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Applications of Integral Calculus in Engineering

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***Abstract:*-- In this chapter we are going to study about the history and the applications of integral calculus. Isaac Newton and Gottfried Leibniz independently discovered calculus in the mid- 17 century. Integration represents the inverse operation of differentiation. Integral calculus is used to improve the important infrastructures. Integral calculus is often used to create the most robust design. At the end of this chapter we will come to know about the basic applications of integral calculus in engineering field which are:- Average function value, Area between two curves, Volume of solid of revolution/ Methods of rings, Work done.**

***keywords:*--Definite integral, Fundamental theorem of calculus, Line integral, Average function value, Area between two curves, Volume of solid of revolution/ Methods of rings, Work done**

INTRODUCTION

In mathematics, an integral assigns umbers to functions in a way that can describe displacement, area, volume, and other concepts that arise by combining infinitesimal data. Integration is one of the two main operations of calculus, with its inverse, differentiation, being the other. Given a function f of a real variable x and an interval [a, b] of the real line, the definite integral.

$$\int_a^b f(x)dx$$

Is defined informally as the signed area of the region in the *xy*-plane that is bounded by the graph of *f*, the *x*-axis and the vertical lines $x = a$ and $x = b$. The area above the *x*-axis adds to the total and that below the *x*-axis subtracts from the total.

Roughly speaking, the operation of integration is the reverse of differentiation. For this reason, the term integral may also refer to the related notion of the ant derivative,a function *F* whose derivative is the given function *f*. In this case, it is called an *indefinite integral* and is written

$$F(x) = \int f(x)dx$$

The integrals discussed in this article are those termed *definite integrals*. It is the fundamental theorem of calculus that connects differentiation with the definite integral: if *f* is a continuous real-valued function defined on a closed interval [*a*, *b*], then, once an anti derivative *F* of *f* is known, the definite integral of *f* over that interval is given by:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

The principles of integration were formulated independently by Isaac Newton and Gottfried Leibniz in the late 17th century, who thought of the integral as an infinite sum of rectangles of infinitesimal width. Bernhard Riemann gave a rigorous mathematical definition of integrals. It is based on a limiting procedure that approximates the area of a curvilinear region by breaking the region into thin vertical slabs. Beginning in the nineteenth century, more sophisticated notions of integrals began to appear, where the type of the function as well as the domain over which the integration is performed has been generalised. A line integral is defined for functions of two or three variables, and the interval of integration [a, b] is replaced by a certain curve connecting two points on the plane or in the space. In a surface integral, the curve is replaced by a piece of a surface in the three-dimensional space.

HISTORY:

PRE- CALCULUS INTEGRATION: The first documented systematic technique capable of determining integrals is the method of exhaustion of the ancient Greek astronomer Eudoxus (ca. 370 BC), which sought to find areas and volumes by breaking them up into an infinite number of divisions for which the area or volume was known. This method was further developed and employed by Archimedes in the 3rd century BC and used to calculate areas for parabolas and an approximation to the area of a circle.

A similar method was independently developed in China around the 3rd century AD by Liu Hui, who used it to find the area of the circle. This method was later used in the 5th century by Chinese father-and-son mathematicians ZuChongzhi and ZuGeng to find the volume of a sphere (Shea 2007, Katz 2004, pp. 125–126).

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The next significant advances in integral calculus did not begin to appear until the 17th century. At this time, the work of Cavalieri with his method of Indivisibles, and work by Fermat, began to lay the foundations of modern calculus, with Cavalieri computing the integrals of xn up to degree n = 9 in Cavalieri's quadrature formula. Further steps were made in the early 17th century by Barrow and Torricelli, who provided the first hints of a connection between integration and differentiation. Barrow provided the first proof of the fundamental theorem of calculus. Wallis generalized Cavalieri's method, computing integrals of x to a general power, including negative powers and fractional powers

Newton and Leibniz:

The major advance in integration came in the 17th century with the independent discovery of the fundamental theorem of calculus by Newton and Leibnitz. The theorem demonstrates a connection between integration and differentiation. This connection, combined with the comparative ease of differentiation, can be exploited to calculate integrals. In particular, the fundamental theorem of calculus allows one to solve a much broader class of problems. Equal in importance is the comprehensive mathematical framework that both Newton and Leibniz developed. Given the name infinitesimal calculus, it allowed for precise analysis of functions within continuous domains. This framework eventually became modern calculus whose notation for integrals is drawn directly from the work of Leibnitz.

Formalization:

While Newton and Leibniz provided a systematic approach to integration, their work lacked a degree of rigour. Bishop Berkeley memorably attacked the vanishing increments used by Newton, calling them "ghosts of departed quantities". Calculus acquired a firmer footing with the development of limits. Integration was first rigorously formalized, using limits, by Riemann. Although all bounded piecewise continuous functions are Riemann-integrable on a bounded interval, subsequently more general functions were considered—particularly in the context of Fourier analysis—to which Riemann's definition does not apply, and Lebesgue formulated a different definition of integral, founded in measure theory (a subfield of real analysis). Other definitions of integral, extending Riemann's and Lebesgue'sapproaches were proposed. These approaches based on the real number system are the ones most common today, but alternative approaches exist, such as a definition of integral as the standard part of an infinite Riemann sum, based on the hyper real number system

Historical notation:

Isaac Newton used a small vertical bar above a variable to indicate integration, or placed the variable inside a box. The vertical bar was easily confused with .x or x′, which are used to indicate differentiation and the box notation, was difficult for printers to reproduce, so these notations were not widely adopted.

