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Mathematical Modelling Of Traffic Flow on Highway

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***Abstract:--* This paper intends a mathematical model for the study of traffic flow on the highways. This paper develops a discrete velocity mathematical model in spatially homogeneous conditions for vehicular traffic along a multilane road. The effect of the overall interactions of the vehicles along a given distance of the road was investigated. We also observed that the density of cars per mile affects the net rate of interaction between them. A mathematical macroscopic traffic flow model known as light hill, Whitham and Richards (LWR) model appended with a closure non-linear velocity-density relationship yielding a quasi-linear first order (hyperbolic) partial differential equation as an initial boundary value problem (IBVP) was considered. The traffic model IBVP is a finite difference method which leads to a first order explicit upwind by difference scheme was discretized.**

***keywords:--* Mathematical modeling, traffic flow, Homogeneous conditions, Multilane road, Velocity-density, Quasi-linear first order (hyperbolic) partial differential equation, Finite difference method.**

I. INTRODUCTION

Mathematical modeling is the process of converting a real-world problem into a mathematical problem and solving them with certain circumstances and interpreting those solutions into real world.

The sample of flow chart for solving real world problem by using mathematical modeling is explicated in below fig.1

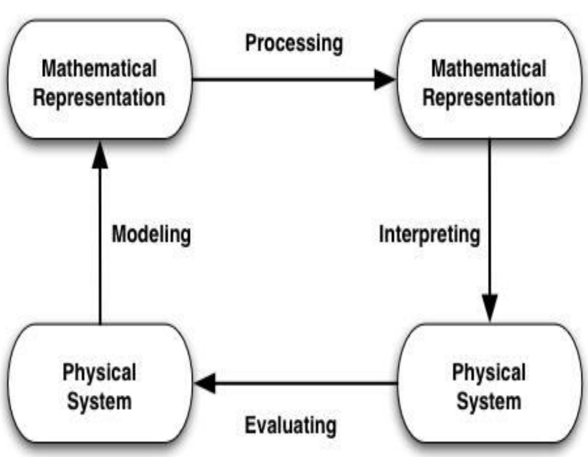

Fig.1

II. HISTORY

Attempts to produce a mathematical theory of traffic flow date back to the 1920s, when Frank Knight first produced an analysis of traffic equilibrium, which was refined into Wardrop's first and second principles of equilibrium in 1

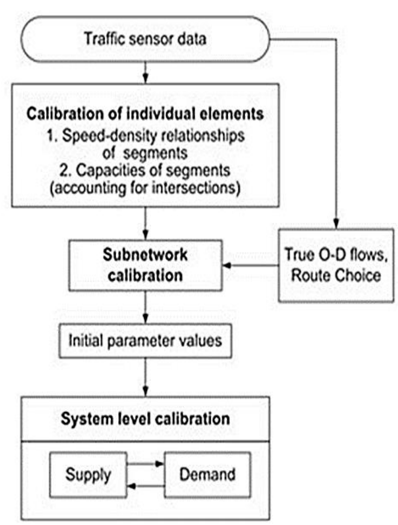

Fig.2

Nonetheless, even with the advent of significant computer processing power, to date there has been no satisfactory

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general theory that can be consistently applied to real flow conditions. Current traffic models use a mixture of empirical and theoretical techniques. These models are then developed into traffic forecasts, to take account of proposed local or major changes, such as increased vehicle use, changes in land use or changes in mode of transport (with people moving from bus to train or car, for example), and to identify areas of congestion where the network needs to be adjusted

