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Influence of thermal radiation on MHD boundary layer flow in a Newtonian liquid with temperature dependent properties over an exponential stretching sheet

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***Abstract*— The paper presents a study of a forced flow and heat transfer of an electrically conducting Newtonian fluid in the presence of a magnetic field due to an exponentially stretching sheet. Thermal radiation term is incorporated in the temperature equation. The governing coupled, non-linear, partial differential equations are converted into coupled, non-linear, ordinary differential equations by a similarity transformation and are solved numerically using shooting method. The influence of various parameters such as the Prandtl number, Chandrasekhar number, variable viscosity parameter, heat source (sink) parameter, radiation parameter and suction/injection on velocity and temperature profiles are presented and discussed.**

***Key words:* Stretching sheet, Variable viscosity, Shooting Method, Heat source.**

INTRODUCTION

Flows due to a continuously moving surface are encountered in several important engineering applications .viz, in the polymer processing unit of a chemical engineering plant, annealing of copper wires, glass fiber and drawing of plastic films. Sakiadis [1-3] initiated the theoretical study of these applications by considering the boundary layer flow over a continuous solid surface moving with constant speed. This problem was extended by Erickson et al. [4] to the case where the transverse velocity at the moving surface is non-zero with heat and mass transfer in the boundary layer accounted for. Crane [5] studied the steady two-dimensional boundary layer flow caused by the stretching sheet, which moves in its own plane with a velocity which varies linearly with the axial distance. There after various aspects of the above boundary layer problem on continuous moving surface were considered by many researchers (Vleggar [6], Gupta and Gupta [7], Grubka and Bobba [8], Chen and Char [9] and Siddheshwar et al. [10]).

Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. During this process of drawing the strips are sometimes stretched. The properties of final product depend on the rate of cooling. Pavlov [11] examined the flow of an electrically conducting fluid caused solely by the stretching of an elastic sheet in the presence of a uniform magnetic field. Chakrabarthi and Gupta [12] considered the flow and heat transfer of an electrically conducting fluid past a porous stretching sheet. Anderson [13] presented an analytical solution of the magnetohydrodynamic flow using a similarity transformation for the velocity and temperature fields. In all the above mentioned studies the physical properties of the ambient fluid were assumed to be constants. However, it is well known that these physical properties of the ambient fluid may change with temperature (Herwig and Wickern [14], Takhar et al. [15], Pop et al. [16], Subhash Abel et al. [17], Pantokratoras [18], Ali [19], Andersson and Aaresth [20], Prasad et al. [21], Sekhar and Chethan [22]).

Magyari and Keller [23] studied the heat and mass transfer on the boundary layer flow due to an exponentially stretching surface. Elbashbeshy [24] added new dimension to the study on exponentially stretching surface. Partha et al. [25] have examined the mixed convection flow and heat transfer from an exponentially stretching vertical surface in quiescent liquid using a similarity solution. Heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet were investigated by Khan and Sanjayanand [26-27]. Sajid and Hayat [28] considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. Sekhar and Chethan [29] analyzed the flow and heat transfer due to an exponentially stretching continuous surface in the presence of Boussinesq-Stokes suspension. Later various investigations were made on the stretching sheet problems in different directions ([30]-[39]).

Suction/injection of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction whereas injection acts in the opposite manner. The process of suction/injection has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag.

The radiative effects have important applications in physics and engineering. The radiation heat transfer effects on different flows are very important in space technology and high temperature processes. Thermal radiation effects play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent.

In the present work, we study the MHD boundary layer flow and heat transfer characteristics of a Newtonian fluid past an exponentially stretching sheet, when viscosity is a function of temperature and in the presence of thermal radiation.

MATHEMATICAL FORMULATION

We consider a steady, two-dimensional boundary layer flow of an incompressible, weakly electrically conducting Newtonian fluid due to a stretching sheet. The liquid is at rest and the motion is affected by pulling the sheet at both ends with equal force parallel to the sheet and with speed u, which varies exponentially with the distance x from the origin.

The boundary layer equations governing the flow and heat transfer in a Newtonian fluid over a stretching sheet, assuming that the viscous dissipation is negligible, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left\{\frac{\mu(t)}{\rho}\frac{\partial u}{\partial y}\right\}-\frac{{\mu_m}^2\sigma {H_0}^2}{\rho}u, \qquad (1.2)$$

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 t}{\partial y^2}+Q_s\left(t-t_\infty\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}, \qquad (1.3)$$

Here u and v are the components of the liquid velocity in the x and y directions, respectively, t is the temperature of the sheet, t_∞ is the temperature of the fluid far away from the sheet, μ is the dynamic viscosity, µm is the magnetic permeability, H_0 is the applied magnetic field, ρ is the density, σ is the electric conductivity of the fluid, k is the thermal conductivity, C_p is the specific heat at constant pressure Q_s is the heat source coefficient and q_r is the radiative heat flux.

