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SIGMA V Elements in Lattice Modules

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***Abstract*- Let L be a compactly generated multiplicative lattice with 1 compact in which every finite product of compact elements is compact and M be a module over L which is also a compactly generated in which the largest element is compact. In this paper we define Ap, in the lattice module M and obtain many properties where p is prime element of L. Finally we define - element in a lattice module M and obtain it's properties.**

***Index Terms*— Prime element, Primary element, Lattice Modules, Baer element, *- element, closed element.**

I. INTRODUCTION

A multiplicative lattice L is a complete lattice provided with commutative, associative and join distributive multiplication in which the largest element 1 acts as a multiplicative identity. An element a ∈L is called proper if a < 1. A proper element p of L is said to be prime if ab ≤ p implies a ≤ p or b ≤ p. If a ∈ L, b ∈ L, (a : b) is the join of all elements c in L such that cb ≤ a. A proper element p of L is said to be primary if ab ≤p implies a ≤ p or b^n ≤ p for some positive integer n. If a ∈ L, then $\sqrt{a}$ = ∨ { x ∈ L_* / x^n ≤ a, n ∈ Z+ }. An element a ∈ L is called a radical element if a = $\sqrt{a}$. An element a ∈ L is called compact if a ≤ $\vee_\alpha b_\alpha$ implies a ≤ $b_{\alpha_1} \vee b_{\alpha_2} \vee \ldots \vee b_{\alpha_n}$ for some finite subset $\{\alpha_1, \alpha_2 \ldots, \alpha_n\}$. Throughout this paper, L denotes a compactly generated multiplicative lattice with 1 compact in which every finite product of compact element is compact. We shall denote by, the set of compact elements of L. A nonempty subset of L_* is called a filter if the following conditions are satisfied.

i) x, y ∈ F implies xy ∈ F
ii)x ∈ F, x ≤y implies y ∈ F

Let F(L_*) denotes a set of all filters of L. For a nonempty subset $\{F_\alpha\}$ ⊆ F(L_*), define ⋓ F_α= {x ≥ $F_1 F_2 \ldots F_n$, $F_i \in F_{\alpha_i}$ for some i = 1,2,…,n}. Then it is observed that, F(L_*) = < F(L_*), ⋓ , ⋂> is a complete distributive lattice with ⋓ as the supremum and the set theoretic ⋂ as infimum. For a ∈ L_* the smallest filter containing a is denoted by [a) and it is given by [a) = {x ∈ L_* |x ≥ a^n for some non-negative integer n}. For a filter F ∈ F(L_*) we denote, 0_F = ∨ {x ∈ L_* | xs = 0 ,for some s ∈ F}. Let M be a complete lattice and L be a multiplicative lattice. Then M is called an L- module or module over L if there is a multiplication between elements of L and M written as aB, where a ∈ L and B ∈ M which satisfies the following properties:

i) $(\vee_\alpha a_\alpha)$A= $\vee_\alpha a_\alpha$A, ∀ a_α ∈ L, A ∈ M
ii)a$(\vee_\alpha A_\alpha)$= $\vee_\alpha aA_\alpha$, ∀a ∈ L , A_α ∈ M
iii)(ab)A = a(bA) ∀ a, b ∈ L, A ∈ M
iv)1B = B
v) 0B = 0_M ∀ a, a_α, b ∈ L and A, A_α ∈ M, where 1 is the supremum of L and 0 is the infimum of L. We denote by 0_M and I_M the least and the greatest element of M. Elements of L will generally be denoted by a, b, c ... and elements of M will generally be denoted by A,B,C... . Let M be an L- module. If N ∈ M and a ∈ L then (N : a)= ∨ { X ∈ M / aX ≤ N }. If A, B ∈ M, then (A : B) = ∨ {x ∈ L | xB ≤ A}. An element A ∈ M is called compact if A ≤ $\vee_\alpha B_\alpha$ implies a ≤ $B_{\alpha_1} \vee B_{\alpha_2} \vee \ldots \vee B_{\alpha_n}$ for some finite subset $\{B_{\alpha_1}, B_{\alpha_2} \ldots, B_{\alpha_n}\}$.A set of compact elements of M will be denoted by M_*. An L- module M is called a multiplication L-module if for every element N ∈ M there exist an element a ∈ L such that N = aI_M see [3]. In this paper a lattice module M will be a multiplication lattice module, which is compactly generated with the largest element I_M compact in which product of a compact element of L and a compact of compact element of M will be compact. A proper element N of M is said to be prime if aX ≤ N implies X ≤ N or aI_M ≤ N that is a ≤ (N : I_M) for every a ∈ L, A ∈ M. If N is a prime element of M then (N : I_M) is a prime element of L. [3]. An element N < I_M in M is said to be primary if aX ≤ N implies X≤N or $a^n I_M$ ≤ N that is a^n ≤ (N : I_M) for some integer n. An element N of M is called a radical if (N : I_M)= $\sqrt{N : I_M}$. If aN = 0_M implies a = 0 or N = 0_M for any a ∈ L and N ∈ M then M is called torsion free L- module. If (0_M : I_M) = 0 then M is called a torsion free faithful L - module. In this paper a module M will be a torsion free faithful , multiplication PG- lattice L- module. For all these definitions and any other undefined term one can refer [1][2].

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Let V be any element of M. An element V ≤ A ∈ M is called σ_V - element if for any x ∈ L_*, V ≤ xI_M ≤ A implies A ∨ (V : x) = I_M.
We write, σ_{VM} = {A ∈ M | A is σ_V - element}.
Theorem (1) Let M be a multiplication Lattice module over a distributive lattice L and A, B ∈ σ_{VM}. Then (A ∧ B) ∈ σ_{VM} and abI_M ∈ σ_{VM}, where A = aI_M, B = bI_M. Proof:-i) Let A, B be σ_V -elements and let x ∈ L_* be such that V ≤ xI_M ≤ A ∧ B. Then xI_M ≤ A and xI_M ≤ B. As A,B ∈ σ_{VM}, A∨(V : x) = I_M and B∨ (V : x) = I_M. But (A ∧ B) ∨ (V : x) = I_M=[A ∨ (V : x)] ∧ [B ∨(V : x)] = I_M ∧ I_M = I_M. Hence A ∧ B is a σ_V -element of M.
ii) If aI_M and bI_M ∈ σ_{VM}, where a, b ∈ L then we prove that abI_M ∈ σ_{VM}. Let x ∈ L_* be such that V ≤ xI_M ≤ abI_M. Then x ≤ ab, so x ≤ a, x ≤ b. Hence xI_M ≤ aI_M and xI_M ≤ bI_M. By hypothesis, aI_M ∨ (V : x) = I_M and bI_M ∨(V : x) = I_M. Since M is a multiplication lattice module and (V : x) ∈ M there exists d ∈ L such that (V : x) = dI_M. We have, aI_M ∨ dI_M = I_M and bI_M ∨ dI_M = I_M. This gives ,
(a ∨ d) I_M = I_M and (b ∨ d) I_M = I_M.
So (a∨d) = 1 and (b∨d) = 1. But (a∨d) = 1 and (b∨d) = 1 implies ab∨d = 1 [2.14 RPD]. Therefore abI_M ∨ dI_M = abI_M ∨ (V : x) = I_M and abI_M is a σ_V - element.