The modern notation for the indefinite integral was introduced by Gottfried Leibniz in 1675 (Burton 1988, p. 359; Leibniz 1899, p. 154). He adapted the integral symbol. ∫, from the letter ſ (long s), standing for summa (written as ſumma; Latin for "sum" or "total"). The modern notation for the definite integral, with limits above and below the integral sign, was first used by Joseph Fourier in Mémoires of the French Academy around 1819–20, reprinted in his book of 1822 (Cajori 1929, pp. 249–250; Fourier 1822, §23).

APPLICATION:

Integrals are used extensively in many areas of mathematics as well as in many other areas that rely on mathematics.

For example, in probability theory, integrals are used to determine the probability of some random variable falling within a certain range. Moreover, the integral under an entire probability density function must equal 1, which provides a test of whether a function with no negative values could be a density function or not.

Integrals can be used for computing the area of a two-dimensional region that has a curved boundary, as well as computing the volume of a three-dimensional object that has a curved boundary.

Integrals are also used in physics, in areas like kinematics to find quantities like displacement, time, and velocity. For example, in rectilinear motion, the displacement of an object over the time interval is given by:

$$x(a) - x(b) = \int_a^b v(t)dt,$$

Where v (t) is the velocity expressed as a function of time.

The work done by a force F(x)(given as a function of position) from an initial position A to a final position B is:

$$W_{A \to B} = \int_A^B F(x)dx$$

Here is a listing of applications covered in this chapter.

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Average Function Value- We can use integrals to determine the average value of a function.

Area Between Two Curves- In this section we'll take a look at determining the area between two curves.

Volumes of Solids of revolution/Methods of Rings- This is the first of two sections devoted to find the volume of a solid of revolution. In this section we look at the method of ring.

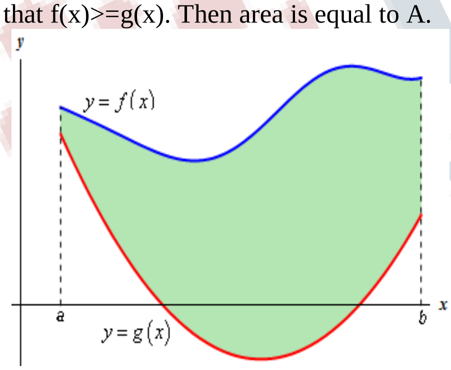
Work- The final application we will look at is determining the amount of work required to move an object.

1)AVERAGE FUNCTION VALUE: The average value of a function f(x) over the interval [a, b] is given by:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

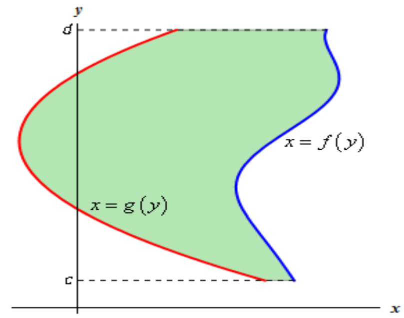
2) AREA BETWEEN CURVES: In this section we are going to look at finding the area between two curves. There are actually two cases that we are going to looking at.

In the first case we want to determine the area between y=*f*(x) and y=g(x) on the interval, we are also going to assume that f(x)>=g(x). Then area is equal to A.



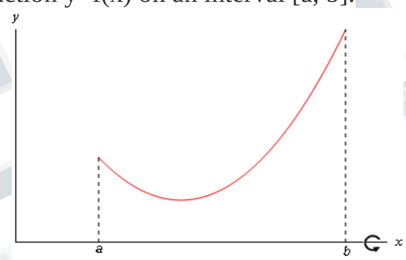
$$A = \int_a^b f(x) - g(x)dx$$

The second case is almost identical to the first case. Here we are going to determine the area between x=f(y) and x=g(y) on the interval [c, d] with f(y)>=g(y).

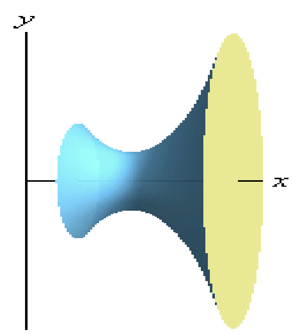


$$A = \int_c^d f(y) - g(y)dy$$

3) Volume of solid of revolution/ Method of rings: In this section we will start looking at the volume of a solid of revolution. We should first define just what a solid of revolution is. To get a solid of revolution we start out with a function y=f(x) on an interval [a, b].



We then rotate this curve about a given axis to get the surface of the solid of revolution. For purposes of this discussion let's rotate the curve a about the x-axis, although it could be any vertical or horizontal axis. Doing this for the curve above gives the following three dimensional regions.