The coefficient of viscosity is assumed to be a reciprocal function of temperature and it is of the form

$$\mu(t)=\frac{\mu_\infty}{1+\delta(t-t_\infty)}$$

If $\frac{1}{\mu}$ is expanded in Taylor's series about $t=t_\infty$ then the scalar appearing in the above expression can be written as $\delta=\left[\frac{\partial}{\partial t}\left(\frac{1}{\mu}\right)\right]_{t=t_\infty}$.

Here μ_∞ is the coefficient of viscosity far away from the sheet.

Using Rosseland approximation for radiation we can write

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial t^4}{\partial y}$$

The following boundary conditions are used.

$$u=U_w(x)=U_0e^{\frac{2x}{L}},\ v=v_c,\ \begin{cases} t=t_w=t_\infty+Ae^{\frac{x}{L}} & in\ PEST\ case \\ -k\left(\frac{\partial t}{\partial y}\right)_w=De^{\frac{3x}{2L}} & in\ PEHF\ case\end{cases}\ at\ \ y=0,$$

$$u\to 0, t\to t_\infty \quad as \quad y\to\infty. \qquad (1.4)$$

$$t_w-t_\infty=\begin{cases} Ae^{\frac{x}{L}} & in\ PEST\ case \\ \frac{DL}{k\sqrt{Re}}e^{\frac{x}{2L}} & in\ PEHF\ case\end{cases}$$

where t_w is the temperature of the sheet, U_0 is the reference velocity and L is the reference length.

We now make the equations and boundary conditions dimensionless using the following definition:

$$(X,Y)=\frac{\left(x,y\sqrt{Re}\right)}{L},\ (U,V,V_c)=\frac{\left(u,v\sqrt{Re},v_c\sqrt{Re}\right)}{U_0},\ T=\frac{t-t_\infty}{\Delta t}$$

(1.5)

where $Re=\frac{U_0 L}{\nu}$ is the Reynolds number and $\Delta t=t_w-t_\infty$ is the sheet-liquid temperature difference.

The boundary layer equations (1.1) - (1.3) on using Eq. (1.5) take the following form.

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0,$$

(1.6)

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\mathrm{V}}{(1+\mathrm{V}T)^2}\frac{\partial T}{\partial Y}\frac{\partial U}{\partial Y}+\frac{1}{(1+\mathrm{V}T)}\frac{\partial^2 U}{\partial Y^2}-QU,$$

(1.7)

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}+UT=\frac{1}{Pr}\left(1+\frac{4}{3}R\right)\frac{\partial^2 T}{\partial Y^2}+H_s T,$$

(1.8)

where

$\mathrm{V}=\delta\,\Delta t$ is the variable viscosity parameter,

$Q=\frac{\mu_m{}^2\sigma H_0{}^2}{\alpha}$ is the Chandrasekhar number,

$R=\frac{4\sigma^* T_\infty{}^3}{k^* K}$ is the radiation parameter,

$Pr=\frac{\mu C_p}{k}$ is the Prandtl number and

$H_s=\frac{Q_s}{\alpha}$ is the heat source (sink) parameter.

The boundary conditions (1.4) take the form

$$U=e^X,\ V=V_c,\ \begin{Bmatrix} T=1 & in\ \ PEST \\ \frac{\partial T}{\partial Y}=-e^X & in\ \ PEHF \end{Bmatrix}\ \ at\ \ Y=0,$$

$$U\to 0,\ T\to 0\ \ as\ \ Y\to\infty.$$

(1.9)

We introduce the stream function $\psi(X,Y)$ as

$$U=\frac{\partial\psi}{\partial Y},\quad V=-\frac{\partial\psi}{\partial X},$$

(1.10)

Using Eq. (1.10), the boundary layer equations Eqs. (1.7) and (1.8) can be written as

$$(1+\mathrm{V}T)\frac{\partial^3\psi}{\partial Y^3}-\mathrm{V}\frac{\partial T}{\partial Y}\frac{\partial^2\psi}{\partial Y^2}+(1+\mathrm{V}T)^2\frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2}-(1+\mathrm{V}T)^2\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial X\partial Y}$$
$$-(1+\mathrm{V}T)^2 Q\frac{\partial\psi}{\partial Y}=0,$$

