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An EOQ Model for Imperfect Items Deterioration and Time Dependent Rates

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Abstract: **In the presented article we have developed an economic order quantity model at infinite planning horizon. Deterioration is taken into consideration. Demand and production rates are assumed to be time dependent. Production process is not reliable hence a percent of produced items is defective. A mathematical formulation has been done to find the optimum value of order quantity and total average cost. A numerical example is also given to illustrate the theoretical results. Finally the sensitivity analysis is reported to find the effect of different parameters.**

Keywords: **Defective Items, Economic Order Quantity Model, Inventory, Time Dependent Production Rate**

I. INTRODUCTION

Deterioration of items is a frequent and natural phenomenon which cannot be ignored. In realistic scenario the life cycle of seasonal product, fruits, food items, electric component, volatile liquid, food etc are short and finite usually can undergo deterioration. Thus the item may not serve the purpose after a period of time and will have to be discarded as it cannot be used to satisfy the future demand of customers. The term deterioration means spoilage, vaporization and obsolescence, loss of character or value in a product along time. At first Wagner and Whitin (1958) dealt with an inventory model for deteriorating items at the end of the shortage period. Researchers have been progressively modifying the existing models by using the deterioration function of various types; it can be constant type or dependent on time. .In our proposed model we have considered the Weibull distribution as the function of deterioration. In past few decades it is observed by Berrotoni [1]that both the leakage failure for the dry batteries and the life expectancy of ethical drugs could be expressed in Weibull distribution. Covert and Philip [2] has influenced by the work of Berrotoni to develop an inventory model for deteriorating items with variable rate of deterioration .They have explained two parameter Weibull distribution to contemplate deterioration as distribution of time. Misra [4] proposed an inventory model with two parameter Weibull distribution and finite rate of replenishment. The research has been summarized in different survey papers Goyal and Giri (2001) , Raafat (1991) , Goyal et al. (2013).

Hill [4] first time considered increases linearly at the beginning and then after maturation it becomes a constant, a stable stage till the end of the inventory cycle.

Deng developed a note on the inventory models the deteriorating items with ramp-type demand rate by exploring two cases where the time point occurs before and after tjjhe point where the demand is stabilized. Skouri et al. (2011) studied with ramp type demand rate and time dependent deterioration rate with unit production cost and shortages. This type of demand patterns examined by Wu and Ouyang (2000), Girl et al. (2003), Manna and Chaudhuri [7], Panda [9] Chen (2006) etc. Two types of backlogging accumulated such as constant type and time dependent partial backlogging rate dependent on the wailing time up to the next replenishment have been studied extensively by many researches such as Abad [15], Chang and Dye (1999), Wang [17] Wu et al. [18] Singh and Singh [19] Dye [20] . However in market structure another important factor is shortages which no retailer would prefer, and in practice are partially backlogged and partially lost.

Present work is without shortage model where (a) the demand rate is stabilized after the production stopping time and after the production is stopping time and (b) Deterioration rate is constant .In the proposed model we at first have the demand rate which is realistic as any new brands product launch in the market the demand rate linearly depends on time and is stabilized after the production stopping the time and before the time when inventory level reaches zero.

II MATHEMATICAL FORMULATION OF THE MODEL

2 .1 Assumptions and Notations

The following notations and assumptions are considered to develop the inventory model

2.1.1 Notations

D – Demand rate (units/unit time)
K- Unit Production cost (units /unit time)
c_1 – Holding cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $c_{01}+\frac{c'_1}{n^{\alpha 2}}$, $\alpha_2 > 0$
c_3 – Deterioration cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $c_{03}+\frac{c'_3}{n^{\alpha 2}}$, $\alpha_2 > 0$
c_4 – Shortage cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $c_{04}+\frac{c'_4}{n^{\alpha 2}}$, $\alpha_2 > 0$
c_5 – Lost sale cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $c_{05}+\frac{c'_5}{n^{\alpha 2}}$, $\alpha_2 > 0$
X – Total average cost for a production cycle
r- Inflationary rate
δ – Backlogging rate

