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On Fuzzy $g^{\#\#}$-Continuous Maps and Fuzzy $g^{\#\#}$ -Homeomorphism Mappings in Fuzzy Topological Spaces

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Abstract:— **The aim of this paper is to introduce new class of Fuzzy sets, namely $g^{\#\#}$-closed fuzzy set for Fuzzy topological spaces .This new class is properly lies between the class of α-closed Fuzzy set and the class of $g^{\#}$-closed fuzzy set, we also introduce application of $g^{\#\#}$ -closed fuzzy sets, the concept of fuzzy $g^{\#\#}$-continuous, fuzzy$g^{\#\#}$-irresolute mapping, fuzzy $g^{\#\#}$-closed maps, fuzzy $g^{\#\#}$-open maps and fuzzy $g^{\#\#}$ -homeomorphism in Fuzzy topological spaces are also introduced, studied and some of their properties are obtained.**

Classification:--**2000 Math Subject classification 54A40**

Keywords: **-- and phrases:$g^{\#\#}$- closed fuzzy sets, $fg^{\#\#}$-continuous, $fg^{\#\#}$-irresolute, $fg^{\#\#}$-open,$fg^{\#\#}$-closed mapping and $fg^{\#\#}$ -homeomorphism.**

I. INTRODUCTION

Prof. L.A. Zadeh's [19] in 1965 introduced of the concept of 'fuzzy subset', in the year 1968, C L. Chang [4] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Subsequently many researchers like, C.K. Wong[18], R.H. Warren [17],R. Lowen[7], A.S. Mashhour[11], K.K. Azad[1], M. N. Mukherjee[12],G. Balasubramanian &P. Sundaram [2] and many others have contributed to the development of fuzzy topological spaces.

The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by C.L.Chang[4] and R.H.Warren [17] are included. Fuzzy topological spaces and some basic concepts and results on fuzzy topological spaces from the works of C.L.Chang [4] , R.H.Warren [17], and C.K.Wong[18] are presented. And some basic preliminaries are included. N.Levine [7] introduced generalized closed sets (g-closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general and fuzzy topology.

Dr. Sadanand Patil [13,14&15] in the year 2009 and R. Devi and M. Muthtamil Selvan[5] in the year 2004, are introduced and studied g-continuous maps. The class of $g\#$- closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of $g*$- closed fuzzy sets. The class of $g*$- closed fuzzy sets is properly placed between the class of closed fuzzy sets and the class of g- closed fuzzy sets.

II. PRELIMINARIES:

Throughout this paper $(X,T),(Y,\sigma)\&(Z,\eta)$or (simply$X,Y,\&Z$) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset A of a space (X,T). $cl(A), int(A)\&C(A)$ denotes the closure, interior and the compliment of A respectively.

Definition 2.01: A fuzzy set A of a $fts(X,T)$ is called:

1) a semi-open fuzzy set, if $A\leq cl(int(A))$ and a semi-closed fuzzy set, if $int(cl(A))\leq 0$ [13]
2) a pre-open fuzzy set, if$A\leq int(cl(A))$ and a pre-closed fuzzy set, if $cl(int(A))\leq A$ [13]
3) a α-open fuzzy set, if$A\leq int(cl(int(A)))$ and a α-closed fuzzy set, if $cl(int(cl(A)))\leq A$ [14]

The semi closure (respectively pre-closure, α-closure) of a fuzzy set A in a $fts(X,T)$is the intersection of all semi closed (respectively pre closed fuzzy set, α-closed fuzzy set) fuzzy sets containing A and is denoted by $scl(A)$ (respectively $pcl(A), \alpha cl(A)$).