(1.11)

$$\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial T}{\partial Y}+UT=\frac{1}{Pr}\left(1+\frac{4}{3}R\right)\frac{\partial^2 T}{\partial Y^2}+H_s T.$$

(1.12)

The corresponding boundary conditions in terms of the stream function can be written as

$$\frac{\partial\psi}{\partial Y}=e^X,\ \frac{\partial\psi}{\partial X}=-V_c,\ \begin{Bmatrix} T=1 & in\ \ PEST \\ \frac{\partial T}{\partial Y}=-e^X & in\ \ PEHF \end{Bmatrix}\ \ at\ \ Y=0,$$

$$\frac{\partial\psi}{\partial Y}\to 0,\ T\to 0\ \ as\ \ Y\to\infty.$$

(1.13)

The following similarity transformation will now be used on Eqs. (2. 11) and (2.12).

$$\psi(x,\eta)=f(\eta)e^X,$$
$$T(X,Y)=\begin{Bmatrix}\theta(\eta)\ \ in\ \ PEST\ \ case\\ \phi(\eta)\ \ in\ \ PEHF\ \ case\end{Bmatrix},$$
$$\eta=Y\ e^X.$$

(1.14)

Using the transformations given by Eq. (1.14) in Eq. (1.11) and (1.12), we get the following boundary value problems.

(i) PEST:

$$(1+\mathrm{V}\theta)f'''-\mathrm{V}\theta' f''+(1+\mathrm{V}\theta)^2\left(f f''-2f'^2-Q_x f'\right)=0,$$

(1.15)

$$\left(1+\frac{4}{3}R\right)\theta''+Pr\left(f\theta'-f'\theta\right)+Pr\,H_{sx}\,\theta=0,$$

(1.16)

$$f(0)=-V_{cx},\ \ f'(0)=1,\ \ \theta(0)=1,$$
$$f'(\infty)\to 0,\ \ \theta(\infty)\ \to 0.$$

(1.17)

(ii) PEHF:

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$$(1+ \mathrm{V}\Phi) f''' - \mathrm{V}\Phi' f'' + (1+ \mathrm{V}\Phi)^2 \left(f f'' - 2 f'^2 - Q_x f' \right) = 0,$$
(1.18)

$$\left(1+\frac{4}{3}R\right)\phi'' + Pr\left(f \phi' - f'\phi \right) + Pr\, H_{sx}\, \phi = 0,$$
(1.19)

$$f(0) = -V_{cx},\ f'(0) = 1,\ \Phi'(0) = -1,$$
$$f'(\infty) \to 0,\ \Phi(\infty) \to 0.$$
(1.20)

where $V_{cx} = \frac{V_c}{e^X}$ is the local suction/injection parameter, $Q_x = \frac{Q}{e^{2X}}$ is the local Chandrasekhar number and $H_{sx} = \frac{Q_s L}{U_0 e^{2X}}$ is the local heat source (sink) parameter.

Here, primes denote the differentiation with respect to η.

METHOD OF SOLUTION

The boundary value problems due to an exponential stretching sheet are solved numerically by shooting method. We adopt the shooting method with Runge-Kutta-Fehlberg-45 scheme to solve the boundary value problems in PEST and PEHF cases mentioned in the previous section. The coupled non-linear Eqs. (1.15) and (1.16) in the PEST case are transformed to a system of five first order differential equations as follows:

$$\frac{df_0}{dY} = f_1,$$
$$\frac{df_1}{dY} = f_2,$$
$$\frac{df_2}{dY} = \frac{\mathrm{V}\theta_1}{(1+\mathrm{V}\theta_0)} f_2 + (1+\mathrm{V}\theta_0)\left(-f_0 f_2 + 2 f_1^2 + Q_x f_1\right),$$
$$\frac{d\theta_0}{dY} = \theta_1,$$
$$\frac{d\theta_1}{dY} = \frac{Pr\, f_1 \theta_0 - Pr\, f_0 \theta_1 - Pr\, H_{sx}\, \theta_0}{\left(1+\frac{4}{3}R\right)}.$$
(1.21)

Subsequently the boundary conditions in Eq. (1.17) take the form

$$f_0(0) = -V_{cx},\ f_1(0) = 1,\ f_1(\infty) \to 0,$$
$$\theta_0(0) = 1,\ \theta_0(\infty) \to 0.$$

(1.22)

Here $f_0 = f(\eta)$ and $\theta_0 = \theta(\eta)$.