- ❖ Traffic flow theory involves the development of mathematical relationships among
- ❖ the primary elements of the traffic stream
    - Flow
    - Density
    - Speed
- ❖ These relationships help traffic engineer in planning, designing and evaluating the effectiveness of implementing traffic engineering measures on a highway system.
- ❖ Traffic flow is generally constrained along a one-dimensional pathway (e.g. a travel lane)
- ❖ A time space diagram shows graphically the flow of vehicles along a pathway over time
- ❖ Time is displayed along the horizontal axis, and distance is shown along the vertical axis
- ❖ Traffic flow in a time-space diagram is represented by the individual trajectory lines of individual vehicles
- ❖ Vehicles following each other along a given travel lane will have parallel trajectories and trajectories will cross when one vehicle passes another
- ❖ Time space diagram are useful tools for displaying and analyzing the traffic flow characteristics of a given roadway segment over time
- ❖ There are three main variables to visualize a traffic stream: speed(v), density (indicated k; number of vehicles per unit of space) and flow (indicated q; the number of vehicles per unit of time)
- ❖ Mathematical model usually describes a system by set of variables and equations and establish relationship between these variables. The values of these variables can be partially anything. Real or integer numbers, Boolean values of strings for example. These variables represent some properties of the system, for example, event occurrence (yes/no)
- ❖ Mathematical formulation
- ❖ Mathematical modeling is the use of a simplified representation of a real-world system, process or theory. Mathematical models are developed in order to enhance our ability to understand, predict, and possibly control the behavior of the system being modeled.
- ❖ Akinrelere and Ayeni described mathematical modeling to be translation of physical problems using mathematical representation or equation and solves then using the result in the problems. To achieve the aim some deduction must be made. This takes place in at least three stages namely:
- Formulate a model
- Deduction of mathematical implication within the model
- Interpretation of former steps in real form

Mathematical models may also involve diagrams and graph circuit diagrams, or stock market index performance charts (Fig.3)

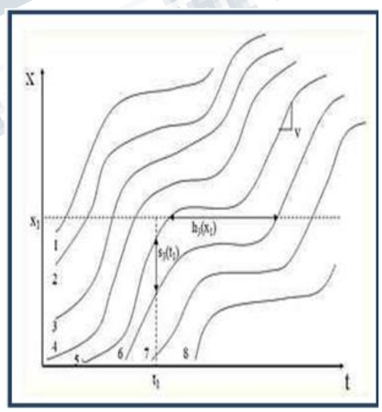

Fig.3

III. MODELLING OF TRAFFIC FLOW

Road transportation is the safest means of modern movement within a city or community around the world. It is the in which if a car breakdown the passenger can easily highlight. Change the car or walk the rest of the journey

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We consider the flow of car on a long highway under the assumptions that the cars do not enter or leave the highway at anyone of the points. This number with a variable number of cars from none up to the maximum carrying capacity of the road.

The number of cars on the stretch from A to B can be modelled by taking the X-axis along the highway and assume that the traffic flows in the positive direction. Supposeρ(x,t) is the density representing the number of cars per unit length at the point x on highway at any time t q(x,t)is the flow (velocity) of cars per unit time. This equation is used converse the momentum cars and to show how the density of the traffic will affect the speed at which the car travel from one station to another to achieve this, they have to use the roadways alongside with other millions of people given rise to millions of vehicles on the roadways, these vehicles interact with each other an impact overall movement of traffic which pose great challenges to the traffic-flow .in an attempt to solve these daily challenges on roadways. This is explained in Fig.4.

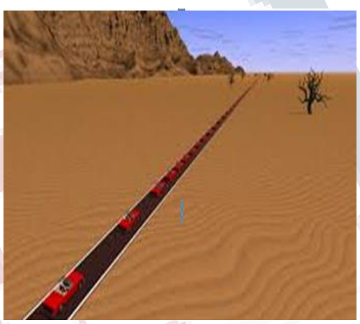


Fig.4

We assume conversation law which states that: change in the total amount of physical quantity contained in a space must be equal to flux of the quantity across the boundary of that region.

