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Some New Fuzzy g**- Open Sets, Fuzzy g** - Irresolute and Fuzzy g** - Homeomorphism Mappings in Fuzzy Topological spaces

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***Abstract:*— The aim of this paper is to introduce new class of Fuzzy sets, namelyg**-closed fuzzy set for Fuzzy topological spaces. This new class is properly lies between the class of closed Fuzzy set and the class of g-closed fuzzy set, we also introduce application of g** -closed fuzzy sets, the concept of fuzzy g**-continuous, fuzzy g**-irresolute mapping, fuzzy g**-closed maps, fuzzy g**-open maps and fuzzy g** -homeomorphism in Fuzzy topological spaces are also introduced, studied and some of their properties are obtained.**

***Classification:* 2000 Math Subject classification 54A40**

***Keywords and phrases:* fg^{**}- closed fuzzy sets, fg^{**}-continuous, fg^{**}-irresolute, fg^{**}-open, fg^{**}-closed mapping and fg^{**} -homeomorphism.**

I. INTRODUCTION

Proof. L.A. Zadeh's [19] in 1965 introduced of the concept of 'fuzzy subset', in the year 1968, C L. Chang [4] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Subsequently many researchers like, C.K. Wong[18], R.H. Warren [17],R. Lowen[7], A.S. Mashhour[11], K.K. Azad[1], M. N. Mukherjee[12],G. Balasubramanian &P. Sundaram [2] and many others have contributed to the development of fuzzy topological spaces. The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by C.L.Chang[4] and R.H.Warren [17] are included. Fuzzy topological spaces and some basic concepts and results on fuzzy topological spaces from the works of C.L.Chang [4] , R.H.Warren [17], and C.K.Wong[18] are presented.

And some basic preliminaries are included. N.Levine [7] introduced generalized closed sets (g-closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general and fuzzy topology.Dr. Sadanand Patil [13,14&15] in the year 2009 and R. Devi and M. Muthtamil Selvan[5] in the year 2004, are introduced and studied g-continuous maps. The class of $g\#$- closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of $g*$- closed fuzzy sets. The class of $g*$- closed fuzzy sets is properly placed between the class of closed fuzzy sets and the class of g- closed fuzzy sets.

II. PRELIMINARIES:

Throughout this paper $(X,T),(Y,\sigma)\&(Z,\eta)$or (simply$X,Y,\&Z$) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset A of a space $(X,T).cl(A),int(A)\&C(A)$ denotes the closure, interior and the compliment of A respectively.

Definition 2.01: A fuzzy set A of a $fts(X,T)$ is called:

1) a semi-open fuzzy set, if $A\le cl(int(A))$ and a semi-closed fuzzy set, if $int(cl(A))\le 0$[13]
2) a pre-open fuzzy set, if$A\le int(cl(A))$ and a pre-closed fuzzy set, if $cl(int(A))\le A$ [13]
3) a α-open fuzzy set, if$A\le int(cl(int(A)))$ and a α-closed fuzzy set, if $cl(int(cl(A)))\le A$ [14]

The semi closure (respectively pre-closure, α-closure) of a fuzzy set A in a $fts(X,T)$is the intersection of all semi closed (respectively pre closed fuzzy set, α-closed fuzzy set) fuzzy sets containing A and is denoted by $scl(A)$ (respectively $pcl(A), \alpha cl(A)$).

Definition 2.02: A fuzzy set A of a $fts(X,T)$ is called:

1) a generalized closed (g-closed) fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$and U is open fuzzy set in(X,T)).[2]
2) a g^*-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T).[7]
3) a sg -closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T).[7]
4) a gs-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T).[7]
5) a gsp-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T).[7]
6) a α-generalized closed (αg-closed) fuzzy set, if αcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T) . [13,14 &15]
7) a generalized α closed (gα-closed) fuzzy set, if αcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T) . [13,14 &15]

Complement of g-closed fuzzy (respectively gp-closed fuzzy set, $g*$-closed fuzzy set, sg-closed fuzzy set, gs-closed fuzzy set, gsp-closed fuzzy set and -closed fuzzy set) sets are called g-open (respectively gp-open fuzzy set, g*-open αgfuzzy set, sg-open fuzzy set, gs-open fuzzy set, gs-open fuzzy set and αg-open fuzzy set) sets.

