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# Modelling and analysis of Geom/GI/1/K Queue With Finite Number of Vacations

[1] K . Sikdar , [2] K . Anitha
Department of Mathematics
BMS Institute of Technology & Management (Affiliated to VTU, Belgaum-18) Post Box No. 6443, Doddaballapura Main Road Yelahanka, Bangalore-560064, India

*Abstract:-* **Discrete-time queueing models have gained prominence in recent years due to its wide applications. These have received much interest due to the emerging broadband integrated services digital network(B-ISDN) which provides the transfer of messages in the form of video, voice and data through high speed local area networks (LAN). The asynchronous transfer mode (ATM) is adopted as the network transport technique in the implementation of B-ISDN. In this paper, we deliberate a finite buffer discrete-time Geo/G/1 queue where the server takes finite number of at most L (≥0) vacations. The server takes vacations whenever the system is empty. After the vacation, the server checks out the system whether to resume the service or to go for another vacation or to be dormant. In this system, jobs arrive according to Bernoulli process and service, vacation times are arbitrarily distributed. We adopt the supplementary variable method and the imbedded Markov chain techniques to attain the queue length distributions at the service completion, vacation termination and arbitrary epochs. The analysis of actual waiting time under the First-Come-First-Served (FCFS) queueing discipline is also carried out. The objective of this paper is to create awareness and better utilization of the queueing theories involved in the analysis of a discrete-time model with at most L vacations**

## I. INTRODUCTION

The asynchronous transfer mode (ATM) is adopted as the network transport technique in the implementation of B-ISDN. It transmits the messages in small, fixed length packets called cells. There are two types of polices such as arrival-first (AF) or departure- first (DF) management policies. AF and DF policies can also be termed as late arrival system with delayed access (LAS-DA) and early arrival system (EAS) respectively, see Gravey and Hébuterne [6], Hunter [7], Bruneel and Kim [1].

There has been much emphasis on finite buffer discrete-time queues as they are more pragmatic in real life situations. For example, in telecommunication networks,messages/packets are stored in the system if a server is busy. In such situations, one of the main concerns of the system designer is the estimation of blocking probability which, in general, is kept small to avoid loss of packets. One could find a lot of finite buffer discrete-time queueing models due to its relevance, see Chaudhry [3], Chaudhry and Gupta [4], Goswami and Sikdar [5], Sikdar and Samanta [13]. Takagi [14] has given a detailed analysis of such models. In the past few years, researchers are analysing discrete-time queues with vacations. Such models have vast applications in communications, computer network, production and stochastic systems. After the completion of service to the jobs who were in the queue, the server can stay in the system which is void now and wait for the next arrival. But there are cases when the server can also vacate for a length of time if he finds the queue empty. This phenomenon is termed as vacation. After coming back from vacation, if the server finds one or more jobs in the queue, he will serve them one by one until the queue is empty and after which he will depart for the next vacation. But on the other hand if the server finds no job in the queue after coming back, then for a single vacation he will remain dormant till at least one job joins the queue. However for multiple vacations, the server takes another vacation and stay on like this until he finds one job waiting upon return from vacation. In the past, several researchers have analyzed various discrete-time queues with single/multiple vacation policies, Chang and Choi [2], Samanta [9], Samanta et al [10, 11], Tian and Zhang [16], Tang et al [15], Wang [18], Zhang and Tian [19]. Takagi [14] has given a detailed analysis of such models. Recently, a study has been done on the discrete-time Geo/G/1 queue with randomized vacations and at mostJ vacations by Wang [17]. Such a model has potential application in practical systems. This encouraged usto study further a discrete-time Geo/G/1/K queueing system where the server deactivates and takes at mostL (≥ 0) vacations whenever the system is void. This article is different from the previous paper as we analysea queue of size K. After the vacation, the server checks out the system whether to resume the service or togo for another vacation or be dormant. This kind of discretetime system is very relevant in communicationnetwork, production management and so on. In a set up with potential clients or internal communication inoffices, email has to be passed on across the network using office automation system. Most of the communicationgrids are coordinated with full synchronization. All the data transmissions, requests and receptions advance in fixed time intervals and therefore the sending of emails, pre-processing and processing of requests will also followin a discretetime way in this system. When the jobs send mails to the server it will process the mails only

 

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ifit is dormant. Otherwise the emails thus acquired will be in a queue. The time taken to process an email willbe random and it is assumed to follow a general distribution time. Also the server requires maintenance for itssmooth functioning. This can be done when the server is idle. When one maintenance is done, the server cango for the next one if there are no emails or start processing the emails again or remain dormant. One may alsonote that this paper is an extension of the analysis done by Samanta et al [12]. We have focused on the study of finite number of L(≥ 0) vacations. The server could anticipate the number of vacations in advance and hence during vacation, the server could render other jobs. If L=0, the server will not be having any vacation at all and if L = 1, the serverwill take only a single vacation whereas the server takes multiple vacations when L > 1.

