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On Fuzzy weakly g**-Continuous Maps and Fuzzy weakly g**-Irresolute Mappings in Fuzzy Topological spaces

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***Abstract:*— The aim of this paper is to introduce new class of Fuzzy sets, namely wg**-closed fuzzy set for Fuzzy topological spaces. This new class is properly lies between the class of closed Fuzzy set and the class of wg-closed fuzzy set, we also introduce application of wg**-closed fuzzy sets, the concept of fuzzy wg**-continuous, fuzzy wg**-irresolute mapping, fuzzy wg**-closed maps, fuzzy wg**-open maps and fuzzy wg**-homeomorphism in Fuzzy topological spaces are also introduced, studied and some of their properties are obtained.**

Classification: **2000 Math Subject classification 54A40**

***Keywords:*— fwg**-closed fuzzy sets, fwg**-continuous, fwg**-irresolute, fwg**-open, fwg**-closed mapping and fwg**-homeomorphism.**

I. INTRODUCTION

Prof. L.A. Zadeh's [19] in 1965 introduced of the concept of 'fuzzy subset', in the year 1968, C L. Chang [4] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Subsequently many researchers like, C.K. Wong[18], R.H. Warren [17],R. Lowen[7], A.S. Mashhour[11], K.K. Azad[1], M. N. Mukherjee[12],G. Balasubramanian &P. Sundaram [2] and many others have contributed to the development of fuzzy topological spaces. The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by C.L.Chang [4] and R.H.Warren [17] are included.

Fuzzy topological spaces and some basic concepts and results on fuzzy topological spaces from the works of C.L.Chang [4], R.H.Warren [17], and C.K.Wong [18] are presented. And some basic preliminaries are included. N.Levine [7] introduced generalized closed sets (g-closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general and fuzzy topology. Dr. Sadanand Patil [14, 15 &16] in the year 2009 and R. Devi and M. Muthtamil Selvan[5] in the year 2004, are introduced and studied g-continuous maps.

The class of wg**- closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of wg- closed fuzzy sets. The class of wg**-closed fuzzy sets is properly placed between the class of closed fuzzy sets and the class of wg- closed fuzzy sets.

II. PRELIMINARIES

Throughout this paper (X, T), (Y,σ) & (Z,η) or (simply X, Y & Z) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset A of a space (X, T). cl (A) , int(A) & C(A) denotes the closure, interior and the compliment of A respectively.

Definition 2.01: A fuzzy set A of a fts (X, T) is called:

1) a semi-open fuzzy set, if $A \le cl(int(A))$ and a semi-closed fuzzy set, if $int(cl(A)) \le 0$ [13]
2) a pre-open fuzzy set, if $A \le int(cl(A))$ and a pre-closed fuzzy set, if $cl(int(A)) \le A$ [13]
3) a α-open fuzzy set, if $A \le int(cl(int(A)))$ and a α-closed fuzzy set, if $cl(int(cl(A))) \le A$ [14]

The semi closure (respectively pre-closure, α-closure) of a fuzzy set A in a fts (X, T) is the intersection of all semi closed (respectively pre closed fuzzy set, α-closed fuzzy set) fuzzy sets containing A and is denoted by scl(A) (respectively pcl(A) , αcl(A)).

Definition 2.02: A fuzzy set A of a fts (X, T) is called:

1) a generalized closed (g-closed) fuzzy set, if cl(A) ≤ U ,whenever A ≤ U and U is open fuzzy Set in (X, T). [2]
2) a weakly-generalized-closed (wg-closed) fuzzy Set, if cl(A) ≤ U , whenever A ≤ U and U is open fuzzy set in (X, T).[14]
3) a weakly-generalized* closed (wg*-closed) fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X,T). [14 ,15&16]

Complement of g-closed fuzzy (respectively wg-closed fuzzy set and wg*-closed fuzzy set) sets are called g-open (respectively wg-open fuzzy set and wg*-open fuzzy set) sets.