Definition 2.03: Let X ,Y be two fuzzy topological spaces. A function f:X→Y is called

1) Fuzzy continuous (f-continuous)[13,14,15] if f^{-1}(B) is open fuzzy set in X ,for every open fuzzy set B of Y
2) Fuzzy generalized- continuous (fg-continuous) function[13,14,15] if f^{-1}(A) is g-closed fuzzy set in X ,for every closed fuzzy set A of Y
3) Fuzzy generalized semi- continuous (fgs-continuous) function[13,14,15] if f^{-1}(A) is gs-closed fuzzy set in X ,for every closed fuzzy set A of Y
4) Fuzzy generalized semi-pre-continuous (fgsp-continuous) function[13,14,15] if f^{-1}(A) is gsp-closed fuzzy set in X ,for every closed fuzzy set A of Y
5) Fuzzy generalized α-continuous (fgα-continuous) function[13,14,15] if f^{-1}(A) is gα-closed fuzzy set in X ,for every closed fuzzy set A of Y
6) Fuzzy α generalized -continuous (fαg-continuous) function[13,14,15] if f^{-1}(A) is αg-closed fuzzy set in X ,for every closed fuzzy set A of Y
7) Fuzzy g* -continuous (fg*-continuous) function[13,14,15] if f^{-1}(A) is g*-closed fuzzy set in X ,for every closed fuzzy set A of Y
8) Fuzzy generalized c-irresolute (fgc-continuous) function[13,14,15] if f^{-1}(A) is fc-closed fuzzy set in X ,for every g- closed fuzzy set A of Y

Definition 2.04: Let X ,Y be two fuzzy topological spaces. A function f:X → Y is called Fuzzy -open (f-open)[13,14,15] iff f(V) is open fuzzy set in Y ,for every open fuzzy set in X.

1) Fuzzy g-open (fg-open)[13,14,15] iff f(V) is g-open- fuzzy set in Y ,for every open fuzzy set in X.
2) Fuzzy g*-open (fg*-open)[13,14,15] iff f(V) is g-open- fuzzy set in Y ,for every open fuzzy set in X.

III. g** - CLOSED FUZZY SETS

Definition 3.01: A Fuzzy set A of a Fuzzy Topological Space (X, T) is called g** -closed Fuzzy Set If cl(A) ≤ U whenever A≤U & g* -open Fuzzy Set in (X ,T).

Theorem 3.02 : Every closed Fuzzy Set is g**-closed Fuzzy Set.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.03 : Let X= {a b c } and the fuzzy set A and B be defined as follows;

A={a, 0.4},{b,0.5},{c,0.7}, B = {(a,1) , (b,0.9) , (c,,0.8) } .Let t=T = {0, 1, A} is a fts. Note that the fuzzy subset B is g**-closed fuzzy set in (X,T).

Theorem 3.04: Every g**- set is gs-closed fuzzy set in fts X.

Proof : Omitted

The converse of the above theorem need not be true as seen from the following example.

Example 3.05 : Let X = {a , b, c} and the fuzzy sets A and B defined as follows ;

A = { (a, 0.4), (b, 0.5), (c,0.7) }, B = { (a, 0.6), (b, 0.6),(c, 0.5) }, C = { (a, 0.3), (b, 0.4), (c, 0.2) } and D = { (a, 0.3), b, 0.4), (c, 0.2) }. Let T = {0, 1, A}. Then (X, T) is fts.. Here the fuzzy set B is gs-closed fuzzy set but not g**-closed set in (X, T).

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Theorem 3.06 : Every g**-closed fuzzy set is gsp-closed fuzzy set in fts X.
Proof : Omitted

