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Stochastic Perishable Inventory Control in a Service Facility System Maintaining Inventory for Service: Semi Markov Decision Problem

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Abstract:-- **In this article we address the problem of optimally controlling the service rate in a service facility system maintaining perishable inventory for service completion. We consider a finite service facility system having finite waiting space with Poisson arrivals and exponentially distributed service times and lif e time of items. For the given values of maximum inventory and waiting space capacity, we determine the order quantities at various instance of time so that the long run expected cost rate is minimized. The problem is modeled as a semi–Markov Decision Problem. We prove the existence of a stationary optimal policy and solve it by employing value iteration technique. Numerical example is provided to get insight into the system behavior.**

Keywords:-- **Service facility, Service control, Poisson demand, Perishable inventory, Exponential service time, Markov Decision Process.**

I. INTRODUCTION

Maintaining perishable inventory for service completion purpose in service facilities is a major system phenomenon which needs in depth study. Last two decades, many researchers in the field of operations research and resource management contributed many results. (Berman, O.,Sapna, KP., Arivarigan, G., Elango, C., Yadavalli, V., Arumugam, N., Krishnamoorthy, S., and Sivakumar, B). In most of the studies mentioned above the system is considered as a Markov process with finite or infinite state space. The expressions for transition probability functions and the infinitesimal generator matrix of the Markov process are derived. The steady state probability distribution of the states has been found. Then by computing proper system performance measures and imposing respective cost structure, the cost analysis is done to get the optimal parameters of the system.

In all the above said models, system performance measure are computed for the uncontrolled system then a specific cost structure is imposed on the system, to get a optimum parameter values using an optimization criteria. We believe that an integrated approach like Markov Decision Process model is most appropriate to study service facility system (Queues-Inventory) and Maintenance systems. Sapna, K.P., and Berman, O., [1] studied one such system under MDP structure using LPP method to control the service rates. So for in the literature only admission control and service rate control problems are studied under MDP regime .Hild Mohamed et. Al [6] analyzed a Markov decision problem: Optimal control of servers in a service facility holding perishable inventory with impatient customers.

The main contribution of this paper is to prescribe a control policy, which is optimal that yields minimum cost. The specific optimal ordering policy for the inventory system is obtained by considering the ordering cost at each state of the system. The ordering rates are functions of the number of customers and the inventory levels.

In this article we consider a service facility system maintaining perishable inventory under MDP structure. We get a continuous time MDP in which time between decision epochs are exponentially distributed. We can also analyze the model by converting it to an equivalent more easy discrete – time process using uniformization technique. This scale downed processes will behave well to apply long run expected total cost rate criteria to get optimal policy for finite horizon problems with finite states and Randomized Markovian decisions.

Here, we use value iteration method to optimize the expected total cost rate. Section 1 gives a brief introduction and literature review of the problem. In section 2, the model formulation is done with notations. Analysis part of the model is given in section 3. Section 4 deals with long – run Expected cost rate criteria to get the optimal vales of the system parameters.

II. PROBLEM FORMULATION

Consider a service facility system with inventory maintained to satisfy the customers. Assume that the maximum capacity of inventory is S and a finite waiting space N, there exist a forced balking when there are N customers waiting in the system.

Customers arrive for service facility according to a Poisson process with parameter $\lambda(>0)$ and are served according to a FCFS queue discipline. One unit (item) from inventory is used up to serve one customer. The service times follow an exponential distribution with parameter $\gamma(>0)$. Whenever the inventory level reaches to a prefixed level s ($0 \le s < S$), an order for Q=S-s items placed and the lead time is exponentially distributed with parameter $\mu > 0$ respectively. The size of the order is adjusted at the time of replenishment so that immediately after replenishment the inventory level reaches to S. Order decision is made at each level below the reorder levels. The items in stock are of perishable native with perishing rate θ.

Let I(t) and X(t) denote the inventory level and the number of customers in the system at time t. Then $\{(I(t), X(t)): t \ge 0\}$ is a two dimensional stochastic process with state space, $E_1 \times E_2$, where $E_1 = \{0, 1, 2, \ldots, S\}$ and $E_2 = \{0, 1, 2, \ldots, N\}$.

Decision Sets:

The reordering decisions taken at each state of the system (j, r) ∈ E, where, I(t) = j and X(t) = r.

Let A_i (i =1, 2, 3) denotes the set of possible actions where, $A_1 = \{0\}$, $A_2 = \{0, 1\}$, $A_3 = \{2\}$, and $A = A_1 \cup A_2 \cup A_3$.