Definition 2.02: A fuzzy set A of a $fts(X,T)$ is called:

1) A generalized closed (g-closed) fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$and U is open fuzzy set in(X,T)).[2]
2) A generalized pre-closed (gp-closed) fuzzy set, if pcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X,T).[13]
3) Aα-generalized closed (αg-closed) fuzzy set, if αcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T) . [13,14 &15]
4) A generalized α-closed (gα-closed) fuzzy set, if αcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X,T).[13,14&15]
5) A generalized semi pre closed (gsp-closed) fuzzy set, if spcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X,T).[13,14&15]
6) Ag^*-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T).[7]
7) A$g^\#$-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is αg-open fuzzy set in (X,T).[13&14]

Complement of g-closed fuzzy (respectively gp-closed fuzzy set, αg-closed fuzzy set, $g\alpha$-closed fuzzy set, gsp-closed fuzzy set, g^*-closed fuzzy set and $g^\#$-closed fuzzy set) sets are called g-open (respectively gp-open fuzzy set, αg-open fuzzy set, $g\alpha$-open fuzzy set, gsp-open fuzzy set, g^*-open fuzzy set and $g^\#$-open fuzzy set) sets.

Definition 2.03: Let X and Y be two fuzzy topological Spaces, A function $f: X \to Y$ is called:

1) A fuzzy continuous (f-continuous) if $f^{-1}(A)$ is closed fuzzy set in X, for every closed fuzzy set A of Y.[2]
2) A fuzzy α-continuous (fα-continuous) if $f^{-1}(A)$ is α-closed fuzzy set in X, for every closed fuzzy set A of Y.[13]
3) A fuzzy generalized-continuous (fg-continuous) if $f^{-1}(A)$ is g-closed fuzzy set in X, for every closed fuzzy set A of Y.[13]
4) A fuzzy generalized α-continuous (fgα-continuous) if $f^{-1}(A)$ is gα-closed fuzzy set in X, for every closed fuzzy set A of Y.[2]
5) A fuzzy α-generalized continuous (fαg-continuous) if $f^{-1}(A)$ is αg-closed fuzzy set in X, for every closed fuzzy set A of Y.[13]
6) a fuzzy g^*-continuous (fg^*-continuous) if $f^{-1}(A)$ is g^*-closed fuzzy set in X, for every closed fuzzy set A of Y.[13]
7) A fuzzy $g^\#$-continuous (f$g^\#$-continuous) if $f^{-1}(A)$ is $g^\#$-closed fuzzy set in X, for every closed fuzzy set A of Y.[14]

Definition 2.04: A map $f: X \to Y$ is called:

1) fuzzy –open (f-open) iff $f(V)$ is open –fuzzy set in Y for every open fuzzy set in X [13]
2) fuzzy g-open (fg-open) iff $f(V)$ is g-open –fuzzy set in Y for every open fuzzy set in X [13]
3) fuzzy g^*-open (fg^*-open) iff $f(V)$ is g^*-open –fuzzy set in Y for every open fuzzy set in X [13]
4) fuzzy $g^\#$-open ($fg^\#$-open) iff $f(V)$ is $g^\#$-open –fuzzy set in Y for every open fuzzy set in X [13]

$g^{\#\#}$- CLOSED FUZZY SETS IN FUZZY TOPOLOGICAL SPACES

Definition 3.01: A fuzzy setA of FTS (X,T) is called $g^{\#\#}$-closed fuzzy set, if $\alpha cl(A) \leq U$ whenever $A \leq U \& U$ is αg-open fuzzy set in (X,T)

Theorem 3.02: Every closed fuzzy set is $g^{\#\#}$-closed set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example3.03: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0.2), (b, 0.3), (c, 0.8)\}$, $B = \{(a, 1), (b, 0.8), (c, 0.9)\}$. Let T = { 0,1, A } in a fts, then $(X,T)$is a fts.Therefore the fuzzy subset B is $g^{\#\#}$-closed fuzzy set but not a closed fuzzy set.

Theorem 3.04: Every α-closed fuzzy set is $g^{\#\#}$-closed set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.05: In the example 3.03. Therefore the fuzzy subset B is $g^{\#\#}$-closed fuzzy set but not aα-closed fuzzy set.

Theorem 3.06: Every $g^{\#\#}$-closed fuzzy set isg^*-closed fuzzy set in $ftsX$.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

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Example 3.07: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows,

Let $A = \{(a, 0.3), (b, 0.5), (c, 0.8)\}, B = \{(a, 0.6), (b, 0.4), (c, 0.8)\}$. Let T = { 0,1, A } in a fts, then (X, T) is a fts. Therefore the fuzzy subset B is g^*-closed fuzzy set but not a $g^{\#\#}$-closed set in (X, T).