The converse of the above theorem need not be true as seen from the following example.
Example 3.07 : In the example 3.05 , (X, T) is a fts. Here the fuzzy set B is gsp-closed fuzzy set but not g**-closed fuzzy set in(X, T).
Theorem 3.08 : Every g**-closed fuzzy set is sg-closed fuzzy set in fts X.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.
Example 3.09 : In the example 3.05 , (X, T) is a fts. Here the fuzzy set B is sg-closed fuzzy set but not g**-closed fuzzy set in (X, T).
Theorem 3.10: Every g**-closed fuzzy set is g*-closed fuzzy set in fts X.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example
Example 3.11 : In the example 3.05 , (X, T) is a fts. Here the fuzzy set B is g*-closed fuzzy set but not g**-closed fuzzy set in (X, T).
Theorem 3.12 : Every g**-closed fuzzy set is g-closed fuzzy set in fts X.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.
Example 3.13 : In the example 3.05 , (X, T) is a fts. Here the fuzzy set B is g-closed fuzzy set but not g**-closed fuzzy set in (X, T).
Theorem 3.14: Every g**-closed fuzzy set is αg-closed fuzzy set in fts X.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.
Example 3.15 : In the example 3.05 , (X, T) is a fts. Here the fuzzy set B is αg-closed fuzzy set but not g**-closed fuzzy set in (X, T).
Theorem3.16 : Every g**-closed fuzzy set is gα -closed fuzzy set in fts X.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.
Example 3.17: In the example 3.05 , (X, T) is a fts. Here the fuzzy set B is αg-closed fuzzy set but not g**-closed fuzzy set in (X, T).

Theorem 3.18 : In a fts X, if a fuzzy set A is both g*-open fuzzy set and g**-closed fuzzy set, then A is closed set.
Proof: Omitted.
Theorem 3.19: if A is g**-closed fuzzy set and cl (A)∧(1–cl(A))=0, then there is no non-zero g*-closed fuzzy set F , such that F≤cl(A)∧(1–A).
Proof : Omitted.
Theorem 3.20: If a fuzzy set A is g**-closed fuzzy set in X such that A ≤ B ≤ cl(A), then B is also a g**-closed fuzzy set in X.
Proof : Omitted.
Theorem 3.21 : A Finite union of g**-closed fuzzy set is a g**-closed fuzzy set.
Proof:Omitted.

We introduce g-open fuzzy set**.

Definition 3.22 : A fuzzy set A of a fts (X, T) is called g**-open fuzzy (briefly g**-open fuzzy set) set if its complement 1–A is g**-closed fuzzy set.

We have the following characterization.
Theorem 3.23 : A fuzzy set A of a fts X is g**-open iff F ≤ int(A). Whenever F is g*-closed fuzzy set and F ≤ A.
Proof : Omitted.
Theorem3.24 : Every open fuzzy set is a g**-open fuzzy set.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.
Example 3.25 : Let X = {a, b, c}. Define the fuzzy sets A and B as fallows.
A= {(a, 0.4), (b, 0.5), (c,0.7)}, B={(a, 0), (b, 0.1), (c, 0.2)}. Then (X,T) is a the fts with the fuzzy topology T = {0, 1, A}. Here the fuzzy set B is g**-open fuzzy set but not a open fuzzy set in X.
Theorem 3.26 : In a fts , Every g*-open fuzzy set is a gs-open fuzzy set.
Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.
Example 3.27: Let X={a,b,c}. Define the fuzzy sets A and B as follows. A={(a,.4),(b,.5),(c,.7)}, B={(a,0),(b,.1),(c,.2)}. Then (X,T) is a fts with the fuzzy topology T={0,1,A}.Here the fuzzy set B is g** open fuzzy set but not a open fuzzy set in X. fuzzy set

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1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is gs-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.28: In a fts X, Every g**-open fuzzy set is a gsp-open fuzzy set.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example3.29 : In the example 3.27, fuzzy set 1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is gsp-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.30 : In a fts , Every g*-open fuzzy set is a sg-open fuzzy set.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.31: In the example 3.27, fuzzy set 1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is sg-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.32 : In a fts , Every g**-open fuzzy set is a g*-open fuzzy set.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.33 : In the example 3.27, fuzzy set 1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is g*-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.34 : In a fts , Every g*-open fuzzy set is a g-open fuzzy set.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.35 : In the example 3.27, fuzzy set 1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is g-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.36 : In a fts , Every g*-open fuzzy set is a αg-open fuzzy set.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 3.37 : In the example 3.27, fuzzy set 1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is αg-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.38 : In a fts , Every g*-open fuzzy set is a gα-open fuzzy set.

Proof : Omitted.

The converse of the above theorem need not be true as seen from the following example.

