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Effects of Radiation on MHD Flow Past an Impulsively Started

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***Abstract*— In this paper, we have investigated the effects of radiation on unsteady magneto-hydrodynamic (MHD) fluid flow past an impulsively started infinite isothermal vertical plate taking into account viscous dissipation. The fluid is considered a gray, absorbing-emitting radiation but non-scattering medium. The dimensionless boundary layer equations of the flow are solved subject to boundary conditions using the Ritz FEM. The obtained numerical results for velocity and skin-friction are presented through the graphs and tables. It has been found that an increase in the radiation parameter decreases the fluid velocity and with increasing Eckert number and Grashof number increases the fluid velocity. The skin-friction increases with increase in the radiation parameter and decreases with increase in the Grashof number and time parameter.**

***Keywords*: MHD, radiation parameter, magnetic parameter, vertical plate, Ritz FEM.**

INTRODUCTION

Magneto-hydrodynamics has an important application in several engineering problems such as MHD power generators in the boundary layer control aerodynamics, nuclear reactors cooling and also in the petroleum industries. Dissipation of energy is significant when considering the unsteady MHD flow through an infinite isothermal vertical plate. Radiative-convective heat transfer flows find numerous applications in glass manufacturing, high temperature aerodynamics, fire dynamics and spacecraft re-entry etc. Bestman and Adjepong [1] have studied the unsteady hydro-magnetic free convection flow with radiative heat transfer in a rotating fluid. Chamkha [2] presented the thermal radiation and buoyancy effects on hydro-magnetic flow over an accelerating permeable surface with heat source or sink. Ganesan and Loganadan [3] studied the radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving cylinder. Radiation effects on the MHD mixed free-forced convective flow past a semi-infinite moving vertical plate for high temperature differences was studied by Azzam [4]. Chamkha et. al [5] have studied the radiative free convection non-Newtonian fluid flow past a wedge embedded in a porous medium. Mohamoud [6] presented the thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Shanker et. al [7] have studied the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption by Galerkin finite element method. Radiation effects on MHD flow thorough porous media past an impulsively started vertical plate with variable heat and mass transfer was presented by Rajput and Kumar [8]. Gebhart [9] have shown the importance of viscous dissipation in natural convective flows. Gebhart and Mollendorf [10] have studied the viscous dissipation effects in external natural flows. Soundalgekar [11] presented the viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction. Computational analysis of coupled radiation-convection dissipation non-gray gas flow in a non-Darcy porous medium using the Keller-Box implicit difference scheme was presented by Takhar et. al [12].

The object of the present paper is to study the influence of radiation on unsteady magneto-hydrodynamic (MHD) flow past an impulsively started infinite isothermal vertical plate taking into account viscous dissipation effect. The problem is governed by the system of non-linear partial differential equations, whose exact solutions are difficult to obtain, if possible. So that, the Ritz FEM has been adopted for its solution, which is more economical from a computational point of view. The behaviors of the velocity field and skin-friction have been discussed for variations in the governing parameters.

II. MATHEMATICAL ANALYSIS

We consider magneto-hydrodynamic (MHD) flow of a viscous incompressible gray, absorbing-emitting fluid past an impulsively started infinite isothermal vertical plate taking into account viscous dissipation effect. The coordinate system is taken the axis along the plate in the upward direction and the axis is normal to the plate. The radiation heat flux in the direction is considered negligible as compared to that in the direction. A transverse magnetic field of uniform strength is applied normal to the plate. The magnetic Reynolds number is small so that the induced magnetic field is neglected. Under the usual Boussinesq's approximation, the flow governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial u'}{\partial y'} + g\beta\left(T' - T_\infty^{'}\right) - \frac{\sigma B_0^2}{\rho} u' \qquad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \qquad (2)$$

The corresponding physical boundary conditions are:

$$\begin{aligned} &t' \le 0; \quad u' = 0, \quad T' = T_\infty^{'} \quad \text{for all } y' \ge 0, \\ &t' > 0; \quad u' = u_0, \quad T' = T_w^{'} \quad \text{at } y' = 0, \\ &\qquad\quad u' = 0, \quad T' \to T_\infty^{'} \quad \text{for } y' \to \infty \end{aligned} \qquad (3)$$

where u' is the velocity component along the plate, g is the acceleration due to gravity, ν is the kinematic coefficient of viscosity, T' is the temperature of the fluid, $T_\infty^{'}$ is the temperature of the fluid far away from the plate, ρ is the density, C_p is the specific heat at constant pressure, k is the thermal conductivity, β is the volumetric coefficient of thermal expansion, t' is the dimensional time, q_r is the radiative heat flux, $T_w^{'}$ is the temperature at the plate and u_0 is the velocity of the moving plate.