Theorem 3.08: In any $ftsX$, Every $g^{\#\#}$-closed fuzzy set is g-closed fuzzy set in $ftsX$.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.09: In the example 3.07.Therefore the fuzzy subset B is g -closed fuzzy set but not a $g^{\#\#}$-closed set in (X, T).

Theorem 3.10: In any $ftsX$, Every $g^{\#\#}$-closed fuzzy set is $g\alpha$-closed fuzzy set in $ftsX$.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.11: In the example 3.07. Therefore the fuzzy subset B is $g\alpha$ -closed fuzzy set but not a $g^{\#\#}$-closed set in (X, T).

Theorem 3.12: In any $ftsX$, every $g^{\#\#}$-closed fuzzy set is αg-closed fuzzy set in $ftsX$.But Converse need not be true.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.13: In the example 3.07.Therefore the fuzzy subset B is αg -closed fuzzy set but not a $g^{\#\#}$-closed set in (X, T).

Theorem 3.14: In any $ftsX$, every $g^{\#\#}$-closed fuzzy set is $\boldsymbol{g}^{\#}$-closed fuzzy set in $ftsX$.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.15: In the example 3.07.Therefore the fuzzy subset B is $g^{\#}$-closed fuzzy set but not a $g^{\#\#}$-closed set in (X, T).

Theorem 3.16: In any $ftsX$,if a fuzzy set A is both αg-open fuzzy set & $g^{\#\#}$-closed fuzzy set. Then A is closed fuzzy set.

Proof: Omitted

Theorem 3.17: If A is $g^{\#\#}$-closed fuzzy set in X and $cl(A) \cap (1 - cl(A)) = 0$ .Then there is no non zero αg-closed fuzzy set F, such that $F \leq cl(A) \cap (1 - A)$

Proof: suppose F is any αg-closed fuzzy set in X, such that $F \leq cl(A) \cap (1 - A)$, Now $F \leq (1 - A)$,this implies that $A \leq (1 - F)$, where $1 - F$ is αg-closed fuzzy set. Then $\alpha cl(A) \leq 1 - F$,

As A is $g^{\#\#}$-closed fuzzy set, this implies that $F \leq 1 - \alpha cl(A)$. Then $F \leq \alpha cl(A) \& F \leq 1 - \alpha cl(A)$ therefore $F \leq \alpha cl(A) \cap (1 - \alpha cl(A)) = 0$. Then $F = 0$, hence Theorem proved.

Theorem 3.18: If a fuzzy set A is $g^{\#\#}$-closed fuzzy set in X, such that $A \leq B \leq \alpha cl(A)$. Then B is also $g^{\#\#}$-closed fuzzy set in X.

Proof: Let U be an αg-open fuzzy set in X , such that $B \leq U$. Then $A \leq U$. Since A is $g^{\#\#}$-closed fuzzy set in X. Then by definition $\alpha cl(A) \leq U$.Now $B \leq \alpha cl(A)$, $\alpha cl(B) \leq cl(\alpha cl(A)) = cl(A) \leq U$ that is $\alpha cl(B) \leq U$. Hence B is $g^{\#\#}$-closed fuzzy set in X

Theorem 3.19: A finite union of $g^{\#\#}$-closed fuzzy set is a $g^{\#\#}$-closed fuzzy set.

Proof: Omitted

Definition 3.20: A fuzzy set A of $fts(X, T)$is called $g^{\#\#}$- closed fuzzy sets, if its compliment $1 - A$ is $g^{\#\#}$-closed fuzzy set.

Theorem 3.21: A fuzzy set A of a $fts\, X$ is $g^{\#\#}$-open fuzzy set, iff $F \leq int(A)$ whenever F is αg-closed fuzzy set and $F \leq A$.

Proof: Omitted

Theorem 3.22: Every open fuzzy set in $fts\, X$ is a $g^{\#\#}$-open fuzzy set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.23: Let $X = \{a, b, c\}$ and the fuzzy sets A & B be defined as follows, Let $A = \{(a, 0), (b, 0.4), (c, 0.7)\}$, $B = \{(a, 0.2), (b, 0.5), (c, 0.7)\}$, Let T = { 0,1, A } in a fts, then $(X, T)$is a fts. Thus the fuzzy set B is $g^{\#\#}$-open Fuzzy set but not a open fuzzy set in $fts\, X$.