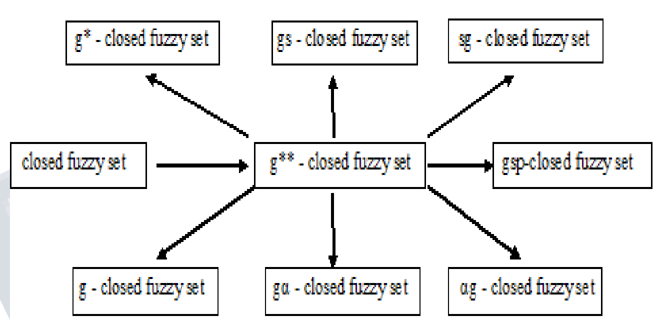
Example 3.39 : In the example 3.27, fuzzy set 1–B={(a, 0.4), (b, 0.4),(c, 0.5)} is gα-open fuzzy set but not g**-open fuzzy set in X.

Theorem 3.40 : If int (A) ≤ B ≤ A and if A is g**-open fuzzy set, then B is g**-open fuzzy set in a fts X.

Proof : Omitted.

Theorem 3.41 : If A ≤ B ≤ X where A is g**-open fuzzy relative to B and B is g**-open fuzzy relative to X, then A is g**-open fuzzy relative to fts X.

Proof : Omitted.



Where A ⟶ B (A —+⟶ B) represents A implies B but not conversely. (A and B are independent).

Theorem 3.42 : Finite intersection of g**-open fuzzy set is a g**-open fuzzy set.

Proof : Omitted.

Theorem 3.43 : If a fuzzy set A is g**-closed fuzzy set and cl (A) ∧ (1–cl (A)) =0, then cl (A) ∧ (1–A) is g**-open set in X.

Proof : Omitted.

Definition 3.45 : For any fuzzy set A in any fts.

fg**-cl (A) = ∧{U:U is g**-closed fuzzy set and A ≤ U }.

fg**- int (A) = {V:v is g**-open fuzzy set and A ≥V }.

Theorem 3.46 : Let A be any fuzzy set in a fts (X, T). Then

And fg**-int (1–A) = 1-fg**-cl (A).

Proof : Omitted.

Theorem 3.47 : In a fts (X,T), a fuzzy set A is g**-closed iff A =fg**-cl (A).

Proof : Omitted.

Theorem 3.48 : In a fts X the following results hold for fuzzy g**-closure.

1) g**-cl(0)=0.
2) g**-ci(A) is **-closed fuzzy set in X.
3) g**-cl(A) ≤g**-cl(B). If A ≤ B.

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4) g**-cl(g**-cl(A)) = g**-cl (A)
5) g**cl(A∨ B) ≥ g**-cl (A) ∨ g**-cl(B)
6) g**cl(A∧ B) ≥ g**-cl (A) ∧ g**-cl(B).

Proof : the easy verification is omitted.

Theorem 3.49 : In a fts X, a fuzzy set A is g**-open fuzzy set iff A =fg**-int (A).

Proof : Omitted.

Theorem 3.50: In a fts X the following results hold for fuzzy g**-interior.

1) g**-int(0)=0.
2) g**-int(A) is **-open fuzzy set in X.
3) g**-int(A) ≤g**-int(B). If A ≤ B.
4) g**-cl(g**-int(A)) = g**-int (A)
5) g**int(A∨ B) ≥ g**-int (A) ∨ g**-int(B)
6) g**int(A∧ B) ≥ g**-int (A) ∧ g**-int(B).

Proof : the easy verification is omit

IV. FUZZY g**-CONTINUOUS MAPPING

In this section the concept of fuzzy g**-continuous , fuzzy g**-irresolute functions and fuzzy g**-homeomorphism , fuzzy g**-open and fuzzy g**-closed mapping in fuzzy topological spaces have been introduced and studied.

Definition 4.01: Let X and Y be two fts. A function $f: X \to Y$ is said to be fuzzy g**-continuous (briefly f g**-continuous) if the inverse image of every open fuzzyset in Y is g**-open fuzzy set in X.

Theorem 4.02:A function $f: X \to Y$ is f g**-continuous iff the inverse image of every closed fuzzy set in Y is wg**-closed fuzzy set in X.

Proof: Omitted.