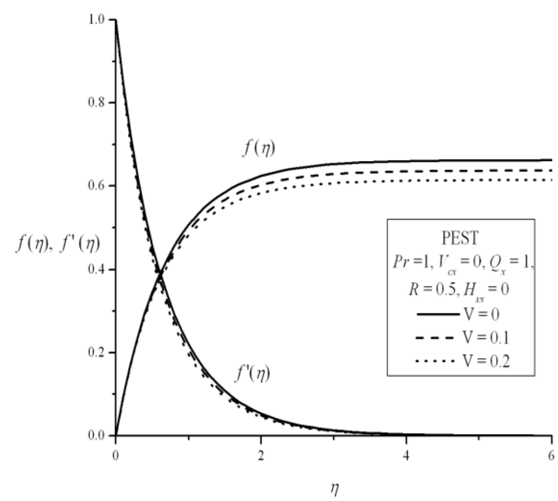
Aforementioned boundary value problem is converted into an initial value problem by choosing the values of $f_2(0)$ and $\theta_1(0)$ appropriately. Resulting initial value problem is integrated using the fourth order Runge-Kutta method. Newton-Raphson method is implemented to correct the guess values of $f_2(0)$ and $\theta_1(0)$. In solving equations (1.21) subjected to boundary conditions (1.22) the appropriate '∞' is determined through the actual computation. Same procedure is adopted to solve the boundary layer equations in PEHF case.

RESULTS AND DISCUSSION

The hydromagnetic boundary layer flow and heat transfer in a weakly electrically conducting Newtonian fluid past an exponentially stretching sheet with temperature dependent viscosity and thermal radiation are investigated. Numerical solution of the problem is obtained by shooting method.

Figures 1 - 3 are the plots of horizontal and transverse velocities for various values of variable viscosity parameter (V), suction/injection parameter (V_{cx}) and Chandrasekhar number (Q_x).

The effect of variable viscosity parameter V on the velocity profiles $f(\eta)$ and $f'(\eta)$ with η is depicted in Fig. 1. It is observed the both the horizontal and transverse velocity profiles decreases with increasing values of V.



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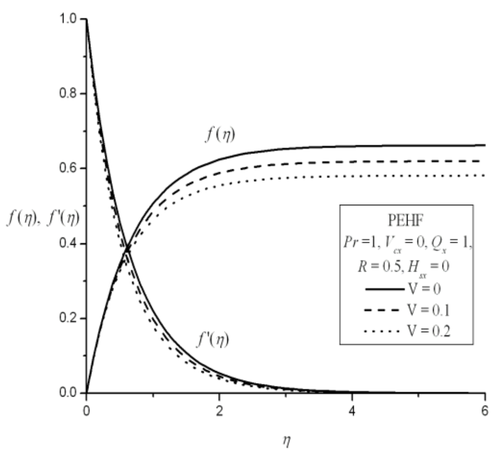
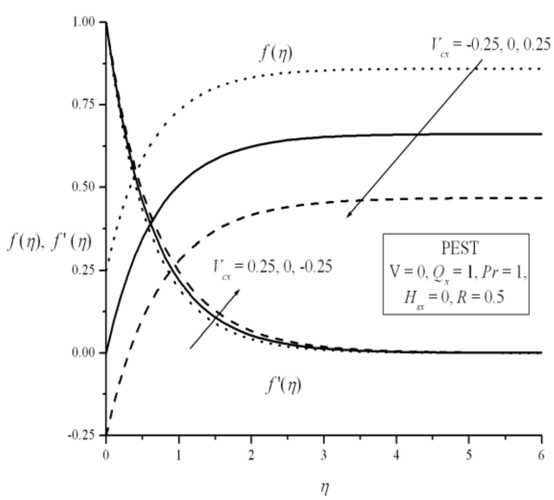


(b)

Figure 1: Plot of velocity for different values of variable viscosity parameter in PEST and PEHF cases.

Figure 2 presents the effects of suction or injection on the horizontal and transverse velocity. With the increasing values of V_{cx} , the horizontal velocity increases. i. e. suction $(V_{cx} < 0)$ causes to decrease the velocity of the fluid in the boundary layer region. This is because in case of suction, the heated fluid is pushed towards the wall where the buoyancy forces act to retard the fluid due to high influence of the viscosity. The same principle operates but in opposite direction in case of injection $(V_{cx} > 0)$.