The paper is structured as follows. In the next section, we formulated the system and introduce the notations to describe the model parameters. In Section 2, the analytic analysis of the model is presented. We also provided waiting time distribution and some important performance measures in Section 3 and 4, respectively.

## II. MODEL FORMULATION AND NOTATIONS

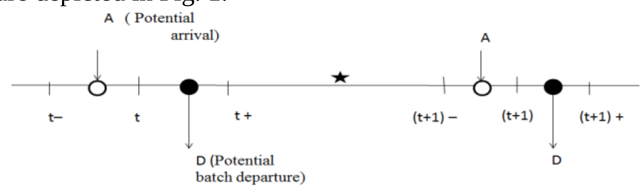
Let us consider a finite-buffer Geom/GI/1/K queueing system where K is the capacity of the system including the one who is in service. It is assumedthat the system has a clock such that time is slotted in intervals of equal length, separated by slot boundaries. Let $L \geq 0$ be a specified integer. The server may take a sequenceof finite number (at most L) of vacations in his idle period. Whenever the systembecomes empty at service completion epoch, the server leaves for a vacation. If no jobs are found in the queue when the server returns from the vacation, he again leaves for another vacation. This pattern continues until server completedLth vacations. If no jobs are found by the end of the Lth vacations, the server stays in the dormant period until one job arrives.We assumethat jobs arrive at the system according to a Bernoulli process with parameter $\lambda$. Theservice [vacation] times $S\,[V]$ are independently identically distributed random variables (i.i.d.r.vs) with probability mass function $s_n = P(S = n), n \geq 1\; [v_n = P(V = n), n \geq 1]$, corresponding probability generating function (p.g.f.) $S(z)\;[V(z)]$ and mean service [vacation] time is

$E(S)\;[E(V)]$. Let $\rho'$be defined as the carried load, i.e. the probability that the server is busy at an arbitrary time, where the offered load $\rho$ is defined as usual to be $\rho = \lambda E(S)$. Here we discuss the models for late arrival system with delayed access (LAS-DA).

## III. DESCRIPTION OF THE Geom/G/1/K QUEUE WITH L NUMBER OF VACATIONS

### 3.1 The LAS-DA system

Let us assume that the time axis is slotted into intervals of equal length with the length of a slot being unity. Further, let the time axis be marked by 0,1, 2, …, t,… and assume that the potential arrivals and departures occur in the time interval (t-,t) and (t,t+), respectively. For the sake of understanding, various time epochs at which events (arrival/departure) occur are depicted in Fig. 1.




O:Potential arrival epoch
: Potential departure epoch
:Outside observer's epoch
$(t+;\ (t\ +\ 1)-)$: Outside observer's interval
$t\,+$: Epoch after a potential departure
$t-$: Epoch prior to a potential arrival

Fig. 1. Various time epochs in late arrival system with delayed access (LAS-DA).

The state of the system just before a potential arrival (at $t-$) is described by the following random variables: the number of jobs in the queue ($N_{t-}$); the remaining service time of the job in service ($U_{t-}$); the remaining vacation time of the server ($V_{t-}$) and the state of the server ( $\xi_{t-}$ ) as $\xi_{t-} = [-1]\{l\}(-2)$ the server is [busy] {on$l$th $(1 \leq l \leq L)$ vacation} (in dormancy). Let us define the joint probabilities as

$$\alpha_n(u,t-) = P(N_{t-} = n,\ U_{t-} = u, \xi_{t-} = -1),\quad 0 \leq n \leq K, u \geq 0,$$

$$\beta_{n,l}(u,t-) = P(N_{t-} = n,\ U_{t-} = u, \xi_{t-} = l),\quad 0 \leq n \leq K, 1 \leq l \leq L, u \geq 0,$$