Definition 2.03: Let X, Y be two fuzzy topological spaces. A function f: X→Y is called

1) Fuzzy continuous (f-continuous) [14 ,15&16] if $f^{-1}(B)$ is open fuzzy set in X , for every open fuzzy set B of Y
2) Fuzzy generalized- continuous (fg-continuous) function [14 ,15&16] if $f^{-1}(A)$ is g-closed fuzzy set in X , for every closed fuzzy set A of Y
3) Fuzzy g*-continuous (fg*-continuous) function[14 ,15&16] if $f^{-1}(A)$ is g*-closed fuzzy set in X , for every closed fuzzy set A of Y

Definition 2.04: Let X, Y be two fuzzy topological spaces. A function f: X→Y is called

1) Fuzzy -open (f-open) [14, 15&16] iff f (V) is open fuzzy set in Y, for every open fuzzy set in X.
2) Fuzzy g-open (fg-open) [14, 15&16] iff f (V) is g-open- fuzzy set in Y, for every open fuzzy set in X.
3) Fuzzy g*-open (fg*-open) [14, 15&16] iff f(V) is g-open- fuzzy set in Y, for every open fuzzy set in X.

III. Weakly g** CLOSED FUZZY SETS

Definitions 3.01: A fuzzy set A of fuzzy topological space in (X, T) is called weakly g** closed fuzzy sets if cl(int(A)) ≤ U whenever A ≤ U and U is g*- open fuzzy set in (X,T).

Theorem 3.02: Every closed fuzzy set is weakly g** closed fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.03: Let X={a, b, c} and the fuzzy sets A and B be defined as follows
A={(a,0.4),(b,0.5),(c,0.7)}, B={(a,1),(b,0.9),(c,0.8)}.
Let T= {0, 1, A}.Then (X, T) is a fts. Note that the fuzzy subset B is weakly g** closed fuzzy set in (X, T) but not a closed fuzzy set in (X, T).

Theorem 3.04: Every g** - closed fuzzy set is weakly g** - closed fuzzy set in (X, T).

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.05: Let X={a,b,c} fuzzy sets A and B be defined as follows A={(a,0.2),(b,0.5),(c,0.3)} and B={(a,0.5),(b,0.2),(c,0.3)}. Consider T= {0, 1, A}.Then (X, T) is fts. The fuzzy set B is wg*-closed but not g*closed fuzzy set in X.

Theorem 3.06: Every weakly g** closed fuzzy set is weakly g-closed fuzzy set in fts X.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.07: In the example 3.05, The fuzzy set B is wg-closed but not wg**-closed fuzzy set.

Theorem 3.08: Every weakly g** closed fuzzy set is weakly g*-closed fuzzy set in fts X.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.09: In the 3.05, The fuzzy set B is wg*-closed but not wg**-closed fuzzy set.

Theorem 3.10: If a fuzzy set A of a fts X is both open and wg**-closed fuzzy set then it is a closed fuzzy set.

Proof: Suppose a fuzzy set A of fts X is both open and wg**-closed. Now A ≤ A, A is open and so g*-open. Then we have cl(int A) ≤ A which implies cl(A) ≤ A Since A is open. Since A ≤ cl(A), we have cl(A) = A. Thus A is closed fuzzy set.

Theorem 3.11: If a fuzzy set A is both open and wg**-closed then it is both regular open and regular closed fuzzy set.

Proof: Omitted.

Theorem 3.12: If a fuzzy set A of fts X is open and wg**-closed then A is g*-closed.

Proof: Omitted.

Theorem 3.13: If a fuzzy set A of fts X is open and wg- closed then A is wg* closed.

Proof: Suppose A is open and wg-closed. Let A ≤ U where U is g-open. Since A is wg-closed we have A ≤ A, A is open implies cl(int A) ≤ A ≤ U. That is cl(int A) ≤ U and hence A is wg*-closed.

Theorem 3.14: If A is wg**-closed fuzzy set and cl(int A) ∧ (1−cl(int A))=0 then cl(int A) ∧ (1-A) has no non zero g-closed fuzzy set.

Proof: Suppose F is any g-closed fuzzy set such that F ≤ cl(int A) ∧ (1−A). Now F ≤ 1−A, which implies that A ≤ 1−F, 1−F is g-open. Since A is wg*-closed, cl(int A) ≤ 1−F, Which implies F ≤ 1− cl(int A).Thus F ≤ cl (int A) and F ≤ 1− cl(int A).Therefore F ≤ cl(int A) ∧ (1-cl(int A)) = 0. Which implies that F = 0. Hence the result follows.