In this case, the time rate of change of the total number of cars in any segment $A \leq x \leq B$ of highway is given by:

$$\frac{d}{dt}\int_{x1}^{x2} \ell(x,t)\, dx = \int_{x1}^{x2} \frac{d\ell}{dt} dx \qquad \textbf{(1)}$$

The flow of cars entering the segment A minus the flow of cars leaving the segment B is given by,

$$q(x_1,t) - q(x_2,t) \qquad \textbf{(2)}$$

The rate of change in **(1)** must be equal to the net flux across A & B given by **(2)**. Thus, we have the conversation equation of the form

$$\frac{d}{dt}\int_{x1}^{x2} \ell(x,t)dx = q(x_1,t) - q(x_2,t) \Rightarrow \int_{x1}^{x2} \frac{\partial \ell}{\partial t} dx = -\int_{x1}^{x2} \frac{\partial q}{\partial x} dx \qquad \textbf{(3)}$$

Then (3) becomes

$$\int_{x1}^{x2} \left(\frac{\partial \ell}{\partial t} + \frac{\partial q}{\partial x}\right) dx = 0 \qquad \textbf{(4)}$$

Since the integrated in **(3)** and **(4)** continuous and it holds for every segment (A, B), then it follows that the integrated must vanish to order to have the partial differential equation given by

$$\frac{\partial \ell}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad \textbf{(5)}$$

We now introduce additional assumptions which are supported by both theoretical and experimental findings:

- Measuring the velocity and position of each individual's car on the road is too difficult. So, this model views the distribution of cars by looking at the density of the cars which is the number cars per mile on the road. We assume that the density is the only property of the cars which matters.
- The second assumptions follow from the first. Only the density of the car matters.
- Therefore, the average velocity of the cars at any point depends on the density of the cars.
- According to these assumptions, the flow rate q depends on x and t only through the density, that is, q=Q(ℓ) for same function Q this relation seems to be reasonable in the sense that the density of cars surroundings a car indeed controls the speed limits, weather conditions, and road characteristics.

IV. MODELS PREDICTING THE LOAD OF TRANSPORTATION NETWORKS

Main Principles of Load Modelling:

The traffic flows are formed by individual movements of the traffic participants or users of the transportation network. In the general case, by the movements are meant not only the trips by various kinds of transport, but pedestrian movements as well. The main factors defining the quantity of movements and their distribution over the urban transportation network are as follows:

(1) Flow-generating factors, that is, distribution of the traffic-generating objects such as place of residence, places of work, cultural and welfare facilities, and so on.

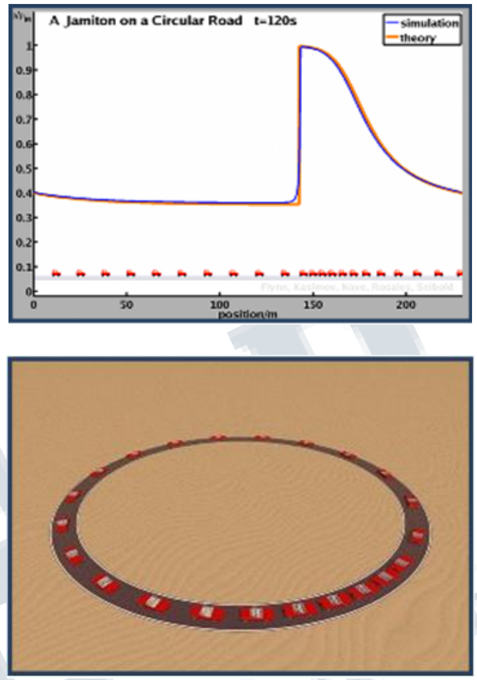
(2) Characteristics of the transportation network such as the quantity and quality of streets and roads, parameters of traffic organization, routes and transportation capacities of the public conveyances, and so on.

(3) Behavioural factors such as population mobility, preferences in choosing the transportation means and routes, and so on.