Theorem 4.03: Every f-continuous function is f g**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.04: Let X=Y={a,b,c} and the fuzzy sets A,B and C be defined as follows. A={(a,0),(b,0.1),(c,0.2)}, B={(a,0.4),(b,0.5),(c,0.7)}, C={(a,1),(b,0.9),(c,0.8)}. Consider T={0,1,B} and σ ={0,1,A}. Then (X,T) and (Y,σ) are fts. Define $f: X \to Y$ by f(a) =a, f(b) =b and f(c) =c. Then f is f wg**-continuous but not f-continuous as the fuzzy set C is closed
fuzzy set in Y and $f^{-1}(C)$ =C is not closed fuzzy set in X but g**-closed fuzzy set in X. Hence f is f g**-continuous

Theorem 4.05: Every f g**-continuous function is fg-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.06 : Let X=Y={a,b,c} and the fuzzy sets A,B,C and D be defined as follows. A={(a,0.2),(b,0.5),(c,0.3)}, B={(a,0.8),(b,0.5),(c,0.7)}, C={(a,0.5),(b,0.2),(c,0.3)}and D={(a,0.5),(b,0.8),(c,0.7)}. Consider T={0,1,A} and σ ={0,1,A,B}. Then (X,T) and (Y,σ) are fts. Define $f: X \to Y$ by f(a) =b, f(b) =a and f(c) =c. Then f is fg-continuous but not f g**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fg-continuous

Theorem 4.07: Every f g**-continuous function is fg*-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.08: In the example 4.06, Then f is fg*-continuous but not fg**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fg*-continuous.

Theorem 4.09: Every f g**-continuous function is fsg-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.10: In the example 4.06, Then f is fsg-continuous but not f g**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fsg-continuous

Theorem 4.11: Every f g**-continuous function is fgs-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.12: In the example 4.06,Then f is fgs-continuous but not fg**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fgs-continuous.

Theorem 4.13: Every f g**-continuous function is fαg-continuous.

Proof: Omitted..

The converse of the above theorem need not be true as seen from the following example.

Example 4.14: In the example 4.06, Then f is fαg-continuous but not f g**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fαg-continuous.

Theorem 4.15: Every f g**-continuous function is fgα-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.16: In the example4.06, Then f is fgα-continuous but not fg**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fgα-continuous

Theorem 4.17: Every f g**-continuous function is fgsp-continuous.

Proof: Omitted.

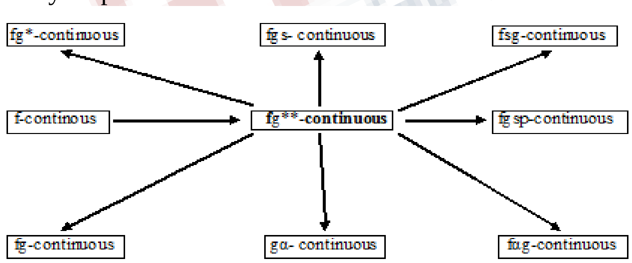
The converse of the above theorem need not be true as seen from the following example.

Example 4.18: In the example 4.06, Then f is fgsp-continuous but not f g**-continuous as the inverse image of closed fuzzy set A in Y is $f^{-1}(A)$ =C which is not g**-closed fuzzy set in X. Hence f is fgsp-continuous.

Theorem 4.19: If $f: X \longrightarrow Y$ is fg**-continuous and $g: Y \longrightarrow Z$ is f-continuous, then gof:X→Z is fg**-continuous.

Proof: Omitted.

Remark 4.20: The following diagram shows the relationship of fg**-continuous maps with ome other fuzzy maps.



Where A ⟶ B (A ⟷ B) represents A implies B but not conversely. (A and B are independent).

Theorem 4.16: Let X_1 and X_2 be fts and P_i:X_1x X_2→X_i(i=1,2)bethe projection mappings. If f:X→X_1 x$X_2$is fg**-continuous then the $P_i$of:X→X_i (i=1,2) is fg**-continuous.

Proof: Omitted.

