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Calculation of the Temperature Distribution and Thermal Deflection in aThin Clamped Circular Plate

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Abstract- **This paper is concerned with the determination of unknown temperature, temperature distribution, and thermal deflection on upper plane surface of a thin clamped circular plate occupying the space D: 0 □ r □ a, 0 □ z □ h by applying finite Hankel transform and Laplace transform techniques.**

***Key words & phrases*: Clamped circular plate, inverse thermoelastic problem, thermal deflection, Hankel transform and Laplace transform.**

I. INTRODUCTION

In [1] and [2] one dimensional transient thermoelastic problems derived the heating temperature and the heat flux on the surface of an isotropic infinite slab. The direct and inverse problems of thermo elasticity of a thin circular plate are considered in [3] to [6] and [9].

In the present paper, an attempt is made to determine the unknown temperature, temperature distribution, and thermal deflection on upper plane surface of a thin clamped circular plate occupying the space D: 0 □ r □ a, 0 □ z □ h with the stated boundary conditions. The finite Hankel transform and Laplace transform techniques have been used to obtain the solution of the problem.

II. STATEMENT OF THE PROBLEM

Consider an isotropic circular plate of thickness h occupying the space $D : 0 \le r \le a,\ 0 \le z \le h$.
The differential equation governing the displacement function U(r,z,t) as [2] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)a_t T \qquad 2.1$$

With Ur = 0 at r = a 2.2

Where ν and at are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and θ is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} = \frac{1}{k}\frac{\partial \theta}{\partial t} \qquad 2.3$$

Subject to the initial condition

$$\theta = 0 \ at \ \ t = o \qquad 2.4$$

The boundary conditions $\theta = 0 \ at \ \ r = 0$ ***2.5***

$$\theta = g(r,t) \quad at\ z = h \ \textit{(Unknown)}$$

2.6

$$\theta + c\frac{\partial \theta}{\partial z} = u(r,t) \ \ at \ \ z = 0 \qquad 2.7$$

The interior condition

$$\theta + c\frac{\partial \theta}{\partial z} = f(r,t) \quad at\ z = \xi \ \text{(Known)} \qquad 2.8$$

Where k is the thermal diffusivity of the material of the plate and c is arbitrary constant.
The equations (2.1) to (2.8) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Hankel transform defined in [4] to equations (2.3) to (2.8) and using (2.5), one obtains

$$\frac{d^2\bar{\theta}}{dz^2} - \lambda_n^2\,\bar{\theta} = \frac{1}{k}\frac{d\bar{\theta}}{dt} \qquad 3.1$$

$$\bar{\theta}(\lambda_n, z, 0) = 0 \qquad \textbf{3.2}$$

$$\left[\bar{\theta}(\lambda_n, z, t)\right]_{z=h} = \bar{g}(\lambda_n, t) \qquad \textbf{3.3}$$

$$\left[\bar{\theta}(\lambda_n,z,t)+c\frac{d\bar{\theta}(\lambda_n,z,t)}{dz}\right]_{z=0}=\bar{u}(\lambda_n,t) \qquad 3.4$$

$$\left[\bar{\theta}(\lambda_n,z,t)+c\frac{d\bar{\theta}(\lambda_n,z,t)}{dz}\right]_{z=\xi}=\bar{f}(\lambda_n,t) \qquad 3.5$$

where $\bar{\theta}$ denotes the finite Hankel transform of T and λ_n is the Hankel transform parameter.

Applying Laplace transform defined in [4] to the equations (3.1), (3.3) to (3.5) using (3.2), one obtains

$$\frac{d^2\bar{\theta}^*}{dz^2}-q^2\bar{\theta}^*=0 \qquad 3.6$$

where $q^2=\lambda_n^2+\frac{s}{k}$

$$\left[\bar{\theta}^*(\lambda_n,z,s)\right]_{z=h}=\bar{g}^*(\lambda_n,s) \qquad \mathit{3.7}$$

$$\left[\bar{\theta}^*(\lambda_n,z,s)+c\frac{d\bar{\theta}^*(\lambda_n,z,s)}{dz}\right]_{z=0}=\bar{u}^*(\lambda_n,s) \qquad 3.8$$

$$\left[\bar{\theta}^*(\lambda_n,z,s)+c\frac{d\bar{\theta}^*(\lambda_n,z,s)}{dz}\right]_{z=\xi}=\bar{f}^*(\lambda_n,s) \qquad 3.9$$

where $\bar{\theta}^*$ denotes the Laplace transform of $\bar{\theta}$ and s is the Laplace transform parameter.