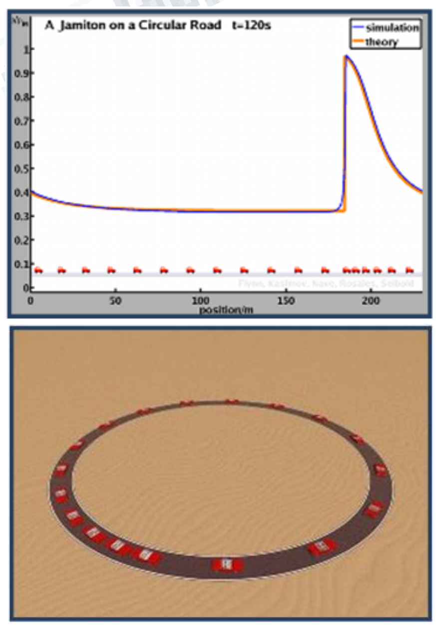
To construct mathematical models, one must describe the above factors in formal terms. Descriptionrelies on the traffic graph whose nodes correspond to the crossroads and stations of the Extrastreet transport, whereas the arcs correspond to the segments of streets and lines of the extra street transport, as well as to the transfers from street to extra street transport. The route graph of public conveyances is an individual component of the transport graph whose nodes are the stops.

A circular road is a particular friendly case for an analysis, since the total number of vehicles is exactly conserved. If the road is not too long, traffic will in general form one single traveling wave, i.e. a single jamiton, and thus a single shock is observed. Below figures and videos show the results of simulations and theoretical predictions for the case of a circular road of length 230m.

In the case of in viscid equations a sharp shock is realized. Here, the final solution is predicted theoretically. The match between theory and numerical results is generally very good. While the in viscid equations allow a simple analysis, using the Rankine-Hugoniot conditions at the shock, the resulting vehicle behaviour is somewhat extreme. Vehicles slowdown from high to low velocity in zero time.



Large number of vehicles (22) Fig.5.0



Medium number of vehicles (18) fig.5.1

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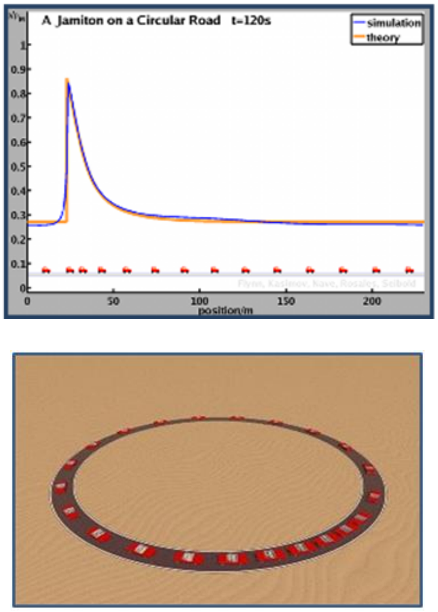


Small number of vehicles (14) fig.5.2

V. Explanation of Multi-Valued Fundamental Diagrams

Our studies show that jamitons have a very specific profile: when plotted in a flow rate vs. density diagram, a jamiton is a straight-line segment, whose slope is the travel velocity of the jamiton on the road. As such, jamitons form a two-parameter family of curves. As a first parameter, one can choose the vehicle density at the sonic point. For each such density (if the associated uniform flow is unstable), one obtains a maximum jamiton curve (infinitely long), and a one-parameter family of sub-jamitons, parameterized by their length (or their shock height, respectively) in Fig.6.