Theorem 3.15: If a fuzzy set A is weakly g** closed fuzzy set in X such that A ≤ B ≤ cl(int A), then B is also a weakly g** closed fuzzy set in X.

Proof: Let U be a g-open fuzzy set in X, such that B ≤ U, then A ≤ U. Since A is weakly g* closed fuzzy set, then by definitions cl(int(A)) ≤ U. Now int B ≤ B ≤ cl(int(A)),which implies cl(int(B)) ≤ cl(cl(int A) = cl(int A) ≤ U. That is cl(int(B)) ≤ U. Hence B is a weakly g** closed fuzzy set.

Theorem 3.16: Let A ≤ Y ≤ X and suppose that A is wg**- closed in fts X. Then A is wg**-closed relative to Y.

Proof: Given that A ≤ Y ≤ X and A is wg**- closed fuzzy set .To prove that A is wg**-closed relative to Y. Let A ≤ Y ∧ G. Then A ≤ G where G is g*-open in X Since A is wg** -closed in X. cl(int A) ≤ G. which implies that cl(int A) ≤ Y ∧ cl(int A) and therefore cl(int A) ≤ Y ∧ G. Hence A is wg**-closed relative to Y.

We introduce weakly g** open fuzzy set

Definition 3.17: A fuzzy set A of the fts (X,T) is called weakly g ** open fuzzy set if its complement 1− A is weakly g** closed fuzzy set.

Theorem 3.18: A fuzzy set A of a fts X is weakly g** open fuzzy set iff F ≤ int(cl A)
Whenever F is g*-closed fuzzy set and F ≤ A

Proof: Omitted.

Theorem 3.19: Every open fuzzy set is a weakly g** open fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.20: Let X= {a,b,c}. Define the fuzzy sets A and B as follows. A={(a,0.4),(b,0.5),(c,0.7)}, B={(a,0),(b,0.1),(c,0.2)}. Then (X, T) is a fts with the fuzzy topology T= {0, 1, A}. Here the fuzzy set B is weakly g** open fuzzy set but not a open fuzzy set in X.

Theorem 3.21: If a fts every wg**-open fuzzy set is wg-open.

Proof: Omitted.

The converse of the above theorem need not be true as shown from the following example.

Example 3.22: In the example 3.20, Here the fuzzy set B is weakly g closed fuzzy set but not a wg* closed fuzzy set in X.

Theorem 3.23: If int (cl (A)) ≤ B ≤ A and if A is weakly g** open fuzzy set, B is weakly g** open fuzzy set in a fts X.

Proof: We have int(cl (A)) ≤ B ≤ A .Then (1− A) ≤ (1−B) ≤ cl(int(1−A)) and since (1−A) is weakly g ** closed fuzzy set and by theorem 2.19 .we have (1−B) is weakly g** closed fuzzy set in X. Hence B is weakly g** open fuzzy set is fts X.

Theorem 3.24: Every g*-open fuzzy set is wg**-open.

Proof: Omitted.

The converse of the above theorem need not be true as shown from the following example.

Example 3.25: In the example 3.20, the fuzzy set 1−B is wg**-open but not g*-open in X.

Theorem 3.26: A Finite union of weakly g** closed fuzzy set is a weakly g** closed fuzzy set.

Proof: Omitted.

Remark 3.27: The intersection of two wg**-open fuzzy sets need not be wg** -open.

Fuzzy wg*-closure (wg* cl) and fuzzy wg*-interior (wg* int) of a fuzzy set are defined as follows.

Definition 3.28: If A is any fuzzy set in a fts, then
wg** cl(A)= ∧{U:U is wg**-closed fuzzy set and A ≤ U}
wg**int(A)= ∨{V:V is wg**-open fuzzy set and A ≥ V}

Theorem 3.29: Let A be any fuzzy set in a fts (X, T) Then
wg** cl(A)=wg**cl(1− A)=1− wg**cl(1− A)=1-wg**int(A) and wg** int(1−A)=1−wg**cl(A)

Proof: Omitted.