Theorem 3.24: In a X , every $g^{\#\#}$- open Fuzzy set is g^*- open Fuzzy set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.25: Let $X = \{a, b, c\}$ and the fuzzy sets A & Bbe defined as follows, Let $A = \{(a, 0), (b, 0.6), (c, 0.7)\}$, $B = \{(a, 0.4), (b, 0.5), (c, 0.7)\}$,$1 - B = \{(a, 0.6), (b, 0.5), (c, 0.3)\}$.

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Let T = { 0,1, A } in a fts, then $(X, T)$is a fts. Thus the fuzzy set B is g^{*}- Open Fuzzy set but not a$g^{\#\#}$-open fuzzy set infts (X, T).

Theorem 3.26: In a X , every $g^{\#\#}$- open Fuzzy set is g- open Fuzzy set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.27: In the example 3.25. Thus the fuzzy set B is g- open Fuzzy set but not a $g^{\#\#}$-open fuzzy set infts (X, T).

Theorem 3.28: In a X , every $g^{\#\#}$- open Fuzzy set is αg- open Fuzzy set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.29: In the example 3.25. Thus the fuzzy set B is αg- open Fuzzy set but not a $g^{\#\#}$-open fuzzy set infts (X, T).

Theorem 3.30: In a X , every $g^{\#\#}$- open Fuzzy set is $g\alpha$- open Fuzzy set.

Proof: Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.31: In the example 3.25.Thus the fuzzy set B is $g\alpha$- open Fuzzy set but not a $g^{\#\#}$-open fuzzy set infts (X, T).

Theorem 3.32: In a X , every $g^{\#\#}$- open Fuzzy set is $g^{\#}$- open Fuzzy set.

Proof: Omitted

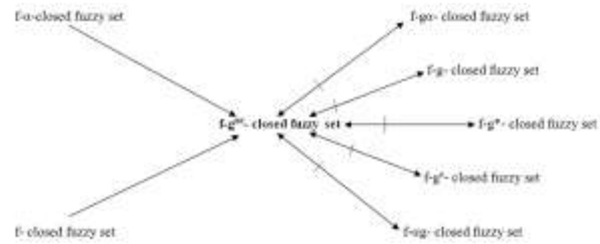
The converse of the above theorem need not be true as seen from the following example.

Example 3.33: In the example 3.07.Thus the fuzzy set B is $g\alpha$- open Fuzzy set but not a $g^{\#\#}$-open fuzzy set infts (X, T).

Theorem 3.34: If $int(A) \leq B \leq A$& if A is $g^{\#\#}$-open fuzzy set. Then B is $g^{\#\#}$-open fuzzy set in a fts (X, T).

Proof: Omitted

Remark 3.35: The above discussions are summarized in the following diagram.



Where A ⟶ B (A ⟷ B) represents A implies B but not conversely. (A and B are independent).

Theorem 3.36: If $A \leq B \leq X$, where A is $g^{\#\#}$-open fuzzy relative to B and B is $g^{\#\#}$-open fuzzy relative to X. Then A is $g^{\#\#}$-open fuzzy relative to fts (X, T).

Proof: Omitted

Theorem 3.37: The finite intersection of $g^{\#\#}$-open fuzzy set is $g^{\#\#}$-open fuzzy set.

Proof: Omitted

Theorem 3.38: If a fuzzy set A is $g^{\#\#}$-closed fuzzy set and $cl(A) \wedge (1 - cl(A)) = 0$. Then $cl(A) \wedge (1 - A)$ is $g^{\#\#}$-open fuzzy set.

Proof: Omitted

4. FUZZY $g^{\#\#}$ -CONTINUOUS MAPPING

In this section the concept of fuzzy $g^{\#\#}$-continuous,fuzzy $g^{\#\#}$-irresolute functions and fuzzy $g^{\#\#}$homeomorphism , fuzzy $g^{\#\#}$-open and fuzzy$g^{\#\#}$-closed mapping in fuzzy topological spaces have been introduced and studied

Definition 4.01: Let X and Y be two fts. A function $f: X \to Y$ is said to be fuzzy $g^{\#\#}$-continuous (briefly $fg^{\#\#}$-continuous) if the inverse image of every open fuzzy set in Y is $g^{\#\#}$-open fuzzy set in X.