2.1.2 Assumptions

(i) Demand rate in ramp type function of time, i.e. demand rate R= f (t) is assumed to be a ramp type function of time f(t)= D_0[t-(t-μ) H(t-μ)] , $D_0 > 0$ and H(t) is a Heaviside's function:

$$H(t-\mu) = \begin{cases} 1 & if\ t \geq \mu \\ 0 & if\ t < \mu \end{cases}$$

(ii) Deterioration varies unit time and it is function of two parameter Weibull distribution of the time, i.e. $\alpha\beta t^{\beta-1}$, $0 < \alpha < 1, \beta \geq 1$, where t denote time of deterioration .
(3) Lead time is zero.
(4) Inflation is considered.
(5) Shortage are Allowed and partially backlogged.
(5) Costs are considered under learning phenomenon.
(6) K= γ f (t) is the production rate where γ (> 1) is a constant.
The unit production cost $v = \alpha_1 R^{-s}$ where $\alpha_1 > 0, s > 0$ and s $\neq$ 2.

α_1 is obviously positive since v and R are both non-negative. Also higher demands result in lower unit cost of production. This implies that v and R are inversely related and hence, must be non-negative i.e. positive.
Now,

$$\frac{dV}{dR} = -\alpha_1 s R^{-(s+1)} < 0.$$

$$\frac{d^2V}{dR^2} = \alpha_1 s(s+1)R^{-(s+2)} > 0.$$

Thus, marginal unit cost of production is an increasing function of R. These results imply that, as the demand rate increases, the unit cost of production decreases at an increasing rate. Due to this reason, the manufacture is encouraged to produce more as the demand for the item increases. The necessity of restriction s$\neq$ 2 asises from the nature of the solution of the problem.

2.2 Mathematical Formulation Of The Model

Case ($\mu \leq t_1 \leq t_2$)
The stock level initially is zero. Production starts just after t=0. When the stock attains a level q at time t= t_1, then the production stops at that time. The time point μ occurs before the point t=t_1, where demand is stabilized after that the inventory level diminishes due to both demand and deterioration ultimately falls to zero at time t = t_2. After time t_2 shortages occurs at t=T, which are partially backlogged and partially lost. Then, the cycle repeats.

Let Q(t) be the inventory level of the system at any time t($0 \leq t \leq t_2$). The differential equations governing the system in the interval [0,t_2]are given by

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = K - F(t) \qquad 0 \leq t \leq \mu \quad (1)$$

with the condition Q(0)=0

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = K - F(t) \qquad \mu \leq t \leq t_1 \quad (2)$$

with the condition $Q(t_1) = q$

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = -F(t) \qquad t_1 \leq t \leq t_2 \quad (3)$$

with the condition $Q(t_1) = q, Q(t_2)$=0

$$\frac{dQ(t)}{dt} = -e^{-\delta(T-t_2)}F(t) \qquad t_2 \leq t \leq T \quad (4)$$

with the condition $Q(t_2)$=0
Using ramp type function F(t), equation (1),(2),(3),(4) become respectively

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = (\gamma - 1)D_0 t \qquad 0 \leq t \leq \mu \quad (5)$$

with the condition Q(0) = 0

$$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = (\gamma - 1)D_0 \mu \qquad \mu \leq t \leq t_1 \quad (6)$$

with the condition $Q(t_1) = q$

$\frac{dQ(t)}{dt} + \propto \beta t^{\beta-1} Q(t) = D_0 \mu$ $\quad t_1 \le t \le t_2$ (7)

With the conditions $Q(t_1)$= q, $Q(t_2)$= 0,

$\frac{dQ(t)}{dt} = -e^{-\delta(T-t_2)} D_0 \mu$ $\quad t_2 \le t \le T$ (8)

with the condition Q(t_2)=0

(5),(6),(7),(8) are first order linear differential equations

For the solution of equation (5) we get

$Q(t)e^{\alpha t^\beta} = (\gamma - 1)\int D_0 t e^{\alpha t^\beta} + C$

=(γ-1) $D_0 \int t[1 + \alpha t^\beta + \frac{\alpha^2 t^{2\beta}}{l^2} + --]t + C$

=(γ -1) $D_0 \int[t + \alpha t^{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2} +$

=(γ -1)D_0 $[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --] + C$ (9)

By using the condition Q (0) = 0 (10)

$Q(t) = (\gamma - 1)D_0 e^{-\alpha t^\beta}[\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t^{2\beta+2}}{2(2\beta+2)} + --],$