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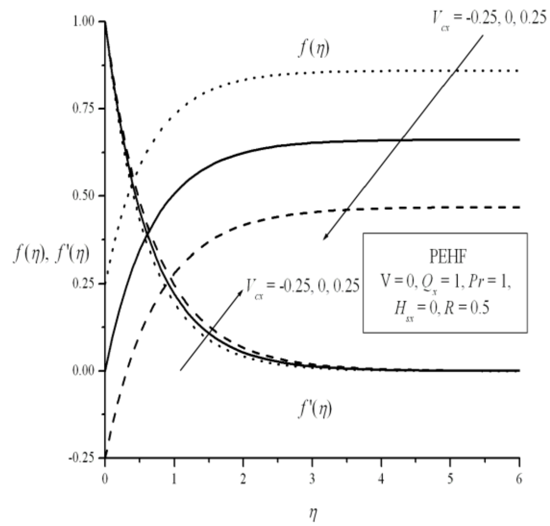

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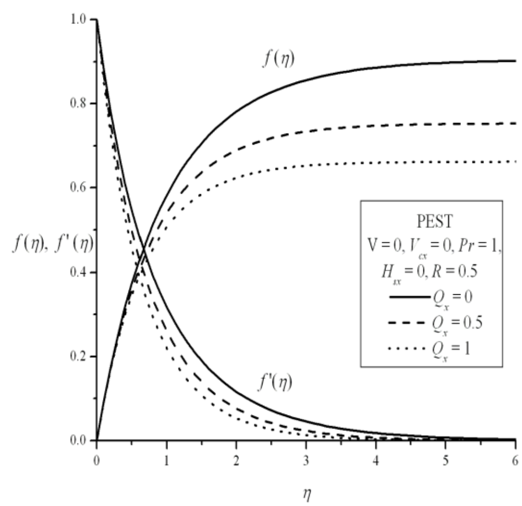
Figure 2: Plot of velocity for different values of suction/injection parameter in PEST and PEHF cases.

Figure 3 show the effect of Chandrasekhar number Q_x on the velocity profiles above the sheet. An increase in Q_x is seen to decrease both velocity components at any point above the sheet. This is because of the retarding effects of the Lorentz force set forth by the magnetic field.



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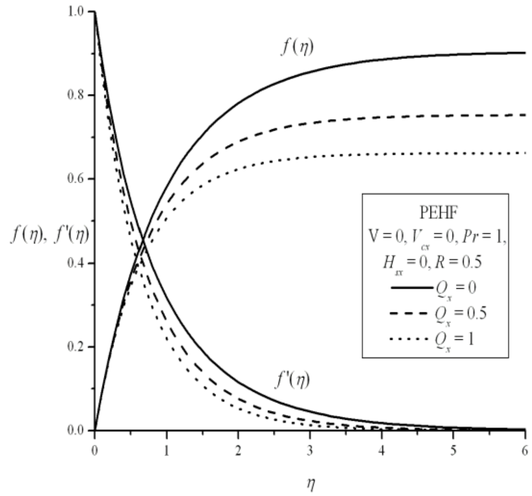

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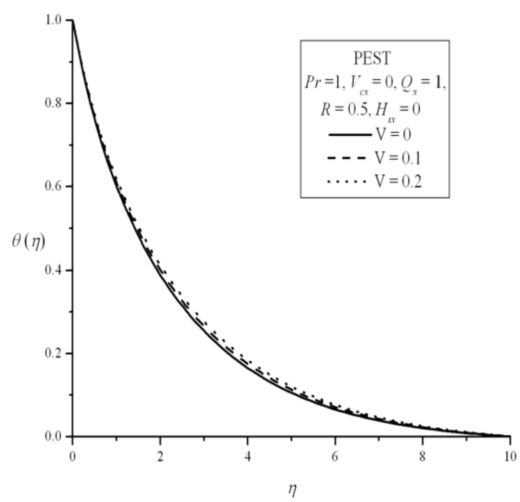
Figure 3: Plot of velocity for different values of Chandrasekhar number (Q_x) in PEST and PEHF cases.

Figures 4 and 5 demonstrate the effect of variable viscosity parameter V and suction/injection parameter V_{cx} on the temperature distribution. The effect of V and injection $(V_{cx} > 0)$ is to increase the thermal boundary layer thickness whereas suction $(V_{cx} < 0)$ reduces it.