$$\gamma_0(t-) = P(N_{t-} = 0, \xi_{t-} = -2)\,.$$

Define the generating functions
$\alpha_n^*(z) = \sum_{u=0}^{\infty} \alpha_n(u) z^u$ and
$\beta_{n,l}^*(z) = \sum_{u=0}^{\infty} \beta_{n,l}(u) z^u,\ |z| \leq 1, 1 \leq l \leq L,\ 0 \leq n \leq K$.One may note that $\alpha_n = \alpha_n^*(1)[\,\beta_{n,l} = \beta_{n,l}^*(1)]$ denotes the probability of $n$ jobs in the queue, when the server is busy [on $l^{th}$ vacation] at arbitrary epoch. The normalization condition is$\sum_{n=0}^{K}\left(\alpha_n + \sum_{l=1}^{L} \beta_{n,l}\right) + \gamma_0 = 1$.

Let $f_j$ and $h_j$ denote the probability that j jobs enter into the system during a service time $S$ of a job and avacation time $V$

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respectively. Hence for $j \geq 0$, we have $f_j = \sum_{k=1}^{\infty} s_k \binom{k}{j} \lambda^j (1-\lambda)^{k-j}$,

$h_j = \sum_{k=1}^{\infty} v_k \binom{k}{j} \lambda^j (1-\lambda)^{k-j}$, and

$f_j = h_j = 0$, for $j > k$ and let

$\hat{f}_i = \sum_{n=i}^{\infty} f_n$, $i \geq 1$, $\hat{h}_K = \sum_{n=K}^{\infty} h_n$.

Now observing the state of the system at two consecutive epochs $t-$ and $(t+1)-$, in steady state we have the following equations for $u \geq 1$.

$\alpha_0(u-1) = (1-\lambda)\alpha_0(u) + (1-\lambda)\alpha_1(0)s_u + \lambda\alpha_0(0)s_u + (1-\lambda)\sum_{l=1}^{L}\beta_{1,l}(0)s_u + \lambda\sum_{l=1}^{L}\beta_{0,l}(0)s_u + \lambda\gamma_0 s_u$, …….(1)

$\alpha_n(u-1) = (1-\lambda)\alpha_n(u) + \lambda\alpha_{n-1}(u) + (1-$ λαn+10su+λαn0su + 1− λl=1Lβn+1,l0su+ $\lambda\sum_{l=1}^{L}\beta_{n,l}(0)s_u$, $1 \leq n \leq K-2$, …….(2)

$\alpha_{K-1}(u-1) = (1-\lambda)\alpha_{K-1}(u) + \lambda\alpha_{K-2}(u) + \lambda\alpha_{K-1}(0)s_u + \alpha_K(0)s_u + \lambda\sum_{l=1}^{L}\beta_{K-1,l}(0)s_u + \sum_{l=1}^{L}\beta_{K,l}(0)s_u$, …….(3)

$\alpha_K(u-1) = \alpha_K(u) + \lambda\alpha_{K-1}(u)$,…….(4)

$\beta_{0,1}(u-1) = (1-\lambda)\beta_{0,1}(u) + (1-\lambda)\beta_0(0)v_u$, …… (5)

$\beta_{0,l}(u-1) = (1-\lambda)\beta_{0,l}(u) + (1-\lambda)\beta_{0,l-1}(0)v_u$, $2 \leq l \leq L$,…….(6)

$\beta_{n,l}(u-1) = (1-\lambda)\beta_{n,l}(u) + \lambda\beta_{n-1,l}(u)$, $1 \leq l \leq L$, $1 \leq n \leq K-1$, …….(7)

$\beta_{K,l}(u-1) = \beta_{K,l}(u) + \lambda\beta_{K-1,l}(u)$, $1 \leq l \leq L$, …….(8)

$\gamma_0 = (1-\lambda)\gamma_0 + (1-\lambda)\beta_{0,L}(0)$. …….(9)

Multiplying (1) to (8) by $z^u$ and summing over u from 1 to $\infty$, we get

$z\alpha_0^*(z) = (1-\lambda)\{\alpha_0^*(z) - \alpha_0(0)\} + (1-\lambda)\alpha_1(0)S(z) + \lambda\alpha_0(0)S(z) + (1-\lambda)S(z)\sum_{l=1}^{L}\beta_{1,l}(0) + \lambda S(z)\sum_{l=1}^{L}\beta_{0,l}(0) + \lambda\gamma_0 S(z)$, ……..(10)