Theorem 4.02: A function $f: X \to Y$ is $fg^{\#\#}$-continuous iff the inverse image of every closed fuzzy set in Y is $g^{\#\#}$-closed fuzzy set in X.

Proof: Omitted.

Theorem 4.03: Every f-continuous function is$fg^{\#\#}$-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.04: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a, 0), (b, 0.1), (c, 0.2)\}$, $B = \{(a, 0.4), (b, 0.5), (c, 0.7)\}$, $C = \{(a, 1), (b, 0.9), (c, 0.8)\}$. Consider $T = \{0,1, B\}$

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and $\sigma = \{0,1,A\}$. Then (X,T) and (Y,σ) are fts. Define $f: X \to Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is $fg^{\#\#}$-continuous but not f-continuous as the fuzzy set C is closed fuzzy set in Y and $f^{-1}(C) = C$ is not closed fuzzy set in X but $g^{\#\#}$-closed fuzzy set in X. Hence f is $fg^{\#\#}$-continuous

Theorem 4.05: Every $f\alpha$-continuous function is $fg^{\#\#}$-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.06: In the example 4.04. As the fuzzy set C is closed fuzzy set in Y and $f^{-1}(C) = C$ is not closed fuzzy set in X but $g^{\#\#}$-closed fuzzy set in X. Hence f is $fg^{\#\#}$-continuous

Theorem 4.07: Every $fg^{\#\#}$-continuous function is fg-continuous.

Proof:Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.08: In the example 4.04. f is fg -continuous but not $fg^{\#\#}$-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not $g^{\#\#}$-closed fuzzy set in X. Hence f is fg-continuous.

Theorem 4.09: Every $fg^{\#\#}$-continuous function is fg^{*}-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.10: In the example 4.04. Then f is fg^{*}-continuous but not $fg^{\#\#}$-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not $g^{\#\#}$-closed fuzzy set in X. Hence f is fg^{*}-continuous

Theorem 4.11: Every $fg^{\#\#}$-continuous function is $f\alpha g$-continuous.

Proof:Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.12: In the example 4.04. f is $f\alpha g$ -continuous but not $fg^{\#\#}$-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not $g^{\#\#}$-closed fuzzy set in X. Hence f is $f\alpha g$ -continuous.

Theorem 4.13: Every $fg^{\#\#}$-continuous function is $fg\alpha$-continuous.

Proof:Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.14: In the example 4.04. f is $fg\alpha$ -continuous but not $fg^{\#\#}$-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not $g^{\#\#}$-closed fuzzy set in X. Hence f is $fg\alpha$ -continuous.

Theorem 4.15: Every $fg^{\#\#}$-continuous function is $fg^{\#}$-continuous.

Proof:Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.16: In the example 4.04. f is $fg^{\#}$ -continuous but not $fg^{\#\#}$-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not $g^{\#\#}$-closed fuzzy set in X. Hence f is $fg^{\#}$ -continuous.

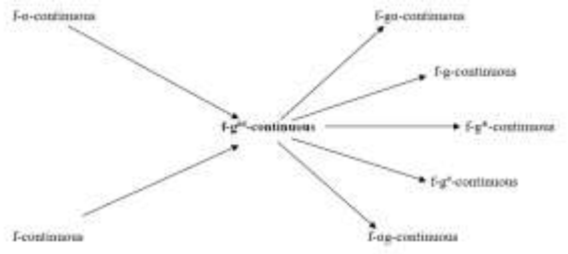
Theorem 4.17: If $f: X \longrightarrow Y$ is $fg^{\#\#}$--continuous and $g: Y \longrightarrow Z$ is f-continuous, then $gof: X \to Z$ is $fg^{\#\#}$-continuous.

Proof: Omitted.

Theorem 4.18: Let X_1 and X_2 be fts and $P_i: X_1\, x X_2 \to X_i$ $(i = 1,2)$be the projection mappings. If $f: X \to X_1\, x\, X_2$is $fg^{\#\#}$- continuous then the $P_i o f: X \to X_i$ $(i = 1,2)$ is $fg^{\#\#}$-continuous.