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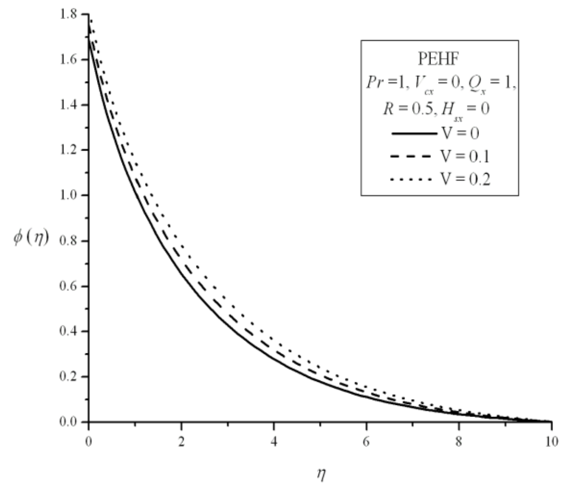

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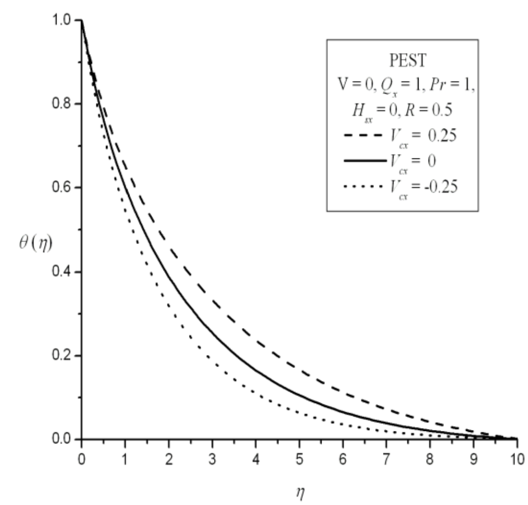
Figure 4: Plot of temperature for different values of variable viscosity parameter (V) in PEST and PEHF cases.



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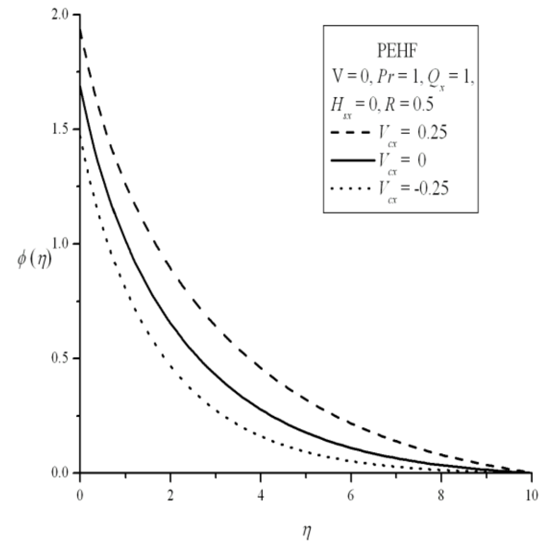

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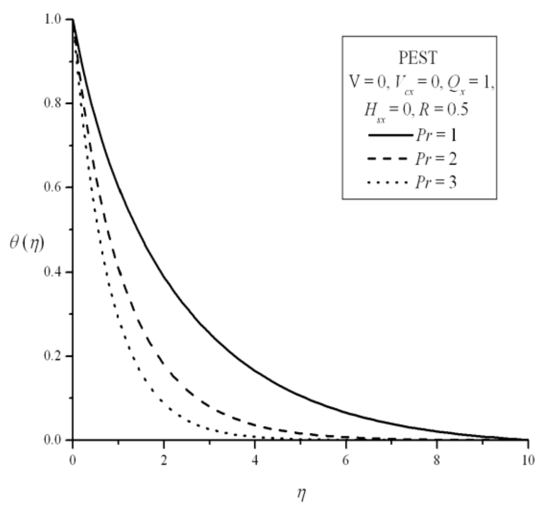
Figure 5: Plot of temperature for different values of suction/injection parameter in PEST and PEHF cases.



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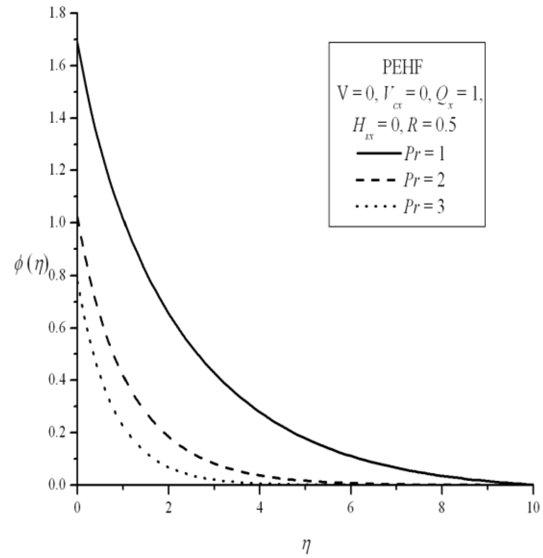