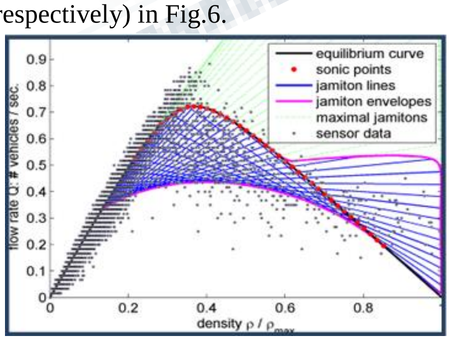


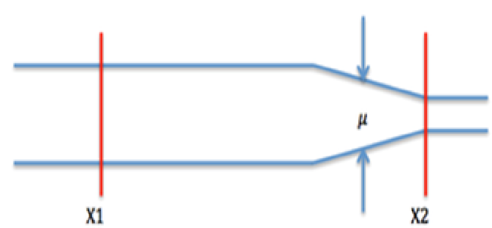
Fig.6

The figure above shows a fundamental diagram, induced by jamitons. The black function is the equilibrium curve that vehicles relax towards. In the regime of densities marked by red dots, uniform traffic flow (of the respective density) is unstable, and jamitons arise. For each red dot (sonic point density), the maximal possible jamiton is marked by a green dotted line segment. Moreover, we calculate how any train of sub-jamitons would appear to a stationary sensor that records flow rate and density in an aggregated fashion (e.g., in intervals of 30 seconds). The resulting averages are given by the blue line segments and their envelopes by the pink curves. Any point enclosed by the pink curves can arise as a sensor measurement of jamitons states. This construction is placed on top of real sensor measurement data (obtained on the southbound direction of I-35W in Minneapolis, MN; data provided by the Minnesota Department of Transportation). A strong qualitative agreement between the jamiton construction and the data is apparent

VI. APPLICATIONS OF TRAFFIC FLOW MODEL

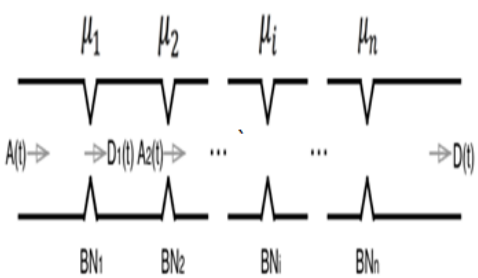
The Bottleneck Model:

One application of the N-curve is the bottleneck model, where the cumulative vehicle count is known at a point before the bottleneck (i.e. this is location X1). However, the cumulative vehicle count is not known at a point after the bottleneck (i.e. this is location X2), but rather only the capacity of the bottleneck, or the discharge rate, μ, is known. The bottleneck model can be applied to real-world bottleneck situations such as those resulting from a roadway design problem or a traffic incident.



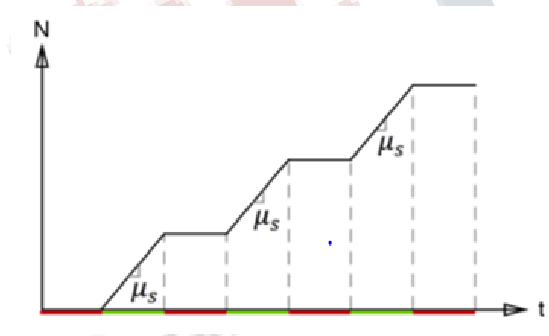
Tandem Queues:

As introduced in the section above, the N-curve is an applicable model to estimate traffic delay during time by setting arrival and departure cumulative counting curve. Since the curve can represent various traffic characteristics and roadway conditions, the delay and queue situations under these conditions will be able to be recognized and modelled using N-curves. Tandem queues occur when multiple bottlenecks exist between the arrival and departure locations. Figure 14 shows a qualitative layout of a tandem-queue roadway segment with a certain initial arrival. The bottlenecks along the stream have their own capacity,'µi [veh/time], and the departure is defined at the downstream end of the entire segment.



Dynamic traffic assignment:

Dynamic traffic assignment can also be solved using the N-curve. There are two main approaches to tackle this problem: system optimum, and user equilibrium. This application will be discussed further in the following section.



Kerner's three-phase traffic theory:

Kerner's three-phase traffic theory is an alternative theory of traffic flow. Probably the most important result of the three-phase theory is that at any time instance there is a range of highway capacities of free flow at a bottleneck. The capacity range is between some maximum and minimum capacities. The range of highway capacities of free flow at the bottleneck in three-phase traffic theory contradicts fundamentally classical traffic theories as well as methods for traffic management and traffic control which at any time instant assume the existence of a particular deterministic or stochastic highway capacity of free flow at the bottleneck.

VII. CONCLUSION

"Traffic flow in highway without mathematical modelling is fish without water" it causes many Real-world problems. Many of the problems which cannot solved by computers can be solved using mathematical modelling. Mathematical modelling plays an Important role in solving real world problems using this paper we can solve many of the traffic problems. Example in India we can mainly see traffic problem which can be easily solved using mathematical modelling. Traffic signals work on computerised programming. This a gigatntic process and amaiable modelling in public system, using government transport without using private transport the might reduce and this article works more efficiently

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