$0 \le t \le \mu$ (11)

for the solution of equation (6) we have

$\int_\mu^t d[e^{\alpha t^\beta} Q(t)] = (\gamma - 1) D_0 \mu \int_\mu^t e^{\alpha t^\beta} dt$

$e^{\alpha t^\beta} Q(t) - e^{\alpha \mu^\beta} Q(\mu) =$

(γ -1) $D_0 \mu \int_\mu^t e^{\alpha t^\beta} dt = (\gamma - 1) D_0 \mu \int_\mu^t 1 + \alpha t^\beta +$

$e^{\alpha t^\beta} Q(t) - e^{\alpha \mu^\beta} Q(\mu) =$ (γ -1) $D_0 \mu[t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} +$----

]

$= (\gamma - 1) D_0 \mu e^{-\alpha t^\beta} [t - \frac{\mu}{2} + \frac{\alpha t^{\beta+1}}{(\beta+1)} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha \mu^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{\alpha^2 \mu^{2\beta+1}}{2(2\beta+1)(2\beta+2)}], \mu \le t \le t_1$

(12)

The solution of equation (7) is given by

Q(t) $e^{\alpha t^\beta} = -D_0 \mu \int e^{\alpha t^\beta} dt + C$

= (13)

Using initial condition Q(t_2)=0 in equation (13) we have,

q= $D_0 \mu e^{-\alpha t_1^\beta} (t_2 + \frac{\alpha t_2{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_2{}^{2\beta+1}}{2(2\beta+1)} + \quad) -$

$D_0 \mu e^{-\alpha t_1^\beta} (t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1{}^{2\beta+1}}{2(2\beta+1)} + --)$

Substitute q in equation (13) the solution of equation (7) is

$Q(t) = D_0 \mu e^{-\alpha t^\beta} [(t_2 - t) + \frac{\alpha}{\beta+1} (t_2{}^{\beta+1} - t^{\beta+1}) + \frac{\alpha^2}{2(2\beta+1)} (t_2{}^{2\beta+1} - t^{2\beta+1}) + - -] \; t_1 \le t \le t_2$

(15)

The solution of equation (8) is q$e^{\alpha t_1^\beta} + Do\mu \left(t_1 + \frac{\alpha t_1 \beta+1}{\beta+1} + \right.$

$\alpha 2t12\beta+122\beta+1+$

$\frac{dQ(t)}{dt} = -D_0 \mu e^{-\delta(T-t_2)}$ $\quad t_2 \le t \le T$

with boundary condition $Q(t_2) = 0$

$Q(t) = -D_0 \mu [t - \delta(T - t_2)t] + c$

By using $Q(t_2)$ =0, we get

$Q(t) = D_0 \mu[(t_2 - t) - \delta(T - t_2)(t_2 - t)]$ (16)

Shortage cost over the period [0,T] is defined as

$\int_{t_2}^t \theta(t)dt = -\int_{t_2}^T D_0 \mu[\{t_2 - t) - \delta[T - t_2](t_2 - tdt e - rt$

$= -D_0 \mu \int_{t_2}^T (1 - rt)[(t_2 - t) - \delta(T - t_2)(t_2 - t)]dt$

(17)

Lost sale cost per cycle is

LS= $D_0 \mu \int_{t_2}^T (1 - e^{-\delta(T-t_2)}) dt$

$= D_0 \mu\delta \left[\frac{3T^2 t_2}{2} - \frac{3t_2^2 T}{2} - \frac{T^3}{2} + \frac{t_2^3}{2} - \frac{5rt_2T^3}{6} - \frac{rt_2^4}{6} + \frac{rTt_2^3}{6} + \right.$

$rT43+rt22T22$ (18)

The total inventory over the period [0, t_2] is

$$\int_0^{t_2} Q(t)dte^{-rt} = \int_0^\mu Q(t)e^{-rt} dt + \int_\mu^{t_1} Q(t)e^{-rt} dt + \int_{t_1}^0 Q(t)e^{-rt} dt$$