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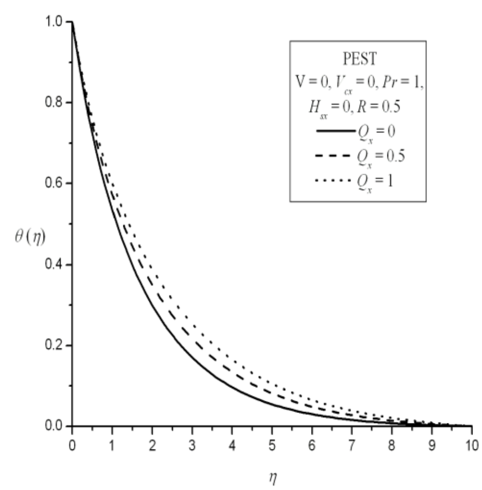
Figure 6: Plot of velocity for different values of Prandtl number in PEST and PEHF cases.

The effect of Prandtl number *Pr* on the temperature field is shown in Fig. 6. It is noticed that the temperature decreases with the increasing value of Prandtl number because thermal boundary layer decreases due to increase in *Pr*.



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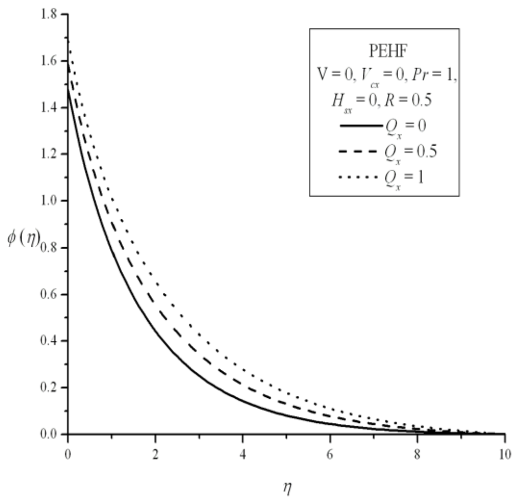

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Figure 7: Plot of velocity for different values of Chandrasekhar number in PEST and PEHF cases.

The effect of Chandrasekhar number Q_x on temperature profiles are shown in Fig. 7. It is noticed that the effect of Q_x is to increase the temperature in the boundary layer. This is because of the fact that the introduction of transverse magnetic field to an electrically conducting fluid gives rise to a resistive type of force known as Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer and to increase the temperature profile. Also, the effect of increasing values of Prandtl number is decrease the temperature distribution in the flow region.

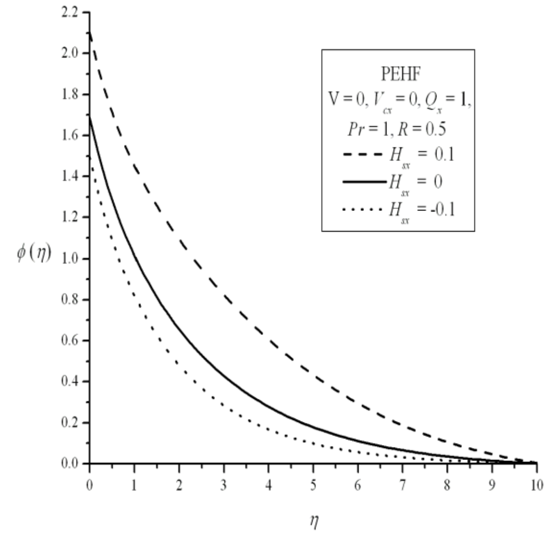

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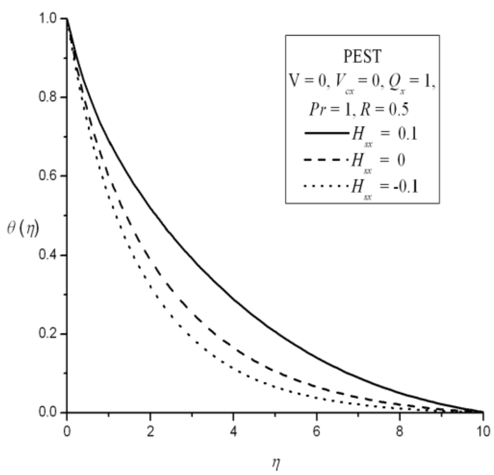
Figure 8: Plot of velocity for different values of heat source/sink parameter in PEST and PEHF cases.