$z\alpha_n^*(z) = (1-\lambda)\{\alpha_n^*(z) - \alpha_n(0)\} + \lambda\{\alpha_{n-1}^*(z) -$ αn−10+1−λSzαn+10+λSzαn0+1−λSzl=1Lβn+1,l0+ $\lambda S(z)\sum_{l=1}^{L}\beta_{n,l}(0)$, $1 \leq n \leq K-2$, ……..(11)

$z\alpha_{K-1}^*(z) = (1-\lambda)\{\alpha_{K-1}^*(z) - \alpha_{K-1}(0)\} + \lambda\{\alpha_{K-2}^*(z) -$ αK−20+λSzαK−10+SzαK0+λSzl=1LβK−1,l0+ $S(z)\sum_{l=1}^{L}\beta_{K,l}(0)$, ………. (12)

$z\alpha_K^*(z) = \{\alpha_K^*(z) - \alpha_K(0)\} + \lambda\{\alpha_{K-1}^*(z) - \alpha_{K-1}(0)\}$, ………(13) $z\beta_{0,1}^*(z) = (1-\lambda)\{\beta_{0,1}^*(z) -$ β0,10+1−λβ00Vz, ……...(14)

$z\beta_{0,l}^*(z) = (1-\lambda)\{\beta_{0,l}^*(z) - \beta_{0,l}(0)\} + (1-\lambda)V(z)\beta_{0,l-1}(0)$, $2 \leq l \leq L$,……...(15)

$z\beta_{n,l}^*(z) = (1-\lambda)\{\beta_{n,l}^*(z) - \beta_{n,l}(0)\} + \lambda\{\beta_{n-1,l}^*(z) -$ βn−1,l0, 1≤l≤L, 1≤n≤K−1, ……...(16)

$z\beta_{K,l}^*(z) = \{\beta_{K,l}^*(z) - \beta_{K,l}(0)\} + \lambda\{\beta_{K-1,l}^*(z) -$ βK−1,l0, 1≤l≤L. ….…...(17)

Lemma 1: Putting $z = 1$ in (14)-(17), adding them and using (9) after some algebraic simplification we obtain the desired result

$\sum_{n=0}^{K}\sum_{l=1}^{L}\beta_{n,l}(0) + \lambda\gamma_0 = (1-\lambda)\alpha_0(0)$……..(18)

Lemma 2: $\rho' = \sum_{n=0}^{K}\alpha_n = E(S)\sum_{n=0}^{K}\alpha_n(0)$ and $1-\rho' = \sum_{n=0}^{K}\sum_{l=1}^{L}\beta_{n,l} + \gamma_0 = E(V)\sum_{n=0}^{K}\sum_{l=1}^{L}\beta_{n,l}(0) + \gamma_0$ …....(19)

Proof: Adding (10) to (13), we get

$$(z-1)\sum_{n=0}^{K}\alpha_n^*(z) = (S(z)-1)\sum_{n=0}^{K}\alpha_n(0).$$

Taking limit as $z \to 1$, yields the first result. Applying similar arguments to the equations (14)-(17) and using(9), which lead to the second result.

### 3.2 Queue length distributions at service completion and vacation terminationepochs

Let $\alpha_n^+$, $\beta_{n,l}^+$ be the probability that there are $n$ jobs in the queue at service completion[$l^{th}$ vacation termination] epoch. Therefore, we have

$\alpha_n^+ = P\{n-1(\geq 0)$or $n$jobs in the queue just prior to service completion epoch/$\leq K$ jobs in the queue just prior to service completion or vacation termination epoch}

$$= \begin{cases} \frac{1}{\sigma}\{(1-\lambda)\alpha_0(0)\} & : \ n = 0, \\ \frac{1}{\sigma}\{(1-\lambda)\alpha_n(0) + \lambda\alpha_{n-1}(0)\} & : 1 \leq n \leq K-1, \\ \frac{1}{\sigma}\{\alpha_K(0) + \lambda\alpha_{K-1}(0)\} & : \ n = K. \end{cases}$$

………(20)

