decrement (*J=1,2*). The net actuarial present value for Z_1 base on fractional age assumption formulated as in equation (3).

$$A_{\dot{x}:\overline{n}|} = E[Z_1]$$

$$= \sum_{k=0}^{17}\sum_{o=1}^{12} 100\,\%.UA.(1{,}05)^k.\frac{1}{(1+i)^k}.\frac{1}{\left(1+\frac{i}{12}\right)^o}\cdot {}_kp_x^{\tau}\cdot {}_{\frac{o}{12}}p_{x+k}^{\tau} \cdot {}_{1-\frac{o}{12}}q_{x+k+\frac{o}{12}}^{(1)}$$

$$+ \sum_{k=0}^{17}\sum_{o=1}^{12} 200\,\%.UA.(1{,}05)^k.\frac{1}{(1+i)^k}.\frac{1}{\left(1+\frac{i}{12}\right)^o}\cdot {}_kp_x^{\tau}\cdot {}_{\frac{o}{12}}p_{x+k}^{\tau} \cdot {}_{1-\frac{o}{12}}q_{x+k+\frac{o}{12}}^{(2)} \qquad (3)$$

Where

- ${}_kp_x^{\tau}\cdot {}_{o/12}p_{x+k}^{\tau}$ is the probability of *x*-aged parents able to survive without decrement; j until age $k + \frac{o}{12}$.
- ${}_{1-\frac{o}{12}}q_{x+k+\frac{o}{12}}^{(j)}$ is the probability of *x*-aged parents experiencing a decrement *j* between the ages $x + k + \frac{o}{12}$ and $x + k + 1$.
- ${}_{o/12}p_{x+k}^{\tau}$ and ${}_{1-o/12}q_{x+k+o/12}^{(j)}$ obtained by applying the Uniform Distribution of Death (UDD) assumption in the fractional age assumption using the TMI III table.

So, the life protections annual premium as follow on equation (4).

$$P_r = \frac{A_{\dot{x}:\overline{n}|}}{\sum_{k=0}^{n-1} v^k . {}_kp_{xy}} \qquad (4)$$

B. The Standard Premium for the Education Stage Fund

This premium in education insurance is used to obtain educational benefits when a child (y) enters elementary school to college. Educational insurance products in this study are modelled based on the level of education in Indonesia. Let n is the education insurance period (maximum 18 years) for children y = 0 years old. Benefits that will be paid when a child enters elementary school is b_{SD} at the point (n-12). Then the benefits of the child when entering secondary school are b_{SMP} when the child enters the junior high school level at point (n-6) and b_{SMA} benefits when the child enters high school at the point (n-3). The last benefit is the b_{Kuliah} is given when the child enters the college level at point (n) as well as additional benefits in the form of an educational scholarship; b_{Kuliah}.

Explanation of this can be seen in Figure 4.

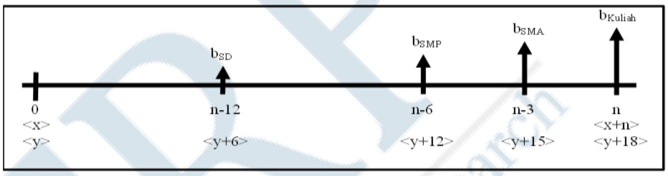


Figure 4. Illustration the Education Stage Fund

Education insurance funds must be sufficient when children enter school later on. Suppose that Insurance money (UA) rises 5% annually by compounding all the benefits of education from elementary school to college. Let the benefits of education funds; b_{school} are as follows.

$$b_{school} = \begin{cases} b_{SD} = 10\% \times UA \times (1{,}05)^{n-12} \\ b_{SMP} = 20\% \times UA \times (1{,}05)^{n-6} \\ b_{SMA} = 30\% \times UA \times (1{,}05)^{n-3} \\ b_{Kuliah} = 150\% \times UA \times (1{,}05)^{n} \end{cases}.$$

