

# Attenuation and its Effect on Measurement of Ultrasonic Radiation Pressure

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**Abstract:**— Attenuation is defined as the decrease in power level at the load caused by inserting a device between a source and load. Effect of Ultrasonic stress has been studied in the solid state devices and its output is observed for discussions. Effect of attenuation in the measurement of Ultrasonic Radiation Pressure has been discussed analytically.

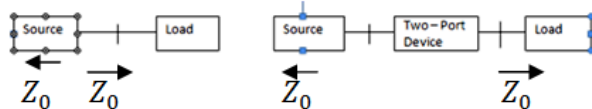
**Keywords:** Statistical Filtering, Attenuation, Ultrasonic Radiation Pressure, Noise, Scattering.

## I. INTRODUCTION

Different kinds of solid state devices have been studied experimentally and its output has been observed in the presence of Ultrasonic Radiation Pressure. Different kinds of mechanisms responsible for the changed characteristics of the solid state devices have been discussed [1-13]. Attenuation of the output ultrasonic radiation pressure waves have been discussed analytically.

### Description:

Attenuation is mostly expressed by a logarithmic scale in decibels (dB) or in nepers. The attenuation of a two port device is defined as:



Attenuation measurement in a matched system Fig 1

### In decibels:

$$A = 10 \log \left[ \frac{\text{power delivered to a matched load by a matched source}}{\text{power delivered to the same load when the two port device is inserted}} \right] \dots (1)$$

### In nepers

$$A = \frac{1}{2} \ln \left[ \frac{\text{power delivered to a matched load by a matched source}}{\text{power delivered to the same load when the two port device is inserted}} \right] \dots (2)$$

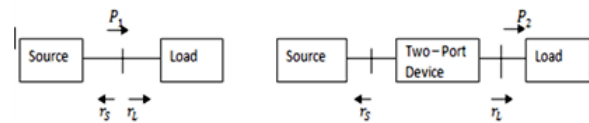
Because

$\log(x) = \frac{\ln(x)}{\ln(10)}$  the following relationship between decibels and nepers is valid. Attenuation in decibels = 8.6858 ×attenuation in nepers.

Attenuation is a property of only two port device. In practical applications neither the source nor the load have an impedance exactly equal to  $Z_0$ , the characteristic impedance of a line. Therefore, source ultrasonic transducer and load (solid state devices, used for study)

have a reflection coefficient  $r_s$  and  $r_L$ , respectively. Let  $P_1$  be the power delivered from the source to the load and  $P_2$  be the power observed by the same load when the two – port device is inserted. Then the loss is defined by:

$$L_1 = 10 \log \frac{P_1}{P_2} \dots \dots \dots (3)$$



Insertion loss measurement in a non-matched system Fig: 2

The insertion loss depends on the property of the device and the reflection coefficients of the source and the load.

When a signal is sent along any transmission path, many different mechanisms degrade it. Because of finite conductivity, every cable shows a resistive loss. Furthermore, the dielectric loss and the skin effect may be significant at higher frequencies, and imperfect screening of cables leads to radiation losses which might be quite important at higher frequencies. Connectors show losses because of non ideal contacts and imperfect impedance matching, which reflect a part of the signal to the transmitter.

Statistical filtering gives good results in the determination of attenuation introduced by different solid materials in the propagation of ultrasonic waves. The method uses different types of correlation theory functions to improve the ratio between the signal containing the needed information on the attenuation, and the noise or perturbation signal.

**International Journal of Science, Engineering and Management (IJSEM)**  
**Vol 2, Issue 1, January 2017**

The energy dissipation mechanisms or the scattering losses from the beam may be expressed in terms of anyone of three parameters, namely, the attenuation factor  $\alpha$ , the logarithmic decrement  $\delta$ , or the dissipation factor  $Q$ .

If we consider an attenuated sine signal, having superposed some random perturbations, we can describe it in the following form:

$$f(t) = e^{-\alpha t} \sin \omega t + \varepsilon(t) \dots \dots \dots (4)$$

where

$\alpha$  - the attenuation coefficient  $\omega$  - angular frequency

$\varepsilon(t)$  - a small random perturbation

The attenuation coefficient  $\alpha$ , can be obtained by means of two consecutive maximum values of eq.(4), at the moments  $t_n$  and  $t_{n+1}$ , for  $\varepsilon(t)=0$ . We will have:

$$\alpha = \frac{\ln[f(t_n)] - \ln[f(t_{n+1})]}{t_{n+1} - t_n} \dots \dots \dots (5)$$

But the random perturbation  $\varepsilon(t)$ , affects the correct values  $f(t_n)$  and  $f(t_{n+1})$ , giving some errors which can be reduced by means of correlation function.