Similarly, the expression of $\beta_{n,l}^+$is given by

$$\beta_{n,l}^+ = \begin{cases} \frac{1}{\sigma}\{(1-\lambda)\beta_{0,l}(0)\} & : n = 0, 1 \leq l \leq L, \\ \frac{1}{\sigma}\{(1-\lambda)\beta_{n,l}(0) + \lambda\beta_{n-1,l}(0)\} & : \ 1 \leq n \leq K-1, \\ & \quad 1 \leq l \leq L, \\ \frac{1}{\sigma}\{\beta_{K,l}(0) + \lambda\beta_{K-1,l}(0)\} & : n = K \ , 1 \leq l \leq L \end{cases}$$

where

$$\sigma = P\left\{\begin{matrix} \leq K \text{ jobs in the queue just prior to service} \\ \text{completion or vacation termination epoch.} \end{matrix}\right\}$$

$= \sum_{n=0}^{K}(\alpha_n + \sum_{l=1}^{L}\beta_{n,l})$ .

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The above results have been obtained by observing the events at epochs $t-$and $t+$ of Fig. 1. It can be seen from(20) - (21) that to get $\alpha_n^+$ and $\beta_{n,l}^+$we need to find out $\alpha_n(0)$ and$\beta_{n,l}(0)$. As $\alpha_n(0)$ and $\beta_{n,l}(0)$ are cumbersome toevaluate directly from (10) - (17), we obtain them using imbedded Markov chain technique. The unknown quantities $\alpha_n^+$ and $\beta_{n,l}^+$can be obtained by solving the system of equations $(\boldsymbol{\alpha}^+,\boldsymbol{\beta}^+) = (\boldsymbol{\alpha}^+,\boldsymbol{\beta}^+)\boldsymbol{P}$with $(\boldsymbol{\alpha}^+,\boldsymbol{\beta}^+)\boldsymbol{e} = 1$, where $\boldsymbol{e}$is a column vector of ones with an appropriate dimension. Here $\boldsymbol{\alpha}^+ = [\alpha_1^+,\alpha_2^+,\alpha_3^+,\ldots,\alpha_n^+]$and$\boldsymbol{\beta}^+ = [\beta_{1,1}^+,\beta_{1,2}^+,\ldots,\beta_{1,L}^+,\ \beta_{2,1}^+,\beta_{2,2}^+,\ldots,\beta_{2,L}^+,\ldots,\beta_{n,1}^+,\beta_{n,2}^+,\ldots,\beta_{n,L}^+]$

are the stationary probability vectorsof the one-step transition probability matrix $\boldsymbol{P}$of order

$(L+1)(K+1)\times(L+1)(K+1)$ as given below. Wesolved the system of equations using the GTH (Grassmann, Taksar and Heyman) algorithm given in Latoucheand Ramaswami ([8], pg 123). The transition probability matrix $\boldsymbol{P}$is given by

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{\mathcal{A}}_{(K+1)\times(K+1)} & \boldsymbol{\mathcal{B}}_{(K+1)\times L(K+1)} \\ \boldsymbol{\mathcal{C}}_{L(K+1)\times(K+1)} & \boldsymbol{\mathcal{D}}_{L(K+1)\times L(K+1)} \end{pmatrix}$$

where

$$\boldsymbol{\mathcal{A}}_{i,j} = \begin{cases} f_{j+1-i}, 1 \le i \le K, & 0 \le j < K, j \ge i-1, \\ \hat{f}_{j+1-i}, 1 \le i \le K, & j = K, j \ge i-1, \\ 0, & otherwise. \end{cases}$$

$$\boldsymbol{\mathcal{B}}_{i,(j,l)} = \begin{cases} h_j, 0 \le j < K, l = 1, & i = 0, \\ \hat{h}_j, \quad j = K, l = 1, & i = 0, \\ 0, & otherwise. \end{cases} \qquad \boldsymbol{\mathcal{C}}_{(i,l),j}$$

$$= \begin{cases} f_j, & i = 0, l = L, 0 \le j < K, \\ \hat{f}_j, & i = 0, l = L, j = K, \\ f_{j+1-i}, & 1 \le i \le K, 0 \le j < K, 1 \le l \le L, j \ge i-1, \\ \hat{f}_{j+1-i}, & 1 \le i \le K, j = K, \quad 1 \le l \le L, j \ge i-1, \\ 0, & otherwise. \end{cases}$$

$$\boldsymbol{\mathcal{D}}_{(i,l),(j,m)} = \begin{cases} h_j, & i = 0, 0 \le j < K, 1 \le l \le L-1, \\ & 1 \le m \le L, m = l+1 \\ \hat{h}_j, & i = 0, \quad j = K, 1 \le l \le L-1, \\ & 1 \le m \le L, m = l+1 \\ 0, & otherwise. \end{cases}$$