Theorem 3.30: In a fts (X, T), a fuzzy set A is weakly g**-closed iff A= wg** -cl(A).

Proof: Let A be a weakly g** -closed fuzzy set in fts (X, T).since A ≤ A and A is weakly g** -closed fuzzy set, A ∈ {f:f is weakly g** -closed fuzzy set and A ≤ f} and A ≤ f implies that
A= ∧ {f:f is weakly g** -closed fuzzy set and A ≤ f} that is A = wg**-cl(A)

Conversely, Suppose that A= wg**-cl(A), that is A = ∧ { f:f is weakly g** -closed fuzzy set and A ≤ f}. This implies that A ∈ {f:f is weakly g** -closed fuzzy set and A ≤ f}. Hence A is weakly g**-closed fuzzy set.

Theorem 3.31: In fts X be the following results hold for fuzzy weakly g**-closer

1) weakly g**-cl(0)=0
2) weakly g**-cl(A) is weakly g**-closed fuzzy set in X
3) weakly g**-cl(A) ≤ weakly g**-cl(B) if A ≤ B
4) weakly g**-cl(weakly g**-cl(A)) =. weakly g**-cl(A)
5) weakly g**-cl(A ∨ B) ≥ weakly g**-cl(A) ∨ weakly g**-cl(B)
6) weakly g**-cl(A ∧ B) ≤ weakly g**-cl(A) ∧ weakly g**-cl(B)

Proof: The easy verification is omitted.

Theorem 3.32: In a fts X, a fuzzy set A is weakly g**-open fuzzy set iff A=wg**-int(A).

Proof: Omitted.

Theorem 3.33: In fts X be the following results hold for fuzzy weakly g**-interior

1) weakly g**-int((0)=0
2) weakly g**-int(A) is weakly g**-open fuzzy set in X
3) weakly g**-int(A) ≤ weakly g**-int(B) if A ≤ B
4) weakly g**-int(weakly g**-int(A)) = weakly g**-int(A)
5) weakly g**-int(A∨B) ≥ weakly g**-int(A) ∨ weakly g**-int(B)
6) weakly g**-int(A∧B) ≤ weakly g**-int(A) ∧ weakly g**-int(B)

Proof: The easy verification is omitted.

Theorem 3.34: In a fts X every weakly g** open fuzzy set is wg-open fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.35: In the example 3.20, the fuzzy subset 1−B={(a,0.4),(b,0.4),(c,0.5)} is wg -open fuzzy set but not weakly g** open fuzzy set in X.

Theorem 3.36: In a fts X, every weakly g** open fuzzy set is wg*-open fuzzy set.

Proof: Omitted.

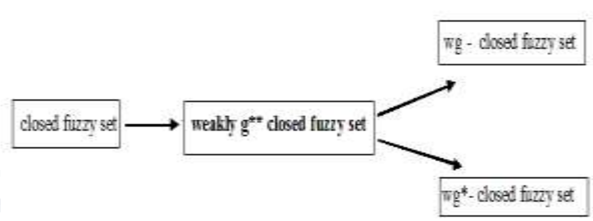
The converse of the above theorem need not be true as seen from the following example.

Example 3.37: In the example 3.20, the fuzzy subset 1−B={(a,0.4),(b,0.4),(c,0.5)} is wg*-open fuzzy set but not weakly g** open fuzzy set in X.

Theorem 3.38: If A ≤ B ≤ X where A is weakly g** open fuzzy relative to B and B is weakly g** open fuzzy relative to X, Then A is weakly g** open fuzzy relative to fts X.

Proof: Omitted.

Remarks 3.39: The following diagram shows the relationships of weakly g** closed fuzzy sets with some other fuzzy sets.



Where A ⟶ B (A ⟷ B)
Represents A implies B but not conversely. (A and B are independent).

IV. FUZZY WEAKLY g** -CONTINUOUS MAPPING

In this section the concept of fuzzy wg**-continuous, fuzzy wg**-irresolute functions and fuzzy wg**-homeomorphism, fuzzy wg**-open and fuzzy wg**-closed mapping in fuzzy topological spaces have been introduced and studied.