Proof: Omitted.

Remark 4.19: The following diagram shows the relationship of $fg^{\#\#}$-continuous maps with some other fuzzy maps.



Where A → B (A ↔ B) represents A implies B but not conversely. (A and B are independent).

Theorem 4.20: Every f-strongly continuous function is $fg^{\#\#}$--continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.21: In the example 4.04. The function f is $fg^{\#\#}$-continuous but not f-strongly continuous, for the fuzzy set C in Y, $f^{-1}(C) = C$ is not both open and closed fuzzy set in X

Theorem 4.22: Every f-perfectly continuous function is $fg^{\#\#}$-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.23: In the example 4.04. The function f is $fg^{\#\#}$-continuous but not f-perfectly continuous as the fuzzy set A is open in Y and $f^{-1}(A) = A$ is not both open and closed fuzzy set in X

Theorem 4.24: Every f-completely continuous function is $fg^{\#\#}$-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.25: In the example 4.04. The function f is $fg^{\#\#}$-continuous but not f-completely continuous as the fuzzy set A is open in Y and $f^{-1}(A) = A$ is not regular-open fuzzy set in X

We introduce the following.

Definition 4.26: A function f: X →Y is said to be fuzzy $g^{\#\#}$-irresolute (briefly $fg^{\#\#}$-irresolute) if the inverse image of every $g^{\#\#}$-closed fuzzy set in Y is $g^{\#\#}$-closed fuzzy set in X.

Theorem 4.27: A function $f: X \to Y$ is $fg^{\#\#}$-irresolute iff the inverse image of every $g^{\#\#}$-open fuzzy set in Y is $g^{\#\#}$-open fuzzy set in X.

Proof: Omitted.

Theorem 4.28: Every $fg^{\#\#}$-irresolute function is $fg^{\#\#}$-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.29: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B, C, D and E be defined as follows.$A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 0), (b, 1), (c, 0)\}$, $C = \{(a, 1), (b, 1), (c, 0)\}$, $D = \{(a, 1), (b, 0), (c, 1)\}$, $E = \{(a, 0), (b, 1), (c, 1)\}$. Consider $T = \{0,1, A, B, C, D\}$and $\sigma = \{0,1, C\}$. Then (X, T) and (Y, σ) are fts. Define $f: X \to Y$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is $fg^{\#\#}$-continuous but not $fg^{\#\#}$--irresolute as the fuzzy set in E is$g^{\#\#}$-closed fuzzy set in Y, but $f^{-1}(E) = C$ is not$g^{\#\#}$-closed fuzzy set in X. Hence f is $fg^{\#\#}$-continuous.

Theorem 4.30: if $f: X \to Y$ is $fg^{\#\#}$-continuous, and $g: Y \to Z$ is f-continuous then $gof: X \to Z$ is$fg^{\#\#}$-continuous.

Proof: Omitted.

Theorem 4.31: Let $f: X \to Y, g: Y \to Z$ be two functions. If f and g are $fg^{\#\#}$--irresolute functions then $gof : X \to Z$ is $fg^{\#\#}$--irresolute functions.

Proof: Omitted.

Theorem 4.32: Let $f: X \to Y, g: Y \to Z$ be two functions. If f is $fg^{\#\#}$-irresolute and g is$fg^{\#\#}$-continuous then $gof: X \to Z$ is $fg^{\#\#}$--continuous.

Proof: Omitted.

Definition 4.33: A function $f: X \to Y$ is said to be fuzzy gc-irresolute (briefly fgc-irresolute) function if the inverse image of every g-closed fuzzy set in Y is g-closed fuzzy set in X.

Theorem 4.34:$f: X \to Y$ be a fgc-irresolute and af-closed map. Then $f(A)$ is a $g^{\#\#}$--closed fuzzy set of Y, for every $fg^{\#\#}$- closed fuzzy set A of X.

Proof: Omitted

We introduce the following.