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Theorem (2) Let A,B ∈ σ_{VM}. Then (A ∨ B) ∈ σ_{VM}.
Proof:- Let x ∈ L_* be such that V ≤ xI_M ≤ (A ∨ B).Since M is a multiplication Lattice module, A = aI_M,B = bI_M, a, b ∈ L. Then (A ∨ B) =(a ∨ b) I_M and xI_M ≤ (a ∨ b) I_M and x ≤ (a ∨ b). Since x ∈ L_* and L is compactly generated, there exist compact element y, z ∈ L such that y ≤ a, z ≤ b, x ≤ y ∨ z. As A,B ∈ σ_{VM}, yI_M ≤ A, zI_M ≤ B, we have, A ∨ (V : y) = I_M, B ∨ (V : z) = I_M. Then (A ∨ B) ∨ [(V : y) ∧ (V : z)] = (A ∨ B) ∨ [V : (y ∨ z)] = I_M, ($\alpha -$ adic - vi) We have x ≤ y ∨ z so V : (y ∨ z) ≤ V : x. Hence (A ∨ B) ∨ [V : x] = I_M. Hence (A ∨ B) ∈ σ_{VM}.
Theorem (3) If A_i ∈ σ_{VM} then $\wedge_i A_i \in \sigma_{VM}$.
Proof:-Let x ∈ L_* be such that V ≤ xI_M ≤ $\wedge_{i\in 1}^{n}(A_i)$.Then xI_M ≤ A_i, for all i and A_i ∨ (V : x) = I_M.Since m is distributive ($\wedge_{i\in 1}^{n}(A_i)$) ∨ (V :x) = [A_1 ∨ (V : x)] ∧ [A_2∨ (V : x)] ∧… ∧ [A_n ∨ (V : x)] =I_M ∧I_M ∧ … ∧ I_M = I_M.

Theorem (4) If A_α ∈ σ_{VM} then $\vee_\alpha A_\alpha \in \sigma_{VM}$.
Proof:- Let x ∈ L_* be such that V ≤ xI_M ≤ $\vee_\alpha(A_\alpha)$. As xI_M is compact, xI_M ≤ A_1 ∨ A_2 ∨ … ∨ A_n. Since M is compactly generated each Ai is the join of compact elements, xI_M ≤ Y_1 ∨ Y_2 ∨ … ∨ Y_n for some Y_i ∈ M_* such that Y_i ≤ A_i, i = 1, 2,…, n. As Y_i ∈ M_* there exist y_i ∈ L_* such that Y_i = $y_i I_M$, 1 ≤ i ≤ n, $y_i I_M$ ≤ A_i. As each A_i is σ_V - element, y_i ∈ L_*, V ≤ $y_i I_M$ ≤ A_i,we have , A_i ∨ (V : y_i) =I_M, i = 1, 2,…, n, but A = $\vee_\alpha(A_\alpha)$ implies A ∨ (V : y_i) = I_M. Hence A ∨ [(V : y_1) ∧ (V : y_2) ∧ … ∧ (V : y_n]= A ∨ [V : (y_1∨y_2 ∨…∨y_n)] = I_M.[_α- adic vi]. Let y = (y_1 ∨ y_2 ∨ … ∨ y_n). Then A ∨ (V : y) = I_M ... We have, xI_M ≤ $y_1 I_M$ ∨$y_2 I_M$ ∨ …∨$y_n I_M$ = (y_1∨y_2 ∨…∨y_n) I_M. This gives x ≤ (y_1 ∨ y_2 ∨ … ∨y_n) = y ∈ L_*. So V : y ≤ V : x, x ∈ L_*.....and A ∨ (V : x) = I_M, where x ∈ L_*. V ≤ xI_M ≤ $\vee_\alpha(A_\alpha)$ = A. Therefore, A =$\vee_\alpha(A_\alpha)$ is a σ_V - element of M.

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Theorem (5) V and I_M are σ_V - elements of M.
Proof:- Let x ∈ L_* . Then obviously V ≤ xI_M ≤ I_M and obviously, I_M ∨ (V : x) = I_M. So I_M is σ_V - elements of M. Let x ∈ L_* be such V ≤ xI_M ≤ V . Hence xI_M = V and V ∨ (V : x) = I_M which shows that V is σ_V - elements of M.

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CONCLUSION

The new terms are defined for lattice module and the properties of prime elements are studied.

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