The radiation flux on the basis of Rosseland approximation can be expressed as;

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \qquad (4)$$

where σ^* is the Stefan-Boltzmann constant and k^* is the spectral mean absorption coefficient of the medium. It is assumed that the temperature differences within the flow are sufficient small such that T'^4 can be expressed as the linear function of temperature T'. It can be established by expanding T'^4 in a Taylor series about a free stream temperature $T_\infty^{'}$ and neglecting higher-order terms, we obtain T'^4 as

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \qquad (5)$$

By using equations (4) and (5), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \qquad (6)$$

To present solutions which are independent of the geometry of the flow regime, we introduce a series of non-dimensional transformations, defined as:

$$u = \frac{u'}{u_0}, y = \frac{u_0 y'}{\nu}, t = \frac{u_0^2 t'}{\nu}, \theta = \frac{T' - T_\infty^{'}}{T_w^{'} - T_\infty^{'}}, P_r = \frac{\mu C_p}{k}, M = \frac{\sigma B_0^2 \nu}{u_0^2},$$

$$E_c = \frac{u_0^2}{C_p (T_w^{'} - T_\infty^{'})}, G_r = \frac{g\beta\nu(T_w^{'} - T_\infty^{'})}{u_0^3}, N = \frac{k^\bullet k}{4\sigma T'^3_\infty}. \qquad (7)$$

Substituting equation (7) into equations (1) and (6), we obtain the following governing equations in non-dimensional form:

$$\frac{\partial u}{\partial t} = G_r\theta + \frac{\partial^2 u}{\partial y^2} - Mu \qquad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(1 + \frac{4}{3N}\right)\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \qquad ($$

where u is the dimensionless velocity, t is the dimensionless time, y is the dimensionless distance, G_r is the Grashof number, M is the magnetic parameter, θ is the dimensionless temperature, P_r is the Prandtl number, N is the radiation parameter and E_c is the Eckert number.

The corresponding boundary conditions in non-dimensional form are:

$$\begin{aligned} &t \le 0; \quad u = 0, \quad \theta = 0 \quad \text{for all } y \ge 0, \\ &t > 0; \quad u = 1, \quad \theta = 1 \quad \text{at } y = 0, \\ &\qquad\quad u = 0, \quad \theta \to 0 \quad \text{as } y \to \infty \end{aligned} \qquad (10)$$

III. SOLUTION OF THE PROBLEM

Equations and are non-linear system of partial differential equations are to be solved under subject to the physically realistic boundary conditions given in equation However, whose exact or approximate solutions are difficult to obtain if possible. Hence, the Ritz finite element method is applied to solve these equations. The algorithm for Ritz finite element method can be summarized by the following steps.

[1] Division of the whole domain into smaller elements of finite dimensions called “finite elements”.
[2] Generation of the element equations using variational formulations.
[3] Assembly of element equations as obtained in step (2).
[4] Imposition of boundary conditions to the equations obtained in step (3).
[5] Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration method. Numerical solutions for the velocity (u) and temperature (θ) are obtained by using $C-$ program. To judge the convergence and stability of the Ritz finite element method, the same $C-$ program was run by making small changes in time t and $y-$directions, no significant change was observed in the values of velocity and temperature. Hence, we conclude that the Ritz finite element method is convergent and stable.

The skin-friction at the plate is given by $\tau = \left(\frac{du}{dy}\right)_{y=0}$

IV. RESULTS AND DISCUSSION

The numerical values for the velocity (u), skin-friction (τ) for different parameters like radiation parameter, Eckert number, magnetic parameter, Grashof number and time parameter are computed to assess the effects of these parameters on the flow. The computed numerical results are presented through the graphs and tables.

Fig.1 depicts the effects of radiation parameter N on the velocity field u at a fixed time $t = 1.0$. It is observed that an increase in the radiation parameter N from 3.0, 5.0 to 10.0, the velocity u decreases. Deceleration of the flow is therefore sustained at considerable distance from the plate towards the free stream as the radiation parameter is increased. The profiles decay monotonically for all values of N from the maximum at the plate to its minimum in the free stream. Fig.2 displays the effects of Eckert number E_c on the velocity field u at a fixed time $t = 1.0$. It is seen that the fluid velocity increases with increase in the Eckert number from 0.1, 0.3 to 0.5. We have plotted the velocity u in Fig.3 for variations in the magnetic parameter M at a fixed time $t = 1.0$. It can be seen that the fluid velocity decreases as the magnetic parameter increases. This is due to the magnetic field exerts a retarding force on the free convective flow. In Fig.4 we have studied the effect of buoyancy via Grashof number (G_r) on the velocity u at a time $t = 1.0$. It is observed that the velocity u increases consistently, as the Grashof number G_r increased from 2.0 to 3.0 and then 4.0. The fluid velocity is accelerated due to the enhancement in buoyancy forces corresponding to an increase in Grashof number i.e., free convection effects. Here, the positive values of G_r corresponds to cooling of the plate by natural convection. Fig.5 shows the effects of the time parameter (t) on the velocity u. It is seen that there is a reasonable increase in the velocity u with increasing in the time parameter. With time the fluid flow is therefore accelerated in the upward direction.