Theorem 4.21: Every f-strongly continuous function is fg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.22: In the example 3.27, the function f is fg**-continuous but not f-strongly continuous, for the fuzzy set C in Y, $f^{-1}(C)$=C is not both open and closed fuzzy set in X

Theorem 4.23: Every f-perfectly continuous function is fg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.24: In the example 3.27, the function f is fg**-continuous but not f-perfectly continuous as the fuzzy set A is open in Y and $f^{-1}(A)$=A is not both open and closed fuzzy set in X

Theorem 4.25: Every f-completely continuous function is fg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.26: In the example 3.27, the function f is fg**-continuous but not f-completely continuous as the fuzzy set A is open in Y and $f^{-1}(A)$=A is not regular-open fuzzy set in X

We introduce the following.

Definition 4.27:a function f: X →Y is said to be fuzzy g**-irresolute (briefly fg**-irresolute) if the inverse image of every g**-closed fuzzy set in Y is g**-closed fuzzy set in X.

Theorem 4.28:A function f: X→Y is fg**-irresolute iff the inverse image of every g**-open fuzzy set in Y is g**-open fuzzy set in X.

Proof: Omitted.

Theorem 4.29: Every fg**-irresolute function is fg**-continuous.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.30: Let X = Y = {a,b,c} and the fuzzy sets A,B,C,D and E be defined as follows.

A= { (a,1),(b,0),(c,0)} , B = {(a,0),(b,1),(c,0)} C= {(a,1),(b,1),(c,0)}, D = {(a,1),(b,0),(c,1)}, E =

{(a,0),(b,1),(c,1)}. Consider T = {0,1,A,B,C,D}and σ = {0,1,C }. Then (X,T) and (Y,σ) are fts. Define f:X→Y by f(a)=b, f(b) = c and f(c) = a. Then f is fwg**-continuous but not fg**-irresolute as the fuzzy set in E is g**-closed fuzzy set in Y, but f^{-1}(E) = C is not g**-closed fuzzy set in X. Hence f is fg**-continuous.

Theorem 4.31:if f:X→Y is fg**-continuous, and g: Y→Z is f-continuous then gof: X→Z is fg**-continuous.

Proof: Omitted.

Theorem 4.32: Let f:X→Y, g:Y→Z be two functions. If f and g are fg**-irresolute functions then gof :X→Z is fg**-irresolute functions.

Proof: Omitted.

Theorem4.33: Let f:X→Y, g:Y→Zbe two functions. If f is fg**-irresolute and g is fg**-continuous then gof:X→Z is fg**-continuous.

Proof: Omitted.

Definition 4.34:A functionf:X→Y is said to be fuzzy gc-irresolute (briefly fgc-irresolute) function if the inverse image of every g-closed fuzzy set in Y is g-closed fuzzy set in X.

Theorem 4.35 :f:X→Y be a fgc-irresolute and a f-closed map. Then f(A) is a g**-closed fuzzy setoff Y, for every wg**-closed fuzzy set A of X.

Proof: Omitted.

We introduce the following.

Definition 4.36 :A function f:X→Y is said to be fuzzy g**-open (briefly fg**-open) if the image of every open fuzzy set in X is g**-open fuzzy set in Y.

Definition 4.38:A function f:X→Y is said to be fuzzy g**-closed (briefly fg**-closed) if the image of every closed fuzzy set in X is g**-closed fuzzy set in Y.

Theorem 4.39: Every f-open map is fg**-open map.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.40: Let X = Y = {a,b,c} and the fuzzy sets A,B, and C be defined as follows.

A= { (a,0),(b,0.1),(c,0.2)} , B = {(a,0.4),(b,0.5),(c,0.7)} C= {(a,1),(b,0.9),(c,0.8)}. Consider T = {0,1,A}and σ = {0,1,B }. Then (X,T) and (Y,σ) are fts. Define f:X→Y by f(a)=a, f(b) = b and f(c) = c. Then f is fwg**-open map but not f-open map as the fuzzy set A open fuzzy set in X,its image f(A) = A is not open fuzzy set in Y which is g**-open fuzzy set in Y.

Theorem 4.41 : Every fg**-open map is fgs-open.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.42 : Let X = Y = {a,b,c} and the fuzzy sets A,B, and C be defined as follows.