The autocorrelation function  $R(\tau)$  of  $f(t)$ , can be well approximated by:

$$R(\tau) = \frac{1}{T} \int_0^T [e^{-\alpha t} \sin \omega t + \varepsilon(t)] \cdot [e^{-\alpha(t+\tau)} \sin \omega(t+\tau) + \varepsilon(t+\tau)] dt \dots \dots \dots (6)$$

By neglecting some terms, having very small values, we can write:

$$R(\tau) \cong e^{-\alpha \tau} \int_0^T e^{-\alpha t} \sin \omega t \cdot \sin \omega(t+\tau) dt \dots \dots \dots (7)$$

After some routine calculation, we find:

$$R(\tau) \cong [e^{-\alpha \tau} \sin(\omega \tau + \varphi)] \cdot \frac{I_2}{T \cos \varphi} \dots \dots \dots (8)$$

with:  $\varphi = \arctg \frac{I_1}{I_2} \dots \dots \dots (9)$

and:

$$I_1 = \frac{e^{-2\alpha T}}{4\alpha(\omega^2 + \alpha^2)} [2\omega^2 - 2\alpha \sin \omega T (\alpha \sin \omega T + \omega \cos \omega T)] \dots (10)$$

$$I_2 = \frac{e^{-2\alpha T}}{4\alpha(\omega^2 + \alpha^2)} [\omega - (\alpha \sin 2\omega T + \omega \cos 2\omega T)] \dots (11)$$

We can see that eq. (4) and eq. (8) represent the same exponential function, which modulate a sine function having the same period, given by the

same angular frequency  $\omega$ . So, their logarithmic decrement  $\alpha$ , will be almost the same, but the factor  $\frac{I_2}{T \cos \varphi}$ , depending also from  $\alpha$ , can introduce some small errors.

The most important fact is that the errors, given by the random perturbations superposed on the signal, can be considerably reduced. Fig.3 shows an attenuated sine signal, with some random perturbations, superposed, expressed by eq. (4).

Fig.3 shows its autocorrelation function. One can see that the random perturbations are pretty well filtered by autocorrelation function, so its error component can be almost eliminated.

This method can be successfully used to improve the precision in attenuation determination obtained for ultrasonic system components. In this case the logarithmic decrement given by two consecutive maximum values, of the modulated sine function, having small differences between them, can be easily affected even by small random perturbation, and so the statistical filtration by means of autocorrelation function can improve considerably the precision in attenuation determination.

It is assumed for the purpose of discussion that the transducer produces a plane stress wave that is attenuated as it propagates through the sample.

Consider a plane stress wave:

$$\sigma(x, t) = \sigma_0 e^{i(\omega t - kx)} \dots \dots \dots (12)$$

where  $\omega$  is the angular frequency and  $k$  is the propagation vector. This requires that  $k^2 v^2 = \omega^2 \dots \dots \dots (13)$

where  $v$  is the propagation velocity. An expression for an attenuated wave is obtained by assuming that either the propagation vector and the velocity are complex, or that the frequency is complex. Taking  $v = v_1 + i v_2 \dots \dots \dots (14)$

$$k = k_1 - i \alpha \dots \dots \dots (15)$$

one obtains the equation of a plane attenuated wave:

$$\sigma(x, t) = \sigma_0 e^{-\alpha x} e^{i(\omega t - k_1 x)} \dots \dots \dots (16)$$

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**Vol 2, Issue 1, January 2017**

The attenuation factor  $\alpha$  is thus defined as the imaginary part of the complex propagation vector. The phase velocity  $v_p$ , which is a real quantity, is given by

$$v_p = \frac{\omega}{k_1} \dots \dots \dots (17)$$

A substitution in Eq. (13) of the quantities defined by (14) and (15) yields, by equating the real and imaginary parts and eliminating  $k_1$ .