Suppose F denote the class of all stationary policies, then a policy *f* (sequence of decisions) can be defined as a function f : E → A, given by f(i, q) = {(k): (i, q) ∈E_i, k ∈ A_i, i = 1, 2, 3}. Let E_1 = {(i, q) ∈ E / f(i, q)= 0,.E_2 = {(i, q) ∈ E / f(i, q)= 0 or 1}, E_3 = {(0, q) ∈ E / f(i, q)= 2},0 represents 'no order', 1 means reorder for 'Q = S-s' items and 2 means compulsory order for S items when inventory level is zero. Objective of the problem is to find the optimal reorder level s so that the long run expected total cost rate is minimum.

2.1 *Notations and Assumptions*:

1. $E_1 \times E_2 = E$ is the state space of theStochastic Process $\{(I(t), X(t)): t \ge 0\}$, where E_1 = {0,1,2,…,S} and E_2={0,1,…,N}2.A_S- decision set corresponding to state s∈E.
2. $C_{(i,q)}(a)$ −cost occurred when action a istaken at state (i, q).

4.$p_{(i,q)}^{(j,r)}(a)$- the transition probability from state $(i, q)$to state(j, r) when action a is taken at state$(i, q) \in E$

5.Inventory levels are reviewed at the time of service completion epoch .

6.Recording policy is (s, S): $Q = S - s$ item ordered when the inventory level reaches s (prefixed level),where $0 \le s < S$.

7. F-the class of stationary policies.

III.ANALYSIS OF THE PROBLEM

Let R denote the stationary policy, which is randomized time invariant and Markovian Policy (MR). From our assumptions it can be seen that $\{(I(t), X(t)): t \ge 0\}$ is denoted as the controlled process $\{(I^R(t), X^R(t)): t \ge 0\}$ when policy R is adopted. Since the process $\{(I^R(t), X^R(t)): t \ge 0\}$ is a Markov Process with finite state space E. The process is completely Ergodic, if every stationary policy gives rise to an irreducible Markov chain. It can be seen that for every stationary policy $f \in F, \{I^f, X^f\}$ is completely Ergodic and also the optimal stationary policy R^* exists, because the state and action spaces are finite.

Let A_j , $j \in E_1$ represent the set of all possible actions for taken the system when it belongs to the set

$$A_j = \begin{cases} \{0, 1\}, & 1 \le j \le s. \\ \{0\}, & s+1 \le j \le S, \quad A = \bigcup_{j \in E_1} A_j. \\ \{2\}, & j = 0. \end{cases}$$

A randomized Markov decision rule from the class F is equivalent to the function $f: E \to A$ given by $p_{d_t} \in \wp(A_j)$, $j \in E_1$, where $d_t$is the Markovian randomized decision rule for t ∈T and randomized (MR).

We denote the set of decision rules at time t by D_t^{MR} .

If $d_t$is the Markovian randomized decision rule, the expected reward satisfies the transition probability relations.

$$p_t((j,r) | (i,q), d_t(i,q)) = \sum_{a \in A_s} p_t((j,r) | (i,q), a) p_{d_t(i,q)}(a).$$

$$r_t(i,q), d_t(i,q) = \sum_{a \in A_s} r_t(i,q,a) p_{d_t(i,q)}(a).$$

For Markovian $\Pi \in \Pi^{MR}$,d_t depends on history analysis through the current state of the process $(i,q) \in E$ so that $p^{\Pi}\{Y_t = a | Z_t = h_t\} = P_{d_t(h_t)}(a)$ where Y_t – denote the action at time t and the history process Z_t defined by Z_1(w)=s_1 and Z_t(w)= {s_1,s_2,s_3,…,s_t} for $1 \le t \le N, N \le \infty$

Randomized Markovian Policy Π

Order size	Q=S-s	Q+1=S-s+1	…	Q+s=S
Probability	p_s	p_{s-1}	…	p_0

Π^{MR} is the randomized Markovian policy. Under this policy Π an action $a \in A(j)$ is chosen with probability $\Pi_a(j)$, whenever the process is in state $j \in E_1$. Whenever $\Pi_a(j) = 0 \, or 1$, the stationary randomized policy Π reduces to a familiar stationary policy.