A= { (a,0.2),(b,0.5),(c,0.3)} , B = {(a,0.8),(b,0.5),(c,0.7)} C= {(a,0.5),(b,0.2),(c,0.3)}. Consider T = {0,1,A}and σ = {0,1,A,B }. Then (X,T) and (Y,σ) are fts. Define f:X→Y by f(a)=b, f(b) = a and f(c) = c. Then the function f is fgs-open map but not fg**-open map as the image of open fuzzy set A in X is f(A) = C open fuzzy set in Y but not g**-open fuzzy set in Y .

Theorem 4.43: Every f-closed map is fg**-closed map..

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example4.44 : Let X = Y = {a,b,c} and the fuzzy sets A,B, and C be defined as follows.

A= { (a,0),(b,0.1),(c,0.2)} , B = {(a,0.4),(b,0.5),(c,0.7)} C= {(a,1),(b,0.9),(c,0.8)}. Consider T = {0,1,A}and σ = {0,1,B }. Then (X,T) and (Y,σ) are fts. Define f:X→Y by f(a)=a, f(b) = b and f(c) = c. Then f is fwg**-closed map but not f-closed map as the fuzzy set C is closed fuzzy set in X, and its image f(C) = C is g**-closed fuzzy set in Y but not closed fuzzy set in Y.

Theorem 4.45:A map f:X→Y is fg**-closed iff for each fuzzy set S of Y and for each open fuzzy set U such that f^{-1}(S) ≤U, there is a g**-open fuzzy set V of Y such that S≤V and f^{-1}(V) ≤U.

Proof: Omitted.

Theorem 4.46: If a map $f: X \to Y$ is fgc-irresolute and fg**- closed and A is g**- closed fuzzy set of X then $f(A)$ is g**- closed fuzzy set in Y.

Proof: Omitted.

Theorem 4.47: If $f: X \to Y$ is f-closed map and $h: Y \to Z$ is fg**- closed maps, then $hof: X \to Z$ is fg**- closed map.

Proof: Omitted.

Theorem 4.48 : Let $f: X \to Y$ be an f-continuous, open and fg**- closed surjection. If X is regularfts then Y is regular.

Proof: Omitted.

Theorem 4.49: If $f: X \to Y$ and $h: Y \to Z$ be two maps such that $hof : X \to Z$ is fg**- closed map.

i) If f is f-continuous and surjective, then h is fg**- closed map.

ii) If h is fg**- irresolute and injective, then f is fg**- closed map.

Proof: Omitted.

Definition 4.50: Let X and Y be two fts. Abijective map $f: X \rightarrow Y$ is called fuzzy-homeomorphism (briefly f-homeomorphism) if f and f^{-1} are fuzzy-continuous.

We introduced the following.

Definition 4.51: A function $f: X \rightarrow Y$ is called fuzzy g**- homeomorphism (briefly g**- homeomorphism) if f and f^{-1} are g**- continuous.

Theorem 4.52: Every f-homeomorphism is fg**-homeomorphism.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.53 : Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 1), (b, 1), (c, 0)\}$, $C = \{(a, 1), (b, 0), (c, 1)\}$. Consider $T = \{0, 1, A\}$and $\sigma = \{0, 1, B\}$. Then (X, T) and $(Y, \sigma)$are fts. Define $\text{f}: \text{X} \rightarrow \text{Y}$by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is fg**- homeomorphism but not f-homeomorphism as A is open fuzzy set in X and its image of $f(A) = A$ is not open fuzzy set in Y. $f^{-1}: Y \rightarrow X$is not f-continuous.

Theorem 4.53: Let $f: X \rightarrow Y$ be a bijective function. Then the following are equivalent:

a) $f$is fg**- homeomorphism.
b) $f$is fg**- continuous and fg**- open maps.
c) $f$is fg**- continuous and fg**- closed maps.

Proof: Omitted.

Definition 4.54: Let X and Y be two fts. A bijective map $f: X \rightarrow Y$ is called fuzzy fg**- c-homeomorphism (briefly fg**- c-homeomorphism) if f and f^{-1} are fuzzy g**- irresolute.

Theorem 4.55: Let X, Y, Z be fuzzy topological spaces and $f: X \rightarrow Y, g: Y \rightarrow Z$ be fg**- c-homeomorphisms then their composition $gof: X \rightarrow Z$ is fg**- c-homeomorphism.

Proof: Omitted.

Theorem 4.56: Every fg**- c-homeomorphism is fg**-homeomorphism.

Proof: Omitted.

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