It is observed that the effect of heat source $(H_{sx} > 0)$ in the boundary layer generates energy which causes the temperature to increase, while the presence of heat sink $(H_{sx} < 0)$ in the boundary layer absorbs the energy which causes the temperature to decrease. These behaviors are seen in Fig. 8.



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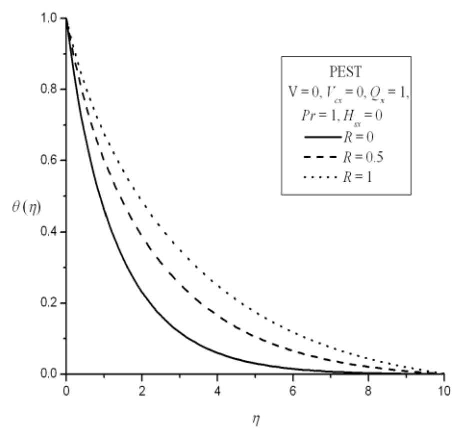

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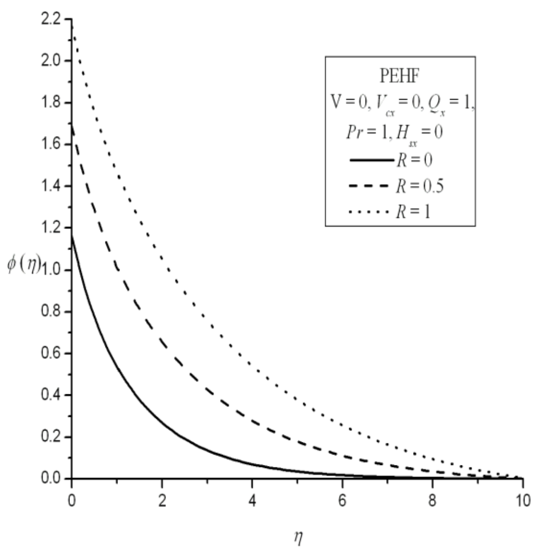

(b)

Figure 9 shows the variation of temperature with η for various values of radiation parameter R. It is clear that thermal radiation R enhances the temperature in the thermal boundary layer region. The increase in radiation parameter means of release of heat energy from the flow region and so the fluid temperature decreases as the thermal boundary layer thickness becomes thinner.

In order to validate our results, we have compared the skin friction $-f''(0)$ rate of heat transfer $-\theta'(0)$ in the absence of variable viscosity $(\mathrm{V}=0)$, suction/injection (V_{cx}) Chandrasekhar number $(Q_x=0)$ and heat source/sink parameter $(H_{sx}=0)$ with the published results and found them to be in good agreement (see Table 1.1 and 1.2).

Table 1.1: Comparison of values of skin friction $-f''(0)$ for various values of V_{cx} with V = Q_x = H_{sx} =R = 0 in case of exponential stretching.

	$-f''(0)$	
V_{cx}	Elbashbeshy (2001)	Present study
0 -0.2 -0.4 -0.6	1.28181 1.37889 1.4839 1.59824	1.281816 1.378894 1.484389 1.598242

Table 1.2: Comparison of values $-\theta'(0)$ for various values of Prandtl number and radiation parameter with V = V_{cx}= Q_x = H_{sx} =R = 0 in case of exponential stretching.

		$-\theta'(0)$				
r	R	Magyari and Keller (2001)	Bidin and Nazar 2009	Ishak 2011	Mukho-padhyay 2013	Present study
	0	0.9548	0.9547	0.9548	0.9547	0.9548
			1.4714	1.4715	1.4714	1.4715
	0.5		1.0735		1.0734	1.0735
	1		0.8627		0.8626	0.8627
	0.5		1.3807		1.3807	1.3807
	1		1.1214		1.1214	1.1214

IV. CONCLUSIONS

1. Increasing values of variable viscosity parameter V reduces the velocity.
2. The effect of variable viscosity parameter V is to increase the temperature in the boundary layer.
3. The temperature in the boundary layer decreases (increases) due to suction (injection).
4. The effect of Prandtl number Pr is to decrease the thermal boundary layer thickness.
5. The heat source parameter H_{sx} increases the heat transfer in both PEST and PEHF cases and the opposite is observed in the case of a sink.
6. Thermal boundary layer thickness increases with the increase in the radiation parameter.

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