Definition 4.35: A function $f: X \to Y$ is said to be fuzzy $g^{\#\#}$-open (briefly $fg^{\#\#}$- open) if the image of every open fuzzy set in X is $fg^{\#\#}$- open fuzzy set in Y.

Definition 4.36: A function $f: X \to Y$ is said to be fuzzy $g^{\#\#}$- closed (briefly $fg^{\#\#}$- closed) if the image of every closed fuzzy set in X is $fg^{\#\#}$- closed fuzzy set in Y.

Theorem 4.37: Every f-open map is $fg^{\#\#}$- open map.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.38: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows.

$A = \{(a, 0), (b, 0.1), (c, 0.2)\}$,

$B = \{(a, 0.4), (b, 0.5), (c, 0.7)\}$,

$C = \{(a, 1), (b, 0.9), (c, 0.8)\}$.Consider $= \{0,1, A\}$and $\sigma = \{0,1, B\}$. Then (X, T) and (Y, σ) are fts. Define $f: X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $fg^{\#\#}$-open map but not f-open map as the fuzzy set A open fuzzy set in X, its image $f(A) = A$ is not open fuzzy set in Y which is $g^{\#\#}$- open fuzzy set in Y.

Theorem 4.39: Every $f\alpha$-open map is $fg^{\#\#}$- open map.

Proof: Omitted.

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The converse of the above theorem need not be true as seen from the following example.

Example 4.40: In the example 4.38. Then f is $fg^{\#\#}$-open map but not $f\alpha$-open map as the fuzzy set A open fuzzy set in X, its image $f(A) = A$ is not open fuzzy set in Y which is $g^{\#\#}$- open fuzzy set in Y.

Theorem 4.41: Every $fg^{\#\#}$- open map is fg- open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.42: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows.
$A = \{ (a, 0.2), (b, 0.5), (c, 0.3)\}$,
$B = \{(a, 0.8), (b, 0.5), (c, 0.7)\} C = \{(a, 0.5), (b, 0.2), (c, 0.3)\}$.Consider $T = \{0,1, A\}$ and $\sigma = \{0,1, A, B\}$. Then (X, T) and (Y, σ) are fts. Define $f: X \to Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then the function f is fg-open map but not $fg^{\#\#}$-open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $fg^{\#\#}$- open fuzzy set in Y .

Theorem 4.43: Every $fg^{\#\#}$- open map is fg^{*}- open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.44: In the example 4.42. Then the function f is fg^{*}-open map but not $fg^{\#\#}$- open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $fg^{\#\#}$- open fuzzy set in Y .

Theorem 4.45: Every $fg^{\#\#}$- open map is $f\alpha g$- open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.46: In the example 4.42.Then the function f is $f\alpha g$-open map but not $fg^{\#\#}$- open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $fg^{\#\#}$- open fuzzy set in Y .

Theorem 4.47: Every $fg^{\#\#}$- open map is $fg\alpha$- open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.48: In the example 4.42.Then the function f is $fg\alpha$-open map but not $fg^{\#\#}$- open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $fg^{\#\#}$- open fuzzy set in Y .

Theorem 4.49: Every $fg^{\#\#}$- open map is $fg^{\#}$- open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.50: In the example 4.42.Then the function f is $fg^{\#}$-open map but not $fg^{\#\#}$- open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $fg^{\#\#}$- open fuzzy set in Y .

Theorem 4.51: Every f-closed map is $fg^{\#\#}$- closed map..

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.52: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows.
$A = \{ (a, 0), (b, 0.1), (c, 0.2)\}$,
$B = \{(a, 0.4), (b, 0.5), (c, 0.7)\}$,
$C = \{(a, 1), (b, 0.9), (c, 0.8)\}$. Consider $T = \{0,1, A\}$ and $\sigma = \{0,1, B\}$. Then (X, T) and (Y, σ) are fts. Define $f: X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $fg^{\#\#}$- closed map but not f-closed map as the fuzzy set C is closed fuzzy set in X, and its image $f(C) = C$ is $g^{\#\#}$-closed fuzzy set in Y but not closed fuzzy set in Y.

Theorem 4.53: Every $f\alpha$-closed map is $fg^{\#\#}$- closed map..