Definition 4.01: Let X and Y be two fts. A function f: X→Y is said to be fuzzy wg**-continuous (briefly fwg**-continuous) if the inverse image of every open fuzzy set in Y is wg**-open fuzzy set in X.

Theorem 4.02: A function f: X→Y is fwg**-continuous iff the inverse image of every closed fuzzy set in Y is wg**-closed fuzzy set in X.

Proof: Omitted.

Theorem 4.03: Every f-continuous function is fwg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.04: Let X=Y= {a,b,c} and the fuzzy sets A,B and C be defined as follows.
A={(a,0),(b,0.1),(c,0.2)}, B={(a,0.4),(b,0.5),(c,0.7)},
C={(a,1),(b,0.9),(c,0.8)}. Consider T= {0, 1, B} and σ = {0, 1, A}. Then (X, T) and (Y,σ) are fts. Define f: X→Y by f (a) =a, f (b) =b and f(c) =c. Then f is fwg**-continuous but not f-continuous as the fuzzy set C is closed fuzzy set in Y and f^{-1}(C) =C is not closed fuzzy set in X but wg**-closed fuzzy set in X. Hence f is fwg**-continuous

Theorem 4.05: Every fwg**-continuous function is fwg- continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.06: Let X=Y= {a,b,c} and the fuzzy sets A,B,C and D be defined as follows.
A={(a,0.2),(b,0.5),(c,0.3)},
B={(a,0.8),(b,0.5),(c,0.7)},
C={(a,0.5),(b,0.2),(c,0.3)}and
D={(a,0.5),(b,0.8),(c,0.7)}. Consider T={0,1,A} and σ ={0,1,A,B}. Then (X, T) and (Y,σ) are fts. Define f: X→Y by f (a) =b, f (b) =a and f(c) =c. Then f is fwg-continuous but not fwg**-continuous as the inverse image of closed fuzzy set A in Y is f^{-1}(A) =C which is not wg**-closed fuzzy set in X. Hence f is fwg-continuous.

Theorem 4.07: Every fwg**-continuous function is fwg*- continuous.

Proof: Omitted.

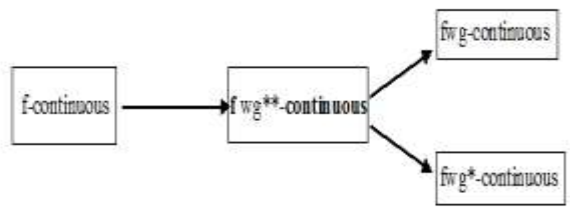
The converse of the above theorem need not be true as seen from the following example.

Example 4.08: In the example 4.06, Then f is fwg*-continuous but not fwg**-continuous as the inverse image of closed fuzzy set A in Y is f^{-1}(A) =C which is not wg**-closed fuzzy set in X. Hence f is fwg*-continuous

Theorem 4.09: If f: X→Y is f wg**-continuous and g: Y → Z is f-continuous, then gof:X→ Z is fwg**-continuous.

Proof: Omitted.

Remark 4.10: The following diagram shows the relationship of fwg**-continuous maps with some other fuzzy maps.



Where A ⟶ B (A ⟵|⟶ B) represents A implies B but not conversely. (A and B are independent).

Theorem 4.11: Let X1 and X_2 be fts and P_i: X1 x X_2→X_i (i=1, 2) be the projection mappings. If f: X→X_1 x X_2 is fwg**-continuous then the $P_i$of:X→X_i (i=1,2) is fwg**-continuous.

Proof: Omitted.

Theorem 4.12: Every f -strongly continuous function is fwg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.13: In the example 3.05, the function f is fwg**-continuous but not f -strongly continuous, for the fuzzy set C in Y, f^{-1}(C) =C is not both open and closed fuzzy set in X

Theorem 4.14: Every f -perfectly continuous function is fwg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.15: In the example 3.05, the function f is fwg**-continuous but not f-perfectly continuous as the fuzzy set A is open in Y and f^{-1}(A) = A is not both open and closed fuzzy set in X

Theorem 4.16: Every f -completely continuous function is fwg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.17: In the example 3.05, the function f is fg**-continuous but not f -completely continuous as the fuzzy set A is open in Y and $f^{-1}(A)$ =A is not regular-open fuzzy set in X

We introduce the following.