The equation (3.6) is a second order differential equation whose solution is given by

$$\bar{\theta}^*(\lambda_n,z,s)=Ae^{qz}+Be^{-qz}$$

3.10

where A and B are arbitrary constants.

Using (3.8) and (3.9) in (3.10), we obtain

$$A=\frac{-1}{(1+cq)(e^{q\xi}-e^{-q\xi})}\left[\bar{u}^*(\lambda_n,s)e^{-q\xi}-\bar{f}^*(\lambda_n,s)\right]$$

and

$$B=\frac{1}{(1-cq)(e^{q\xi}-e^{-q\xi})}\left[\bar{u}^*(\lambda_n,s)e^{q\xi}-\bar{f}^*(\lambda_n,s)\right]$$

Substituting the values of A and B in equation (3.10) and then applying inversion of Laplace transform and finite Hankel transform, one obtains

$$\theta(r,z,t)=\frac{4k\pi(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(1-c^2\lambda_n^2)J_1^2(\lambda_n a)}$$

$$\times\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi z}{\xi}\right)-\left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times\int_0^t\bar{f}(\lambda_n,t')e^{-k(\lambda_n^2+\frac{m^2\pi^2}{\xi^2})(t-t')}dt'$$

$$-\frac{4k\pi(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(1-c^2\lambda_n^2)J_1^2(\lambda_n a)}$$

$$\times\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi(z-\xi)}{\xi}\right)-\left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\bar{u}(\lambda_n,t')e^{-k(\lambda_n^2+\frac{m^2\pi^2}{\xi^2})(t-t')}dt'$$

3.11

$$g(r,t)=\frac{4k\pi(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(1-c^2\lambda_n^2)J_1^2(\lambda_n a)}$$

$$\times\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi h}{\xi}\right)-\left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi h}{\xi}\right)\right]$$

$$\times\int_0^t\bar{f}(\lambda_n,t')e^{-k(\lambda_n^2+\frac{m^2\pi^2}{\xi^2})(t-t')}dt'$$

$$-\frac{4k\pi(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(1-c^2\lambda_n^2)J_1^2(\lambda_n a)}$$

$$\times\sum_{m=1}^{\infty}m(-1)^{m+1}\left[\sin\left(\frac{m\pi(h-\xi)}{\xi}\right)-\left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi(h-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t\bar{u}(\lambda_n,t')e^{-k(\lambda_n^2+\frac{m^2\pi^2}{\xi^2})(t-t')}dt'$$

3.12

where λ_n are the positive roots of the equation $J_0(\lambda_n a)=0$.

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IV.DETERMINATION OF QUASI-STATIC THERMAL DEFLECTION

The differential equation satisfied by the deflection *w(r, t)* is

$$D\nabla_1^{\,4} w = \frac{-\nabla_1^{\,2} M_\theta}{1-\nu}$$

4.1

Where ν is the Poisson's ratio of the plate material , M_T denote the thermal momentum of the plate and D denote the flexural rigidity,

Where $\nabla_1^{\,2} = \dfrac{d^2}{dr^2} + \dfrac{1}{r}\dfrac{d}{dr}$

Since the edge of the circular plate is fixed and clamped,

$$w = \left[\frac{\partial w}{\partial r}\right]_{r=a} = 0$$

4.2

We assume that the solution of equation (4.1) satisfying equation (4.2) as

$$w(r,t) = \sum_{n=1}^{\infty} c_n(t)\left[J_0(\lambda_n r) - J_0(\lambda_n a)\right]$$