In the transition probability matrix $\boldsymbol{P}$, the top left corner ($\boldsymbol{\mathcal{A}}$) refers to a transition from service completion toservice completion, the top right corner ($\boldsymbol{\mathcal{B}}$) refers to a transition from service completion to vacation termination, the bottom left corner ($\boldsymbol{\mathcal{C}}$)refers to a transition from vacation termination to service completion and thebottom right corner($\boldsymbol{\mathcal{D}}$)refers to a transition from vacation termination to vacation termination.

Lemma 3:

Let $\Theta_B[\Theta_I]$be the random variable denoting the busy [server unavailable] period and $E(\Theta_B)$ $[E(\Theta_I)]$ be thecorresponding mean. From the definition of the carried load $\rho'$(the fraction of time that the server is in a busyperiod), we have

$$\rho' = \frac{E(\Theta_B)}{E(\Theta_B)+E(\Theta_I)} \quad \ldots\ldots\ldots.(23)$$

Using Lemma 2 and equation (23), we have

$$\frac{E(\Theta_I)}{E(\Theta_B)} = \frac{1-\rho'}{\rho'} = \frac{E(V)\sum_{n=0}^{K}\sum_{l=1}^{L}\beta_{n,l}(0) + \gamma_0}{E(S)\sum_{n=0}^{K}\alpha_n(0)} \quad \ldots\ldots\ldots(24)$$

Applying equations (9) and (20)-(21) in above, we obtain

$$\frac{E(\Theta_I)}{E(\Theta_B)} = \frac{\lambda E(V)\sum_{n=0}^{K}\sum_{l=1}^{L}\beta_{n,l}^+ + \beta_{0,L}^+}{\lambda E(S)\sum_{n=0}^{K}\alpha_n^+} \ldots\ldots \quad (25)$$

Now dividing the numerator and denominator of the right side expression of equation (24) by $E(\Theta_B)$ and thenusing equation (25), we get the value of $\rho'$.

Lemma 4:

Adding all the terms in equation (24) and using this in the first identity of Lemma 2, immediately yields

$$\sigma = \frac{\rho'}{E(S)\sum_{n=0}^{K}\alpha_n^+}.$$

3.3 Queue length distributions at arbitrary epochs

The arbitrary epoch probabilities are obtained from (9) and setting $z = 1$ in equations (10) - (12), (14) - (16) and then using (20) - (21), we get

$$\gamma_0 = \frac{\sigma}{\lambda}\beta_{0,L}^+, \quad \ldots\ldots\ldots.(26)$$

$$\alpha_0 = \frac{\sigma}{\lambda}\left(\alpha_1^+ - \alpha_0^+ + \sum_{l=1}^{L}\beta_{0,l}^+)\right) + \gamma_0, \ldots\ldots\ldots.(27)$$

$$\alpha_n = \alpha_{n-1} + \frac{\sigma}{\lambda}\left(\alpha_{n+1}^+ - \alpha_n^+ + \sum_{l=1}^{L}\beta_{n+1,l}^+)\right), \quad 1 \le n \le K-1, 1 \le l \le L \ \ldots\ldots\ldots.(28)$$

$$\beta_{0,1} = \frac{\sigma}{\lambda}\left(\alpha_0^+ - \beta_{0,1}^+\right), \quad \ldots\ldots\ldots.(29)$$

$$\beta_{0,l} = \frac{\sigma}{\lambda}\left(\beta_{0,l-1}^+ - \beta_{0,l}^+\right), 2 \le l \le L, \quad \ldots\ldots\ldots(30)$$

$$\beta_{n,l} = \beta_{n-1,l} - \frac{\sigma}{\lambda}\beta_{n,l}^+, 1 \le n \le K-1, 2 \le l \le L. \quad \ldots\ldots\ldots.(31)$$

But $\alpha_K$and $\beta_{K,l}$ ,$1 \le l \le L$can not be obtained from equations (13) and (17), respectively by setting z = 1.