Therefore, the total inventory in [0, t_2] is given by

$D_0 \mu[\frac{t_2^2}{2} - t_2 t_1 + \frac{t_1^2}{2} + \frac{(\alpha)t_2^{\beta+2}\beta}{(\beta+1)(\beta+2)} - \frac{\alpha t_1 t_2^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha^2 t_2^{2\beta+1} t_1}{2(2\beta+1)} + \frac{\alpha^2 t_2^{2\beta+2}(\beta+3)}{2(2\beta+2)(\beta+1)} + \frac{\alpha^2 t_1^{2\beta+2}(5\beta+3)}{2(2\beta+1)(2\beta+2)(\beta+1)} + \frac{\alpha t_2 t_1^{\beta+1}}{(\beta+1)} - \frac{\alpha^2 t_2^{\beta+1} t_1^{\beta+1}}{(\beta+1)^2} - \frac{rt_2^2}{2} + rt_2 t_1 - \frac{rt_1^2}{2} - \frac{r\alpha t_2^{\beta+2}}{(\beta+1)} + \frac{r\alpha t_2^{\beta+1} t_1}{(\beta+1)} + \frac{r\alpha t_1^{\beta+3}\beta}{(\beta+3)(\beta+1)} - \frac{\alpha^2 t_2^{2\beta+2} r}{2(2\beta+1)} + \frac{r\alpha t_2^{\beta+3}(2\beta+3)}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha^2 \beta t_2^{2\beta+3} r}{2(2\beta+1)(\beta+2)(2\beta+3)} - \frac{\alpha^2(5\beta+3) t_1^{2\beta+3} r}{2(2\beta+1)(\beta+1)(2\beta+3)} - \frac{\alpha t_2 t_1^{\beta+2}}{(\beta+2)} + \frac{\alpha^2 t_2^{\beta+1} t_1^{\beta+2}}{(\beta+1)(\beta+2)}]$ (19)

Total number of deteriorated items over the period [0,t_2] is given by

Production in [0,μ]+ Production in [μ,t_1] – Demand in [o,μ] –Demand in [μ, t_2]

$=\gamma \int_o^\mu Dote^{-rt} dt + \gamma \int_o^{t_1} D_0 \mu e^{-rt} dt - D_0 \int_o^\mu te^{-rt} dt - \int_\mu^{t_2} Do\mu e^{-rt} dt$

$$\frac{1}{2}\gamma D_0\mu\left[2t_1-\mu-rt_1^2+\frac{r\mu^2}{3}\right]-\frac{1}{2}D_0\,\mu\left[2t_2-\mu-rt_2^2+\frac{r\mu^2}{3}\right] \quad (20)$$

The cost of production in $[u, u+du]$ *is*

$$\text{Kv du} = \frac{\alpha_1\gamma}{R^{s-1}} \quad (21)$$

Hence the production cost over the period $[o, t_1]$ is given by

$$\int_o^{t_1}\text{Kv}e^{-ru}\,du = \int_0^{\mu}\text{Kv}e^{-ru}\,du + \int_{\mu}^{t_1}\text{Kv}e^{-ru}\,du$$

$$=\alpha_1\gamma {D_0}^{1-s}\left[\frac{(s-1)\mu^{2-s}+(2-s)\mu^{1-s}t_1}{(2-s)}\right]+\alpha_1\gamma {D_0}^{1-s}\left[\frac{\mu^{3-s}}{2}-\frac{{t_1}^2}{2}\mu^{1-s}-\frac{\mu^{1-s}}{(1-s)}\right] \quad (22)$$

The total average inventory cost X is given by

X= Inventory Cost + Deterioration Cost+ Production Cost+ Shortage Cost + Lost Sales Cost (23)

III. CONCLUSION

In this study, an EOQ model with ramp type demand rate and unit production cost under inflationary condition has been developed. The quality and quantity of goods decrease in course of time due to deterioration it is a natural phenomena .Hence consideration of Weibull distribution time varying deterioration function defines a significant meaning of perishable, volatile and failure of any kind of item. Shortages are allowed and partially backlogged. The two considered phenomena viz., learning and inflation play an important role in realistic scenario. A mathematical model has been developed to determine the optimal ordering policy cost which minimizes the present worth of total optimal cost.

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EOQ model deals with the size of order which minimizes various costs which include set up cost ,ordering cost , holding cost etc. But EOQ model for deteriorating item was first considered by Wanger and Whitin and it was developed further by Berrotoni .And again Deng(et al.) developed a note on inventory model with ramp type demand rate.

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