4.3

The term M_θ is defined as

$$M_\theta(r,t) = \alpha E \int_0^h z T(r,z,t)\,dz$$

4.4

Substituting the value of *T(r, z, t)* from equation (3.11) in equation (4.4) one obtains

$$M_\theta(r,t) = \frac{4k\pi\alpha E(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{(1-c^2\lambda_n^{\,2})} \frac{J_0(\lambda_n r)}{J_1^{\,2}(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1}\left[1-(-1)^m\right]\left(\frac{h(z-\xi)}{\pi}\right)$$

$$\times \int_0^t \bar{f}(\lambda_n,t')e^{-k(\lambda_n^{\,2}+\frac{m^2\pi^2}{\xi^2})(t-t')}\,dt'$$

$$-\frac{4k\pi\alpha E(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{(1-c^2\lambda_n^{\,2})} \frac{J_0(\lambda_n r)}{J_1^{\,2}(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1}\left[1-(-1)^m\right]\frac{(\xi - z)}{\pi}$$

$$\times \int_0^t \bar{u}(\lambda_n,t')e^{-k(\lambda_n^{\,2}+\frac{m^2\pi^2}{\xi^2})(t-t')}\,dt'$$

4.5

Using the equations (4.1), (4.5) and the result $\left[\dfrac{d^2}{dr^2} + \dfrac{1}{r}\dfrac{d}{dr}\right]J_0(\lambda_n r) = -\lambda_n^{\,2} J_0(\lambda_n r)$, one obtains

$$c_n(t) = \frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^{\,2}(1-c^2\lambda_n^{\,2})J_1^{\,2}(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1}\left[\frac{(z-\xi)h}{\pi}\right]\left[1-(-1)^m\right]$$

$$\times \int_0^t \bar{f}(\lambda_n,t')e^{-k(\lambda_n^{\,2}+\frac{m^2\pi^2}{\xi^2})(t-t')}\,dt'$$

$$-\frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^{\,2}(1-c^2\lambda_n^{\,2})J_1^{\,2}(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} (-1)^{m+1}\left[\frac{(\xi - z)h}{\pi}\right]\left[1-(-1)^m\right]$$

$$\times \int_0^t \bar{u}(\lambda_n,t')e^{-k(\lambda_n^{\,2}+\frac{m^2\pi^2}{\xi^2})(t-t')}\,dt'$$

4.6

Substituting the value of $c_n(t)$ in equation (4.3), one obtains the quasi-static thermal deflection *w(r, t)* as

$$w(r,t) = \frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^{\,2}(1-c^2\lambda_n^{\,2})} \frac{[J_0(\lambda_n r) - J_0(\lambda_n a)]}{J_1^{\,2}(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\frac{(z-\xi)h}{m\pi}\right]\left[1-(-1)^m\right]$$

$$\times \int_0^t \bar{f}(\lambda_n,t')e^{-k(\lambda_n^{\,2}+\frac{m^2\pi^2}{\xi^2})(t-t')}\,dt'$$

$$-\frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^{\,2}(1-c^2\lambda_n^{\,2})} \frac{[J_0(\lambda_n r) - J_0(\lambda_n a)]}{J_1^{\,2}(\lambda_n a)}$$

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$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\frac{(\xi - z)h}{m\pi}\right]\left[1-(-1)^m\right]$$

$$\times \int_0^t \bar{u}(\lambda_n, t')e^{-k(\lambda_n^2 + \frac{m^2\pi^2}{\xi^2})(t-t')} dt' \qquad 4.7$$

V. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(r,t) = (\xi + h)e^{\xi}(r-a)(1-e^{-t})$,