3.1. Steady State Analysis:

Let $\{(I^R(t), X^R(t)) : t \geq 0\}$ denote the process. $\{(I(t), X(t)) : t \geq 0\}$ in which R is the policy adopted from our assumptions made in the previous section. The controlled process $\{I^R, X^R\}$ where R is the randomized Markovian policy in a Markov process. Under the randomized policy Π, the expected long run total cost rate when policy Π is adopted is given by equation (1)

$$C^{\Pi} = h\bar{I}^{\Pi} + c_1\bar{w}^{\Pi} + c_2\alpha_a^{\Pi} + g\alpha_b^{\Pi} + \beta\alpha_c^{\Pi} + p\alpha_d^{\Pi}$$

h -holding cost / unit item / unit time c_1 – waiting cost / customer / unit time

c_2 – reordering cost / order g - balking cost / customer β - service cost / customer

p-Perishable cost /item $\bar{I}^{\Pi}$ - mean inventory level, α_a^{Π} - reordering rate , $\bar{w}^{\Pi}$ - mean waiting time in system , α_b^{Π} - balking rate, α_c^{Π} - service completion rate, α_d^{Π} - expected perishing rate. Our objective is to find an optimal policy Π^* for which $C^{\Pi^*} \leq C^{\Pi}$ for every MR policy in Π^{MR}. For any fixed MR policy $\Pi \in \Pi^{MR}$ and $(i,q),(j,r) \in E$, define

$$P_{iq}^{\Pi}(j,r,t) = \Pr\{I^{\pi}(t) = j, X^{\pi}(t) = r \mid I^{\pi}(0) = i, X^{\pi}(0) = q\}, \quad (i,q),(j.r) \in E$$

Now $P_{iq}^{\pi}(j,r,t)$ satisfies the Kolmogorov forward differential equation $P_i'(t) = P(t)A$, where A is an infinitesimal generator of the Markov process $\{(I^{\pi}(t), X^{\pi}(t)) : t \geq 0\}$. For each MR policy π, we get an irreducible Markov chain with the state space E and actions space A which are finite,

$$P^{\Pi}(j,r) = \lim_{t \to \infty} P_{iq}^{\Pi}(j,r,t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

exists and is independent of initial state conditions. This implies the balance equations (3) – (14) given below. Transitions in and out of a state give a system of equations. Consider the typical state (j, r) that lies in the range s+1 ≤ j ≤ S-1; 1 ≤ r ≤ N– 1. When (j, r) lies in this range, there is no order pending and hence transition out of this state can be due to either by demand or a service completion or item perish. The corresponding balance equation is given by equation (7). A service completion in state (j+1, r+1) will decrease both inventory level and number of customers by one unit, thus transition made to state (j, r).

When one customer arrives and enters the system (r < N) at state (j, r -1), the new state is (j, r). Considering two different ways of reaching state (j, r) and are reflected on the right hand side of Eq. (7).When one item perish from the inventory (j, r) give to (j-1, r) with rate jθ. Now the system of equations can be written in order as follows,

$$(\lambda + S\theta)P^{\pi}(S,0) = \mu \textstyle\sum_{j=0}^{s} p_j . P^{\pi}(j,0) \quad (3)$$

$$(\lambda + \gamma + S\theta)P^{\pi}(S,r) = \mu \textstyle\sum_{j=0}^{s} p_j . P^{\pi}(j,r) + \lambda . P^{\pi}(S, r-1), \; 1 \leq r \leq N-1 \quad (4)$$

$$(\gamma + S\theta)P^{\pi}(S,N) = \mu \textstyle\sum_{j=0}^{s} p_j . P^{\pi}(j,N) + \lambda P^{\pi}(S, N-1) \quad (5)$$

$$(\lambda + j\theta)P^{\pi}(j,0) = (\gamma + (j+1)\theta)P^{\pi}(j+1,1), \quad s+1 \leq j \leq S-1, \quad (6)$$

$$(\lambda + \gamma + j\theta)P^{\pi}(j,r) = (\gamma + (j+1)\theta)P^{\pi}(j+1, r+1) + \lambda P^{\pi}(j, r-1), \; s+1 \leq j \leq S-1; 1 \leq r \leq N-1 \quad (7)$$

$$(\gamma + j\theta)P^{\pi}(j,N) = \lambda P^{\pi}(j, N-1), s+1 \leq j \leq S-1 (8)$$

$$(\lambda + \mu P_j + j\theta)P^{\pi}(j,0) = (j+1)\theta P^{\pi}(j+1,0) + \gamma P^{\pi}(j+1,1), \; 1 \leq j \leq s, \quad (9)$$