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.54: In the example 4.52.Then f is $fg^{\#\#}$-closed map but not $f\alpha$-closed map as the fuzzy set C is closed fuzzy set in X, and its image $f(C) = C$ is $g^{\#\#}$-closed fuzzy set in Y but not closed fuzzy set in Y.

Theorem 4.55: A map $f: X \to Y$ is $fg^{\#\#}$- closed iff for each fuzzy set S of Y and for each open fuzzy set U such that $f^{-1}(S) \leq U$, there is a $g^{\#\#}$- open fuzzy set V of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

Proof: Omitted.

Theorem 4.56: If a map $f: X \to Y$ is fgc-irresolute and $fg^{\#\#}$- closed and A is $g^{\#\#}$-closed fuzzy set of X, then $f(A)$ is $g^{\#\#}$-closed fuzzy set in Y.

Proof: Omitted.

Theorem 4.57: If $f: X \to Y$ is f-closed map and $h: Y \to Z$ is $fg^{\#\#}$-closed maps, then $hof: X \to Z$ is $fg^{\#\#}$- closed map.

Proof: Omitted.

Theorem 4.58: Let $f: X \to Y$ be an f-continuous, open and $fg^{\#\#}$- closed surjection. If X is regular fts then Y is regular.

Proof: Omitted.

Theorem 4.59: If $f: X \to Y$ and $h: Y \to Z$ be two maps such that $hof : X \to Z$ is $fg^{\#\#}$-closed map.

i) If f is f-continuous and surjective, then h is $fg^{\#\#}$- closed map.

ii) If h is $fg^{\#\#}$- irresolute and injective, then f is $fg^{\#\#}$- closed map.

Proof: Omitted.

Definition 4.60[13,14]: Let X and Y be two fts. A bijective map $f: X \to Y$ is called fuzzy-homeomorphism (briefly f-homeomorphism) if f and f^{-1} are fuzzy-continuous.

We introduced the following.

Definition 4.61: A function $f: X \to Y$ is called fuzzy $g^{\#\#}$-homeomorphism (briefly $fg^{\#\#}$-homeomorphism) if f and f^{-1} are $fg^{\#\#}$- continuous.

Theorem 4.62: Every f-homeomorphism is$fg^{\#\#}$-homeomorphism.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.63: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 1), (b, 1), (c, 0)\}$, $C = \{(a, 1), (b, 0), (c, 1)\}$. Consider $T = \{0,1, A\}$ and $\sigma = \{0,1, B\}$. Then (X, T) and (Y, σ) are fts. Define f: X → Y by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is $fg^{\#\#}$-homeomorphism but not$f$-homeomorphism as A is open fuzzy set in X and its image of $f(A) = A$ is not open fuzzy set in Y. $f^{-1}: Y \to X$ is not f-continuous.

Theorem 4.64: Let $f: X \to Y$ be a bijective function. Then the following are equivalent:

a) f is $fg^{\#\#}$- homeomorphism.

b) f is $fg^{\#\#}$- continuous and $fg^{\#\#}$- open maps.

c) f is $fg^{\#\#}$- continuous and $fg^{\#\#}$- closed maps.

Proof: Omitted.

Definition 4.65: Let X and Y be two fts. A bijective map $f: X \to Y$ is called fuzzy $fg^{\#\#} - c$-homeomorphism (briefly $fg^{\#\#} - c$-homeomorphism) if f and f^{-1} are fuzzy $g^{\#\#}$-irresolute.

Theorem 4.66: Let X, Y, Z be fuzzy topological spaces and $f: X \to Y, g: Y \to Z$ be $fg^{\#\#}$-c-homeomorphisms then their composition $gof: X \to Z$ is $fg^{\#\#} - c$-homeomorphism.

Proof: Omitted.

Theorem 4.67: Every $fg^{\#\#}$-c-homeomorphism is $fg^{\#\#}$-homeomorphism.

Proof: Omitted.

CONCLUSION:

The newly defined concept of $g^{\#\#}$-closed fuzzy set for Fuzzy topological spaces is properly lies between the class of α-closed Fuzzy set and the class of $g^{\#}$-closed fuzzy set. Explicate expression for $fg^{\#\#}$-closure,interior,irresolute,continuous, homeomorphism mapping have been investigated,

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