Definition 4.18: A function f: X→Y is said to be fuzzy wg**-irresolute (briefly fwg**-irresolute) if the inverse image of every wg**-closed fuzzy set in Y is wg**-closed fuzzy set in X.

Theorem 4.19: A function f: X→Y is fwg**-irresolute iff the inverse image of every wg**-open fuzzy set in Y is wg**-open fuzzy set in X.

Proof: Omitted.

Theorem 4.20: Every fwg**-irresolute function is fwg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.21: Let X = Y = {a,b,c} and the fuzzy sets A,B,C,D and E be defined as follows.
A= { (a,1),(b,0),(c,0)} , B = {(a,0),(b,1),(c,0)}
C= {(a,1),(b,1),(c,0)}, D = {(a,1),(b,0),(c,1)},
E = {(a, 0),(b,1),(c,1)}. Consider
T = {0,1,A,B,C,D}and σ = {0,1,C }. Then (X, T) and (Y,σ) are fts. Define f: X→Y by f(a)=b, f(b) = c and f(c) = a. Then f is fwg**-continuous but not fwg**-irresolute as the fuzzy set in E is wg**-closed fuzzy set in Y, but $f^{-1}(E)$ = C is not wg**-closed fuzzy set in X. Hence f is fwg**-continuous.

Theorem 4.22: If f: X→Y is fwg**-continuous, and g: Y→Z is f-continuous then gof: X→Z is f wg**-continuous.

Proof: Omitted.

Theorem 4.23: Let f: X→Y, g: Y→Z be two functions. If f and g are fwg**-irresolute functions then gof: X→Z is fwg**-irresolute functions.

Proof: Omitted.

Theorem 4.24: Let f: X→Y, g: Y→Z be two functions. If f is fwg**-irresolute and g is fwg**-continuous then gof: X→Z is fwg**-continuous.

Proof: Omitted.

Definition 4.25: A function f: X→Y is said to be fuzzy gc-irresolute (briefly fgc-irresolute) function if the inverse image of every g-closed fuzzy set in Y is g-closed fuzzy set in X.

Theorem 4.26: f: X→Y be a fgc-irresolute and a f-closed map. Then f (A) is a wg**-closed fuzzy set of Y, for every wg**-closed fuzzy set A of X.

Proof: Omitted.

We introduce the following.

Definition 4.27: A function f: X→Y is said to be fuzzy wg**-open (briefly fwg**-open) if the image of every open fuzzy set in X is wg**-open fuzzy set in Y.

Definition 4.28: A function f: X→Y is said to be fuzzy wg**-closed (briefly fwg**-closed) if the image of every closed fuzzy set in X is wg**-closed fuzzy set in Y.

Theorem 4.29: Every f-open map is fwg**-open map.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.30: Let X = Y = {a,b,c} and the fuzzy sets A,B, and C be defined as follows.
A= {(a,0),(b,0.1),(c,0.2)} , B={(a,0.4),(b,0.5),(c,0.7)}
C= {(a,1),(b,0.9),(c,0.8)}. Consider
T = {0,1,A}and σ = {0,1,B }. Then (X, T) and (Y,σ) are fts. Define f: X→Y by f(a)=a, f(b) = b and f(c) = c. Then f is fwg**-open map but not f-open map as the fuzzy set A open fuzzy set in X, its image f(A) = A is not open fuzzy set in Y which is wg**-open fuzzy set in Y.

Theorem 4.31: Every fwg**-open map is fwg-open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.32: Let X = Y = {a,b,c} and the fuzzy sets A,B, and C be defined as follows.
A= {(a,0.2),(b,0.5),(c,0.3)},
B = {(a,0.8),(b,0.5),(c,0.7)},
C= {(a,0.5),(b,0.2),(c,0.3)}. Consider
T = {0,1,A}and σ = {0,1,A,B }. Then (X, T) and (Y,σ) are fts. Define f: X→Y by f(a)=b, f(b) = a and f(c) = c. Then the function f is fgs-open map but not fwg**-open map as the image of open fuzzy set A in X is f(A) = C open fuzzy set in Y but not wg**-open fuzzy set in Y .