$$\alpha = \frac{\omega v_2}{(v_1^2 + v_2^2)} = \frac{\omega v_2}{|v|^2} \dots \dots \dots (18)$$

and, by eliminating  $\alpha$ .

$$k_1 = \frac{\omega v_1}{(v_1^2 + v_2^2)} = \frac{\omega v_1}{|v|^2} \dots \dots \dots (19)$$

In certain physical problems, it is convenient to express the attenuation factor  $\alpha$  in terms of a complex elastic constant.

$$c = c_1 + i c_2 \dots \dots \dots (20)$$

From Eqs. (17) – (19) and the relation  $= \left(\frac{c}{\rho}\right)^{\frac{1}{2}}$ , where  $\rho$  is the density one obtains.

$$\alpha = \left(\frac{\omega v_2}{v_p v_1}\right) = \left(\frac{\omega (c_2)^{1/2}}{v_p (c_1)^{1/2}}\right) \dots \dots \dots (21)$$

The foregoing expressions define the attenuation factor  $\alpha$  in terms of the distance  $x$ , where  $\alpha$  has the dimensions of reciprocal length.

A procedure alternative to the preceding for expressing attenuation in terms of time  $t$  is obtained by assuming that  $k$  is real, and that  $v$  and  $\omega$  are complex. Thus,

$$\omega = \omega_1 + i \alpha_1 \dots \dots \dots (22)$$

which gives

$$\sigma(x, t) = \sigma_0 e^{-\alpha_1 t} e^{i(\omega t - kx)} \dots \dots \dots (23)$$

Where  $\alpha_1$  has the dimensions of reciprocal time. A substitution in Eq. (13) of the quantities defined by (14) and (22) yields, in this case,

$$\alpha_1 = k v_2 \dots \dots \dots (24)$$

Since the attenuation is determined by the envelop of the high frequency wave, one can use

$$\sigma(x) = \sigma_0 e^{-\alpha x} \dots \dots \dots (25)$$

to specify the attenuation  $\alpha$ . It is assumed for the present that  $\alpha$  is not dependent on  $x$ .  $\alpha$  may then be defined by (25) or by

$$\alpha = -\frac{1}{\sigma(x)} \frac{d\sigma(x)}{dx} = -\frac{d}{dx} [\log_e \sigma(x)] \dots \dots \dots (26)$$

Another way of expressing the same thing is to write  $\log_e \sigma(x)$  using (25):

$$\log_e \sigma(x) = \log_e \sigma_0 - \alpha x \dots \dots \dots (27)$$

Then, for two different points  $x_1$  and  $x_2$  where  $x_1 < x_2$ , the difference of the expressions at the two points is:

$$\alpha = \frac{1}{x_2 - x_1} \log_e \left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \dots \dots \dots (28)$$

and, since any ratio of two amplitudes such as  $\sigma(x_1)$  and  $\sigma(x_2)$  must, in order to be expressed in decibels or in nepers, be written, respectively, as

$$20 \log_{10} \left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \text{ db}$$

or

$$\log_e \left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \text{ nepers,}$$

then

$$\alpha = \left(\frac{1}{x_2 - x_1}\right) 20 \log_{10} \left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \text{ db/unit length} \dots (29)$$

$$\alpha = \left(\frac{1}{x_2 - x_1}\right) \log_e \left(\frac{\sigma(x_1)}{\sigma(x_2)}\right) \text{ nepers/unit length} \dots (30)$$

$$\alpha \text{ (db/unit length)} = 8.686 \alpha \text{ (nepers/unit length)} \dots \dots \dots (31)$$

Another expression for energy loss is that of logarithmic decrement  $\delta$ , which is defined for a harmonically oscillating system in free decay, and for small damping, as  $\delta = (W/2E)$ , where  $W$  is the energy loss per cycle in the specimen, and  $E$  is the total vibrational energy stored in the specimen per cycle. This definition is, it turns out, equivalent to the relation

$$\delta = \log_e \left(\frac{\sigma_n}{\sigma_{n+1}}\right) \dots \dots \dots (32)$$

where  $\sigma_n$  and  $\sigma_{n+1}$  are the amplitudes of two consecutive cycles. It follows from Eq. (30) that

$$\delta \text{ (nepers)} = \alpha \text{ (nepers/cm)} \lambda \text{ (cm)}$$

$$\delta = \frac{\alpha \text{ (nepers/cm)}}{v \text{ (1/sec)}} v \text{ (cm/sec)} \dots \dots \dots (33)$$