The numerical values of skin-friction (τ) for different material parameters are presented in table 1 and 2, respectively. From table 1, we observe that the skin-friction increases with increasing radiation parameter (N) and decreases with increase in the Grashof number (G_r) and time parameter (t). From table 2, it is seen that an increase in the magnetic parameter (M) increases the skin-friction and increasing Eckert number (E_c) and time parameter (t) decreases the skin-friction.

V. FIGURES AND TABLES

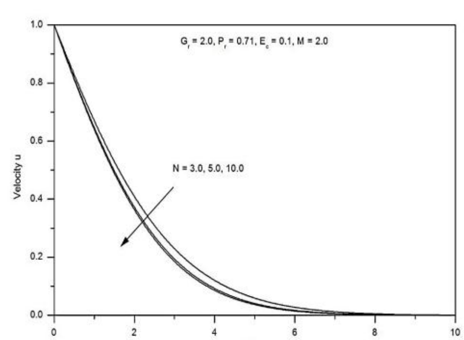

Fig. 1: Effect of radiation parameter on the velocity profiles at

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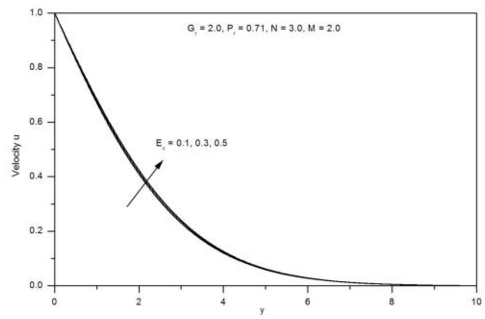

Fig. 2: Effect of Eckert number   on the velocity profiles at

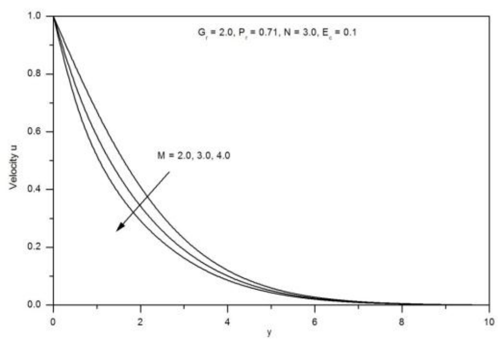

Fig. 3: Effect of magnetic parameter   on the velocity profiles at

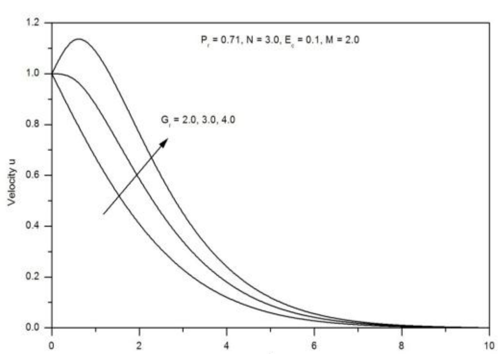

Fig. 4: Effect of Grashof number   on the velocity profiles at

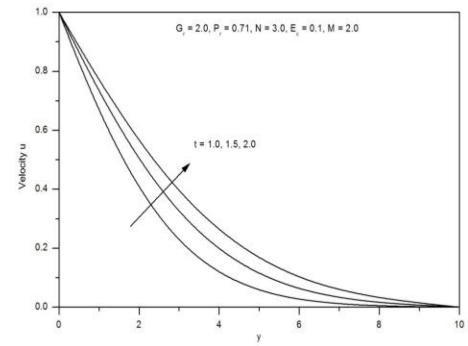

Fig. 5: Effect of time parameter   on the velocity profiles.

Table 1: Numerical values of skin-friction for variations and time

N	$G_r = 2.0$ $t = 1.0$	$G_r = 2.0$ $t = 2.0$	$G_r = 3.0$ $t = 1.0$	$G_r = 3.0$ $t = 2.0$
3	0.346598	0.233168	0.003580	- 2.166262
5	0.364358	0.247446	0.030202	- 0.144824
10	0.379888	0.260090	0.053488	- 0.125856

Table 2: Numerical values of skin-friction for variations and time

E_c	$M = 2.0$ $t = 1.0$	$M = 2.0$ $t = 2.0$	$M = 3.0$ $t = 1.0$	$M = 3.0$ $t = 2.0$
0.1	0.346598	0.233168	0.617520	0.547204
0.3	0.338216	0.224644	0.611620	0.541586
0.5	0.329906	0.216302	0.605760	0.536054

CONCLUSION

The governing equations of the flow have been examined for unsteady MHD fluid flow past an impulsively started infinite isothermal infinite vertical plate in the presence of radiation taking into account viscous dissipation. The dimensionless governing equations of the flow are solved by using the Ritz FEM. We conclude that the velocity increases with increase in the Eckert number and time parameter and with increasing radiation parameter decreases the flow velocity. The skin-friction increases with increase in the radiation parameter and magnetic parameter and decreases with increasing time parameter.

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