Theorem 4.33: Every f-closed map is fwg**-closed map.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.34: Let X = Y = {a,b,c} and the fuzzy sets A,B, and C be defined as follows.
A = {(a,0),(b,0.1),(c,0.2)},
B = {(a,0.4),(b,0.5),(c,0.7)},
C = {(a,1),(b,0.9),(c,0.8)}. Consider
T = {0,1,A}and σ = {0,1,B }. Then (X, T) and (Y,σ) are fts. Define f: X→Y by f(a)=a, f(b) = b and f(c) = c. Then f is fwg**-closed map but not f-closed map as the fuzzy set C is closed fuzzy set in X, and its image f(C) = C is wg**-closed fuzzy set in Y but not closed fuzzy set in Y.

Theorem 4.35: A map f:X→Y is fwg**-closed iff for each fuzzy set S of Y and for each open fuzzy set U such that $f^{-1}(S) \leq U$, there is a wg**-open fuzzy set V of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.
Proof: Omitted.
Theorem 4.36: If a map f: X→Y is fgc-irresolute and fwg**- closed and A is wg**- closed fuzzy set of X, then f(A) is wg**- closed fuzzy set in Y.
Proof: Omitted.
Theorem 4.37: If f: X→Y is f-closed map and h: Y→Z is fwg**- closed maps, then hof: X→Z is fwg**- closed map.
Proof: Omitted.
Theorem 4.38: Let f: X→Y be an f -continuous, open and fwg**- closed surjection. If X is regular fts then Y is regular.
Proof: Omitted.
Theorem 4.39: If f: X→Y and h: Y→Z be two maps such that hof: X→Z is fwg**- closed map.
i) If f is f-continuous and surjective, then h is fwg**- closed map.
ii) If h is fwg**- irresolute and injective, then f is fwg**- closed map.

Proof: Omitted.
Definition 4.40: Let X and Y be two fts. A bijective map f: X→Y is called fuzzy-homeomorphism (briefly f-homeomorphism) if f and f^{-1} are fuzzy-continuous.
We introduced the following.
Definition 4.41: A function f: X→Y is called fuzzy wg**- homeomorphism (briefly wg**-homeomorphism) if f and f^{-1} are wg**- continuous.
Theorem 4.42: Every f-homeomorphism is fwg**-homeomorphism.
Proof: Omitted.
The converse of the above theorem need not be true as seen from the following example.

Example 4.43: Let X=Y= {a,b,c} and the fuzzy sets A, B and C be defined as follows. A={(a,1),(b,0),(c,0)}, B={(a,1),(b,1),(c,0)}, C={(a,1),(b,0),(c,1)}. Consider T= {0,1,A} and σ={0,1,B}. Then (X, T) and (Y,σ) are fts. Define
f: X→Y by f(a)=a, f(b)=c and f(c)=b. Then f is fwg**- homeomorphism but not f-homeomorphism as A is open fuzzy set in X and its image of f(A)=A is not open fuzzy set in Y. f^{-1}:Y→X is not f-continuous.
Theorem 4.44: Let f: X→Y be a bijective function. Then the following are equivalent:
a) f is fwg**- homeomorphism.
b) f is fwg**- continuous and fwg**- open maps.
c) f is fwg**- continuous and fwg**- closed maps.

Proof: Omitted.
Definition 4.45: Let X and Y be two fts. A bijective map f: X→Y is called fuzzy fwg**- c-homeomorphism (briefly fwg**- c-homeomorphism) if f and f^{-1} are fuzzy wg**- irresolute.
Theorem 4.46: Let X, Y, Z be fuzzy topological spaces and f: X→Y, g: Y→Z be fwg**- c-homeomorphisms then their composition gof: X→Z is fwg**- c-homeomorphism.
Proof: Omitted.
Theorem 4.47: Every fwg**- c-homeomorphism is fwg**- homeomorphism.
Proof: Omitted.

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