Finally,

$$\alpha \text{ (db/}\mu\text{sec)} = 8.68 \times 10^{-6} v \text{ (cm/sec)} \alpha \text{ (nepers/cm)} \dots \dots \dots (34)$$

or

$$\alpha \text{ (db/}\mu\text{sec)} = 8.68 \times 10^{-6} v \text{ (sec}^{-1}) \delta \text{ (nepers)} \dots \dots \dots (35)$$

At the same time,

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$$\alpha (db/\mu\text{sec}) = \alpha (db/cm) \times 10^{-6} v \text{ cm/sec.}$$

.....(36)

The dissipation factor Q may be discussed in terms of resonance absorption in a system subjected to forced vibrations. Resonance response over some range of frequencies is observed whenever the applications of a periodic stress results in a periodic strain that is behind in phase with respect to stress.

**Conclusion:** An attempt has been made to study the attenuation of ultrasonic wave analytically.

**REFERENCES**

1. V. R. Singh and Awadhesh Prasad, "Effect of Ultrasonic stress on the sensitivity of Silicon Strain devices and application to acoustic power measurement" Sensors and Actuators A., Elsevier Squoia, Lausanne, Netherlands, Vol. 28, pp 7 – 11, 1991.
2. V. R. Singh and Awadhesh Prasad, "Effect of Ultrasonic Stress on Amplification of an Operational Amplifier Device", Applied Acoustics, UK, Vol 27, pp 69 – 73, 1989.
3. V. R. Singh and Awadhesh Prasad, "Acoustoelectric Effect in semiconductor materials and devices", Chinese Journal of Acoustics, Vol. 9, No. 3, pp 275 – 279, 1990.
4. V. R. Singh, Awadhesh Prasad and Sanjay Yadav, "Ultrasonic Stress Effect on a Germanium Based Junction Transistor", Acustica, Great Britain, Vol. 71, pp 79 – 80, 1990.
5. Awadhesh Prasad and V. R. Singh, "Characteristics of Silicon Laser p-i-n photodiodes in Ultrasonic Field", IETE, New Delhi, Technical Review, Vol. 7, No. 1, pp 64 – 65, 1990.
6. V. R. Singh and Awadhesh Prasad, "Technical Note: Effect of Ultrasonic Stress on offset voltage of operational Amplifier Devices", Noise control Engineering Journal, USA, Vol. 35 No.2, pp 65 – 67, 1990.
7. V. R. Singh and Awadhesh Prasad, "Effect of Ultrasonic Stress on the N-Type Silicon Photodiodes", ITBM, France, Vol. 10 No, pp 567-571, 1989.
8. Awadhesh Prasad, NPL, New Delhi – 110012, Ph. D. Thesis, "Studies on Characterization of Solid State Devices for Scientific, Biomedical and other Applications", Meerut Univeristy, Meerut, UP, Ph. D. Degree awarded on 29.03.1992.
9. Awadhesh Prasad, "Effect of Ultrasonic Stress on the Solid State Devices and Materials", ISST Journal of Applied physics, Vol. 3 No. 1, pp 41 – 47, 2012.
10. Awadhesh Prasad, "Effect of Ultrasonic Stress on the Silicon Based p-n-junction diodes", Vol. 3, No. 1, pp 17 – 20, 2012, ISST Journal of Applied Physics.
11. Awadhesh Prasad, "Effect of Ultrasonic stress on the solid state devices and Materials", Acta Cinecia Indica, Vol. XXXVIIP, No. 2, pp 85 – 88, 2011, Meerut, UP.
12. Awadhesh Prasad, "Ultrasonic Radiation Pressure Effect in Materials and Devices", ISST Journal of Applied Physics, Vol. 3, No . 2, pp 1-6, 2012.
13. Awadhesh Prasad, "Effect of Ultrasonic Stress on Solid State Devices and Materials," ISST Journal of Applied Physics, Vol . 7, pp 34 - 48 No .1, 2016.
14. Rohn Truell, Charles Elbaum and Bruce B. Chick, Ultrasonic Methods in Solid State Physics, Academic Press, New York, 1969.