The provision of educational benefits is adjusted to the time of school enrollment and the child must be a live at that time. Furthermore, the Net Single Premium (PS) is the sum of the present value of all benefits as in equation (5).

$$PS = (b_{SD} \times v^{n-12}) + (b_{SMP} \times v^{n-6}) + (b_{SMA} \times v^{n-3}) + (b_{Kuliah} \times v^{n}). \qquad (5)$$

The discrete life annuity formula for joint life status (x) and (y) is as follows.

$$\ddot{a}_{xy:\overline{n}|} = \sum_{k=0}^{n-1} \upsilon^k . {}_kp_{xy} \qquad (6)$$

where υ^k is the discount factor at point k.

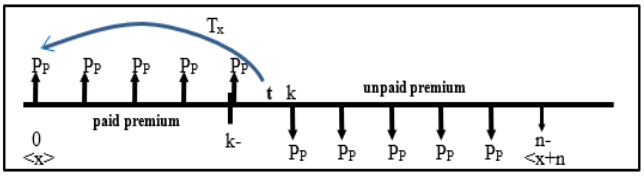
The annual standard premium formula, P_S for education insurance is semi-continuous. And the formula can be written as follows.

$$P_s = \frac{(b_{n-12} \times v^{n-12}) + (b_{n-6} \times v^{n-6}) + (b_{n-3} \times v^{n-3}) + (b_n \times v^n)}{\sum_{k=0}^{n-1} v^k . {}_kp_{xy}}. \qquad (7)$$

C. Waiver of Premium Rider

A waiver of premium rider pays all life insurance premiums due if the insured person becomes disabled. A waiver of premium rider is an optional benefit on many term life insurance policies as specially Education Insurance product, and may also be available on permanent forms of insurance coverage. The waiver of premium rider will provide a benefit only if the insured person becomes totally disabled. If a total disability occurs, the remaining premium charges will no longer be required to be paid, but the policy will remain in force for the length of term that the policy was written for.

Based on figure 5, if a parent has a disability at time t during the insurance period which results in a loss of ability to fulfil the premium payment obligation, this rider will apply and the next insured does not need to pay the premium to the company.



Waiver of premium benefits can be written as follows.

$$b_3 = \begin{cases} P_p.\ddot{a}_{x;\overline{n-k}|}.v^{1-s}, untuk\ k = 0,1,2,\dots,n-1 \\ 0 \qquad ,untuk\ k = n, n+1,\dots \end{cases}$$

Let Z_2 is the present value of insurance benefits b_3. And Z_2 is expressed as

$$Z_2 = b_3.v^{k_x+s}$$

So, the net actuarial present value for Z_2 base on fractional age assumption formulated as in equation (8).

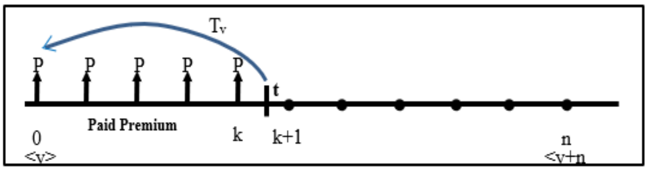
$$P_{Waiver} = \mathrm{E}[Z_2]$$
$$= \sum_{k=0}^{17}\sum_{o=1}^{12} P_p.\ddot{a}_{x;\overline{n-k}|}.v^{1-o/12}.\frac{1}{(1+i)^k}.\frac{1}{\left(1+\frac{i}{12}\right)^o}\cdot{}_k p_x^{\tau}\cdot{}_{\frac{o}{12}}p_{x+k}^{\tau}\cdot{}_{1-\frac{o}{12}}q_{x+k+\frac{o}{12}}^{(3)}. \quad (8)$$

where

- P_p is the base premium in equation (1).
- ${}_k p_x^{\tau}\cdot{}_{o/12}p_{x+k}^{\tau}$ is the probability of x-aged parents able to survive without decrement; j until age $k+\frac{o}{12}$.
- ${}_{1-\frac{o}{12}}q_{x+k+\frac{o}{12}}^{(3)}$ is the probability of x-aged parents experiencing a disability between the ages $x+k+\frac{o}{12}$ and $x+k+1$.
- ${}_{o/12}p_{x+k}^{\tau}$ and ${}_{1-o/12}q_{x+k+o/12}^{(3)}$ obtained by applying the Uniform Distribution of Death (UDD) assumption in the fractional age assumption using the TMI III table.