What we want to do over the course of the next two sections is to determine the volume of this object. In the final the Area and Volume formulas section of the Extras chapter we derived the following formulas for the volume of this solid.

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Where, A(x) and A(y) is the cross-sectional area of the solid. There are many ways to get the cross-sectional area and we'll see two (or three depending on how you look at it) over the next two sections. Whether we will use A(x) or A(y) will depend upon the method and the axis of rotation used for each problem.

$$V = \int_a^b A(x)dx$$

$$V = \int_c^d A(y)dy$$

One of the easier methods for getting the cross-sectional area is to cut the object perpendicular to the axis of rotation. Doing this the cross section will be either a solid disk if the object is solid (as our above example is) or a ring if we've hollowed out a portion of the solid (we will see this eventually).

In the case that we get a solid disk the area is,
A=π(radius)^2
Where the radius will depend upon the function and the axis of rotation.

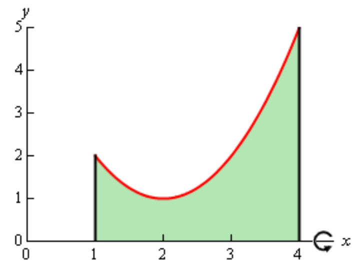
In the case that we get a ring the area is,
A=π((outer radius)^2-(inner radius)^2)
Where again both of the radii will depend on the functions given and the axis of rotation. Note as well that in the case of a solid disk we can think of the inner radius as zero and we'll arrive at the correct formula for a solid disk and so this is a much more general formula to use.

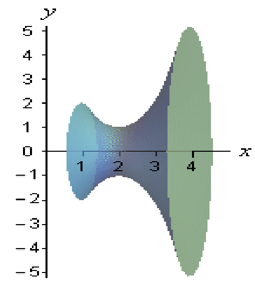
Also, in both cases, whether the area is a function of x or a function of y will depend upon the axis of rotation as we will see.

This method is often called the method of disks or the method of rings.

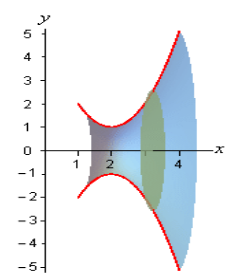
Example 1: Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5, x = 1, x = 4$ and the x-axis about the x-axis.

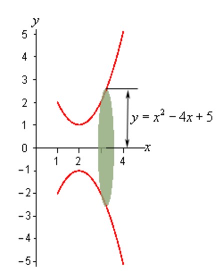
Solution: The first thing to do is get a sketch of the bounding region and the solid obtained by rotating the region abou x- axis. Here are both the sketches:





to get a cross section we cut the solid at any x. Below are a couple of sketches showing a typical cross section. The sketch on the right shows a cut away of the object with a typical cross section without the caps. The sketch on the left shows just the curve we're rotating as well as its mirror image along the bottom of the solid.





In this case the radius is simply the distance from the x-axis to the curve and this is nothing more than the function value at that particular x as shown above. The cross-sectional area is then,

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$A(x) = \pi(x^2-4x+5)^2 = \pi(x^4-8x^3+26x^2-40x+25)$

Next we need to determine the limits of integration. Working from left to right the first cross section will occur at x=1 and the last cross section will occur at x=4 . These are the limits of integration.The volume of this solid is then,

$$V = \int_a^b A(x)dx$$

$$= 3.14\int (x^4 - 8x^3 + 26x^2 - 40x + 25)dx$$

$$= 48.984$$

4) WORK: This is the final application of integral that we'll be looking at in this course. In this section we will be looking at the amount of work that is done by a force in moving an object. In a first course in Physics you typically look at the work that a constant force, F, does when moving an object over a distance of d. In these cases the work is,

W=Fd

However, most forces are not constant and will depend upon where exactly the force is acting. So, let's suppose that the force at any x is given by F(x). Then the work done by the force in moving an object from x=a to x=b is given by,

$$W = \int_a^b F(x)dx$$

Notice that if the force is constant we get the correct formula for a constant force.

$$W = \int_a^b F dx$$

$$= F(b-a)$$

Where b-a is simply the distance moved, or d. So, let's take a look of a example of non-constant forces.

CONCLUSION:

We have seen that in situations where it is impossible to know the function governing some phenomenon exactly; it is still possible to derive a reasonable estimate for the integral of the function based on data points. The idea is to choose a model function going through the data points and integrate the model function. The definition of an integral as a limit of Reimann sums shows that if you chose enough data points, the integral of the model function converges to the integral of the unknown function; so theoretically, numerical integration is on solid ground. We have also seen that there are many practical factors that influence how well numerical integration works. Simple model functions may not emulate the behaviour of the unknown function well. Complicated model functions are hard to work with. Problems with the number of data points, or the way in which the data was collected can have a major impact, and while we have explored some simple ways of estimating how accurate a particular numerical integral will be, this can be quite complicated in general.
Nonetheless, by using common sense, together with a solid grasp of what the integral means and how it is related to the geometry of the function being integrated, a creative scientist, mathematician or engineer can accomplish a great deal with numerical integration.

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