$$(\lambda + \mu P_j + \gamma + j\theta)P^{\pi}(j,r) = (j+1)\theta P^{\pi}(j+1,r) + \gamma P\pi j{+}1{,}r{+}1{+}\lambda P\pi j{,}r{-}1, \quad 1{\leq}j{\leq}s{;}1{\leq}r{\leq}N{-}1, \quad (10)$$

$$(\mu P_j + \gamma + j\theta)P^{\pi}(j,N) = (j+1)\theta P^{\pi}(j+1,N) + \lambda P^{\pi}(j, N-1,), \; 1 \leq j \leq s, \quad (11)$$

$$(\lambda + \mu p_0)P^{\pi}(0,0) = \gamma P^{\pi}(1,1) + \theta P^{\pi}(1,0) \quad (12)$$

$$(\lambda + \mu p_0)P^{\pi}(0,r) = \gamma P^{\pi}(1,r+1) + \theta P^{\pi}(1,r) + \lambda P^{\pi}(0, r-1,), \; 1 \leq r \leq N-1, \quad (13)$$

$$\mu p_0 P^{\pi}(0,N) = \lambda P^{\pi}(0, N-1) + \theta P^{\pi}(1,N) \quad (14)$$

Together with the above set of equations, the total probabilitycondition $\sum_{(j,r)} \mathbf{P}^{\boldsymbol{\pi}}(\mathbf{j},\mathbf{r}) = \mathbf{1}$ **(15)**

gives steady state probabilities $\{P^{\pi}(j, r), (j, r) \in E\}$ uniquely.

3.2 System Performance Measures.

The average inventory level in the system is given by

$$\bar{I}^{\pi} = \sum_{j=1}^{S} j \sum_{r=0}^{N} P^{\pi}(j,r) \quad (16)$$

Mean waiting time in the system is

$$\bar{W}^{\pi} = \sum_{r=1}^{N} \frac{r}{\gamma} \sum_{j=0}^{s} P^{\pi}(j,r) + \sum_{k=0}^{s} \frac{1}{\mu p_k} \sum_{m=1}^{\lceil N/s \rceil} \sum_{r=1}^{ms} m P^{\pi}(j,r). \quad (17)$$

The reorder rate is given by

$$\alpha_a^{\pi} = \mu \sum_{r=0}^{N} \sum_{j=0}^{S} p_j P^{\pi}(j,r) \quad (18)$$

The balking rate is given by

$$\alpha_b^{\pi} = \lambda \sum_{j=0}^{S} P^{\pi}(j,N) \quad (19)$$

The servicecompletion rate is given by

$$\alpha_c^{\pi} = \gamma \sum_{r=1}^{N} \sum_{j=1}^{S} P^{\pi}(j,r). \quad (20)$$

The expected perishable rate is given by $\alpha_d^{\pi} = \sum_{j=1}^{S} \sum_{r=0}^{N} j\theta \, P^{\pi}(j,r)$ (21)

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Now the long run expected cost rate is given by

$$C^{\pi} = h\sum_{j=0}^{S} j\sum_{r=0}^{N} P^{\pi}(j,r) + \frac{c_1}{\gamma}\sum_{r=1}^{N} r\sum_{j=0}^{S} P^{\pi}(j,r) + \frac{c_1}{\mu}\sum_{m=1}^{[N/s]}\sum_{r=1}^{ms}\sum_{j=0}^{S}\frac{m}{p_j}P^{\pi}(j,r) + c_2\mu\sum_{r=1}^{N} r\sum_{j=0}^{s} p_j P^{\pi}(j,r)$$
$$+ g\lambda\sum_{j=0}^{S} P^{\pi}(j,N) + \beta\gamma\sum_{j=1}^{S}\sum_{r=1}^{N} P^{\pi}(j,r) + p\sum_{j=1}^{S}\sum_{r=1}^{N} j\theta P^{\pi}(j,r) \qquad (22)$$

IV. VALUE ITERATION METHOD

For the Semi Markov decision model the formulation of a value-iteration algorithm is not straight forward. However, by the data transformation method, we can convert the Semi-Markov decision model into a discrete -time Markov decision model such that both models have the same average cost for each stationary policy. In the discrete -time model it is no restriction to assume that all $\bar{C}_s(a) = \frac{c_s(a)}{\tau_s(a)}$ are positive. Otherwise add a sufficiently large positive constant to each $\bar{C}_s(a)$. Here, $c_s(a)$is the expected cost incurred until the next decision epoch if action a is chosen in the present state s and $\tau_s(a)$ is the expectedtime until the next decision epoch if action a is chosen in the present state s .