$u(r,t) = (\xi + h)(r-a)(1-e^{-t})$,

$\beta = \dfrac{4k\pi\alpha E(\xi + h)}{D(1-\nu)a^2\xi^2}$, $a = 1$, $\xi = 0.25$, $h = 1$, $t = 1$ sec, $k = 0.86$ in the equation (4.7) to obtain

$$\frac{w(r,t)}{\beta} = \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2(1-\lambda_n^2)} \frac{[J_0(\lambda_n r) - J_0(\lambda_n)]}{J_1^2(\lambda_n)}\left[J_0(\lambda_n) - \int_0^1 J_0(\lambda_n r)dr\right]$$

$$\times \sum_{m=1}^{\infty}(-1)^{m+1}[0.25 - z][1-(-1)^m]$$

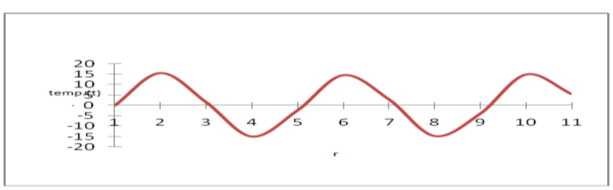
$$\times \left[\frac{1-e^{-0.86(\lambda_n^2 + 157.75m^2)}}{0.86(\lambda_n^2 + 157.75m^2)} + \frac{0.37 - e^{-0.86(\lambda_n^2 + 157.75m^2)}}{1-0.86(\lambda_n^2 + 157.75m^2)}\right]e^{0.25}$$

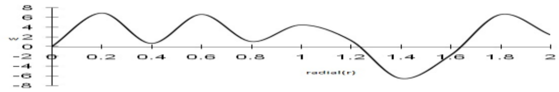
$$-\sum_{n=1}^{\infty} \frac{1}{\lambda_n^3(1-\lambda_n^2)} \frac{[J_0(\lambda_n r) - J_0(\lambda_n)]}{J_1^2(\lambda_n)}\left[J_0(\lambda_n) - \int_0^1 J_0(\lambda_n r)dr\right]$$

$$\times \sum_{m=1}^{\infty}(-1)^{m+1}[z - 0.25][1-(-1)^m]$$

$$\times \left[\frac{1-e^{-0.86(\lambda_n^2 + 157.75m^2)}}{0.86(\lambda_n^2 + 157.75m^2)} + \frac{0.37 - e^{-0.86(\lambda_n^2 + 157.75m^2)}}{1-0.86(\lambda_n^2 + 157.75m^2)}\right]$$

5.1





VI. CONCLUSION

The temperature distribution, unknown temperature and thermal deflection have been determined on upper plane surface of the circular plate with the aids of finite Hankel transform and Laplace transform techniques. The results obtained are in terms of Bessel' function in the form of infinite series. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The expressions that are obtained can be applied to the design of useful structures or machines in engineering applications.

REFERENCES

[1] **Grysa K and Cialkowski M. J**: On a certain inverse problem of temperature and thermal stress fields, Acta Mechanica 36(1980), pp.169-185.

[2] **Grysa K and Kozlowski Z**: One dimensional problem of temperature and heat flux determination at the surfaces of a thermoelastic slab part-I, The Analytical Solutions, NUCL. Engrg 74(1982), pp.1-14.

[3] KHOBRAGADE, N. W AND WANKHEDE, P. C**: AN INVERSE STEADY-STATE PROBLEM OF THIN CIRCULAR PLATE IN MARCHI-FASULO TRANSFORM DOMAIN, THE JOURNAL OF INDIAN ACADEMY OF MATHEMATICS, VOL. 25, NO. 2, (2003).**

[4] KHOBRAGADE, N. W AND WANKHEDE, P. C**: AN INVERSE UNSTEADY-STATE PROBLEM OF THIN CIRCULAR PLATE IN MARCHI-FASULO TRANSFORM DOMAIN, BULLETIN OF THE CALCUTTA MATHEMATICAL SOCIETY, VOL. 95, NO. 5, (2003).**

[5] NOWACKI W.**: THE STATE OF STRESS IN A THICK CIRCULAR PLATE DUE TO TEMPERATURE FIELD.**
Bull.. Sci. Acad. Polon Ser. Tech. 5, (1957), p 227.

[6] **Roychaudhari S. K**: A note on the quasi-static stresses in a thin circular plate due to transient temperature applied along the circumference of a circle over the upper face.
Bull. Acad. Sci. Ser. Tech. 20 (1972), pp. 21-24.

[7] **Wankhede P. C:** On the quasi-static thermal stresses in a circular plate. Indian J. of Pure and Appl. Maths 13,(1982), pp. 1273-1277.