D. Return of Premium

A policy add-on that returns the premiums paid if the insured outlives the term of the policy. If the child dies at time t, the parent is freed from the responsibility of paying the next premium and the standard premium that has been paid is returned to the parent. In calculating this type of premium, it will involve the life chances of parents because parents are considered responsible for paying premiums until the child dies



Return of premium benefits can be written as follows.

$$b_4 = \begin{cases} P_p.\ddot{a}_{xy:\overline{k}|}, untuk\ k = 0,1,2,\dots,n-1 \\ 0 \qquad ,untuk\ k = n, n+1,\dots \end{cases}$$

Let Z_3 is the present value of insurance benefits b_4 and Z_3 is expressed as

$$Z_3 = b_4.v^k$$

So, the net actuarial present value for Z_3 base on fractional age assumption formulated as in equation (9).

$$P_{Return} = E[Z_3] = \sum_{k=0}^{17}\sum_{t=1}^{12} P_p.\ddot{a}_{xy:\overline{k}|}.\frac{1}{(1+i)^k}\cdot{}_k p_{xy}\cdot{}_{\frac{o}{12}}p_{xy+k}\cdot{}_{1-\frac{o}{12}}q^{\ 1}_{xy+k+\frac{o}{12}}.$$

Where

- P_p is the base premium in equation (1).
- ${}_k p_{xy}\cdot{}_{\frac{o}{12}}p_{xy+k}$ is the probability of x-aged parents and y-aged child be able to survive until age $k+\frac{o}{12}$.
- ${}_{1-\frac{o}{12}}q^{\ 1}_{xy+k+\frac{o}{12}}$ is the probability of children die earlier than parents between the ages $y+k+\frac{o}{12}$ and $y+k+1$.

(8)

${}_{\frac{o}{12}}p_{xy+k}$ and ${}_{1-\frac{o}{12}}q^{\ 1}_{xy+k+\frac{o}{12}}$ obtained by applying the Uniform Distribution of Death (UDD) assumption for parents (x) and Balducci assumption for child (y) in the fractional age using the TMI III table.

IV. NET ANNUAL PREMIUM

Net single premiums from educational insurance products is the sum of base premiums and rider premiums. The equation (10) is a premium that must be paid simultaneously by parents to insurance companies.

$$PTB = PS + A_{\acute{x}:\overline{n}|} + P_{Waiver} + P_{Return}, \qquad (10)$$

where the PS formula obtained from the equation (7), $A_{\acute{x}:\overline{n}|}$ from the equation (3), P_{Waiver} from the equation (8) and P_{Return} from the equation (9).

If X and Y independent, so the discrete life annuity formula for joint life status (x) and (y) is as follows.

$$\ddot{a}_{xy:\overline{n}|} = \sum_{k=0}^{n-1} v^k\cdot{}_k p_x\cdot{}_k p_y. \qquad (11)$$

Based on equation (10) and equation (11), so annual net premiums from educational insurance products are formulated in equation (12).

$$P = \frac{PS + A\acute{x}:\overline{n}| + P_{Waiver} + P_{Return}}{\sum_{k=0}^{n-1} v^k\cdot{}_k p_x\cdot{}_k p_y}. \qquad (12)$$

V. CONCLUSION

The role of actuaries and researcher in building and modelling insurance product is becoming increasingly important, specially study about insurance premiums. The premium must be enough to pay for the promised insurance benefits. The reasonable premium to cover claims must be estimated well.

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