4.1 Data Transformation method. First chose a number τ with $0 < \tau \le \min_{(s,a)} \tau_s(a)$. Where s = (q, i, k). Consider now, the discrete time Markov decision model whose basic elements are given by,

$\bar{E} = E \ and \ \bar{A}_s = A_s, s \in \bar{E}$,

$\bar{C}_s(a) = \frac{c_s(a)}{\tau_s(a)}, \ a \in \bar{A}_s \ and \ s \in \bar{E}$,

$$P_{sv} = \begin{cases} \left[\frac{\tau}{\tau_s(a)}\right].p_{sv}(a) & s \ne v, a \in \bar{A}_s, s \in \bar{E} \\ \left[\frac{\tau}{\tau_s(a)}\right]p_{sv}(a) + \left[1 - \left[\frac{\tau}{\tau_s(a)}\right]\right], & s = v, a \in \bar{A}_s, s \in \bar{E} \end{cases}$$

Regarding the choice of τ in the algorithm, it is recommended to take $\tau = \min_{(s,a)} \tau_s(a)$ when the embedded Markov chains $\{X_n\}$ in the Semi Markov model are aperiodic; otherwise $\tau = \frac{1}{2}\min \tau_s(a)$ is a reasonable choice. Let $v(q,i) = \lambda + i\theta + \gamma + p(i)$.Then for action a = 0in state s = (q, i, k)

$$p_{sv}(0) = \begin{cases} \frac{\lambda}{v(q,i)}, & v = (q+1, i) \\ \frac{i\theta}{v(q,i)}, & v = (q, i-1) \\ \frac{\gamma}{v(q,i)}, & v = (q-1, i-1), q \le i \\ 0 & elsewhere. \end{cases},$$

And $\tau_s(0) = \frac{1}{v(q,i)}$.

For action a = 1 in state s = (q, i)

$$p_{sv}(1) = \begin{cases} \frac{\lambda}{v(q,i)}, & v = (q+1, i,) \\ \frac{i\theta}{v(q,i)}, & v = (q, i-1) \\ \frac{\gamma}{v(q,i)}, & v = (q-1, i-1), \\ \frac{p(i)\mu}{v(q,i)}, & v = (q, S), \end{cases},$$

And $\tau_s(1) = \frac{1}{v(q,i)}$.

Finally the one step expected costs $C_2(a)$ are simply given by,

$$c_2(a) = \begin{cases} K + c*(S-1), & s = (q,i) \ and \ a = 1 \\ 0, & otherwise \end{cases}$$

Now, having specified the basic elements of the Semi - Markov decision model, we are in a position to formulate the value iteration algorithm for the computation of a (nearly) optimal acceptance rule. In the data transformation we take,

$$\tau = \frac{1}{\lambda + i\theta + p(i) + \gamma}$$

Using the above specifications, the value iteration scheme becomes quite simple for the allocation problem. Note that the expressions for the one-step transition times $\tau_s(a)$ and the one-step transition probabilities $p_{sv}(a)$ have common denomination and so the ratio $p_{sv}(a)/\tau_s(a)$ has a very simple form. In specifying the value iteration scheme, we distinguish between the auxiliary states(q, i, 0) and the other states.

4.2 Convergence of the bounds

In value iteration for discrete time Markov decision problem, the lower and upper bounds m_n and $M_n$converge to the same limit so that the algorithm will be stopped after finitely many iterations only, if a certain aperiodicity condition is satisfied. The next theorem gives sufficient conditions for the convergence of the Value – Iteration algorithm.

4.3 Theorem: Suppose the weak uni-chain assumption holds and that for each average cost optimal stationary policy the associated Markov chain X_n is aperiodic. Then there are finite constants α > 0 and 0 < β < 1 such that $|M_n - m_n| \le \alpha\beta^n, n \ge 1$. In particular, $\lim_{n\to\infty} M_n = \lim_{n\to\infty} m_n = g^*$. For the proof of the theorem for the special case of Strong aperiodicity assumption.[(i) for each stationary policy R the associated Markov chain $X_n$has no two disjoint closed sets; (ii) $p_{ii}(a) > 0$ for all i **ϵ** I and a **ϵ** A(i)] see Tijms (pp-270, 2003). From the above result, $M_n - m_n \le (1 - \rho)(M_{n-N} - m_{n-N})$ where, $\rho = \frac{\lambda}{c\mu}$. It was shown that

$M_n - m_n$ $n \geq 1$ is non – increasing. Thus we find that $M_n - m_n \leq (1-\rho)^{(n/N)}(M_0 - m_0)$, $n \geq 1$ implying the result.