We can obtain them using Lemma 2 in the sequel

$$\alpha_K = \rho' - \sum_{n=0}^{K-1}\alpha_n \ and$$

$$\sum_{l=1}^{L}\beta_{K,l} = 1 - \rho' - \sum_{n=0}^{K-1}\sum_{l=1}^{L}\beta_{n,l} - \gamma_0$$

Let $X_n$ denotes the probability that there are $n$ jobs in the queue at arbitrary epoch. Then

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$$X_n = \begin{cases} \alpha_0 + \sum_{l=1}^{L} \beta_{0,l} + \gamma_0, & n = 0, \\ \alpha_n + \sum_{l=1}^{L} \beta_{n,l}, & 1 \le n \le K-1, \\ \alpha_K + \sum_{l=1}^{L} \beta_{K,l}, & n = K. \end{cases}$$
……….(32)

### IV. WAITING TIME DISTRIBUTIONS

Let $W_q(z)$ be the probability generating function of actual waiting time of an arrived job. Note that an arrivedjob may be either

served immediately if he sees the system in dormant state, or

served after the completion of the job being served and all the waiting jobs in front of him depart from the system if he sees the server in busy state, or

served after $l^{th}(1 \le l \le L)$vacation period ends and all the waiting jobs in front of him depart from the system if he sees the server in the $l^{th}(1 \le l \le L)$vacation state.

Therefore, the p.g.f of the actual waiting time distribution in the queue is given by

$$W_q(z) = \frac{1}{1 - X_K}\left(\gamma_0 + \sum_{n=0}^{K-1} \alpha_n^*(z)\big(S(z)\big)^n + \sum_{n=0}^{K-1}\sum_{l=1}^{L} \beta_{n,l}^*(z)\big(S(z)\big)^n\right).$$

Thus the expected actual waiting time is given by

$$W_q = \frac{1}{1 - X_K}[E(S)\sum_{n=0}^{K-1} n(\alpha_n + \sum_{l=1}^{L}\beta_{n,l}) + \sum_{n=0}^{K-1}(\alpha_n^{*(1)} + \sum_{l=1}^{L}\beta_{n,l}{}^{*(1)})]$$

where $\alpha_n^{*(1)}$and $\beta_{n,l}{}^{*(1)}$, $1 \le n \le K-1$, $1 \le l \le L$are obtained by differentiating equations (10)-(12) and(14)-(16) w.r.t z at $z = 1$. These are given by

$$\alpha_0^{*(1)}(1) = E(S)[\gamma_0 + \frac{\sigma}{\lambda}\left(\alpha_1^+ + \sum_{l=1}^{L}\beta_{1,l}^+\right) - \frac{1}{\lambda}\alpha_0,$$

$$\alpha_n^{*(1)}(1) = \alpha_{n-1}^{*(1)}(1) + \frac{\sigma E(S)}{\lambda}\left(\alpha_{n+1}^+ + \sum_{l=1}^{L}\beta_{n+1,l}^+\right) - \frac{1}{\lambda}\alpha_n, \quad 1 \le n \le K-1, 1 \le l \le L,$$

$$\beta_{0,1}^{*(1)}(1) = \frac{\sigma E(V)}{\lambda}\alpha_0^+ - \frac{1}{\lambda}\beta_{0,1},$$

$$\beta_{0,l}^{*(1)}(1) = \frac{\sigma E(V)}{\lambda}\beta_{0,l-1}^+ - \frac{1}{\lambda}\beta_{0,l}, 2 \le l \le L,$$

$$\beta_{n,l}^{*(1)}(1) = \beta_{n-1,l}^* - \frac{1}{\lambda}\beta_{n,l}, 1 \le n \le K-1, \quad 2 \le l \le L.$$

### V. PERFORMANCE MEASURE

Here we obtain several performance measures which are useful in evaluating system efficiency.

$L_q$ = average queue length = $\sum_{n=0}^{K} n X_n$

$L_{q2}$= average queue length when the server is busy = $\sum_{n=0}^{K} n\,\alpha_n$

$L_{q1}$= average queue length when the server is on vacation = $\sum_{n=0}^{K}\sum_{l=1}^{L} n\,\beta_{n,l}$

$PBL$ = blocking probability of an arriving job = $X_K$

$W_q(L)$ = average waiting time in the queue using Little's rule = $\frac{Lq}{\lambda}$ where $\lambda' = \lambda(1 - PBL)$

In case ofinfinite buffer queues$\rho$ and $\rho'$are equal where as in finite buffer queues they are different. We have not presented any numerical result due to lack of space.

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