4.4 Value iteration algorithm. Step:0 choose $V_0(s)$ such that $0 < V_0(s) \leq \min\{c_s(a)/\tau_s(a)\}$ for all S. choose a num. τ wit $0 < \tau \leq \min\{\tau_s(a)\}$.

Step: 1 in the states (q, i, 0) the only possible decision is to leave the system alone. Thus,

$$V_n(q,i,0) = \begin{cases} \tau\lambda V_{n-1}(q+1,i) + \tau i\theta V_{n-1}(q,i-1) & \\ +\tau\mu V_{n-1}(q-1,i-1) & for\ q \leq i \\ \tau\lambda V_{n-1}(q+1,i) + \tau i\theta V_{n-1}(q,i-1) & \\ +\tau\mu V_{n-1}(q,S) & for\ q > i \end{cases}$$

Where V_{n-1} (q, i, 1) = 0 and V_{n-1} (q, i, 0) = 0 when q ≤ 0 or i ≤ 0. Then the action a = 1for the states (q, i, 1).

$$V_n(q,i,1) = \min\begin{cases} V(q,i) + \tau\lambda V_{n-1}(q,i) + \tau i\theta V_{n-1}(q,i-1) \\ +\tau\mu V_{n-1}(q-1,i-1) + \{1-\tau V(q,i)\}.V_{n-1}(q,i) \\ \tau\lambda V_{n-1}(q+1,i) + \tau i\theta V_{n-1}(q+1,i-1) \\ +\tau\mu V_{n-1}(q,i-1) + \{1-\tau V(q+1,i)\}.V_{n-1}(q,i) \end{cases}$$

for (q+1) ≤ i.

$$V_n(q,i,1) = \min\begin{cases} V(q,i) + \tau\lambda V_{n-1}(q,i) + \tau i\theta V_{n-1}(q,i-1) \\ +\tau\mu V_{n-1}(q-1,i-1) + \{1-\tau V(q,i)\}.V_{n-1}(q,i) \\ \tau\lambda V_{n-1}(q+1,i) + \tau i\theta V_{n-1}(q+1,i-1) \\ +\tau\mu V_{n-1}(q,S) + \{1-\tau V(q+1,i)\}.V_{n-1}(q,i) \end{cases}$$

for (q+1) > i.

Step: 2 Compute the bounds when S = (q, i, 1),

$m_n = \min\{V_n(S) - V_{n-1}(S)\}$,

$M_n = \max\{V_n(S) - V_{n-1}(S)\}$

The algorithm is stopped when $M_n - m_n \leq (1-\rho)(M_{n-N} - m_{n-N})$, here N = 1.

Numerical example: Consider the following numerical problem with given data set:S = 3, s = 2, N = 4, λ = 2, μ = 3, γ = 4, h = 0.1, $c_j = 3j$; j = 0, 1, 2, g = 5, β(μ) = 2μ

Action(a)\prob. value	p_2	p_1	p_0
0	0.5	0.2	0
1	0.5	0.8	0
2	0.0	0.0	1

We have applied the standard value-iteration algorithm to the numerical data given above.

For each stationary policy the associated Markov chain $\{(X_n, I_n)\}$ is aperiodic.Taking $V_0(i) = 0$ for all I and the accuracy number (tolerance factor) ε $=10^{-4}$, the algorithm is stopped after 26 steps with the stationary policy R(n)=(0,0,1,2), together with the lower and upper bounds $m_n = 0.4336$ and $M_n = 0.4340$. The average cost is given by $\frac{1}{2}(m_n + M_n) = 0.4338$.

V. CONCLUSIONS AND FUTURE RESEARCH:

Analysis of perishable inventory control at service facility is fairly recent system study. Most of the previous work determined optimal ordering policies or system performance measures. We approached the problem in a different way, given a service rate we determine the optimal ordering policy to be employed to minimize the long – run expected cost rate. Thus the optimal inventory control in a perishable environment in the service facility is established. In future we may extend this model to perishable inventory system with discrete time MDP.

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