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The Isotropic and Anisotropic Stellar Masses for a Charged Matter

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***Abstract*- In this paper we compare the relativistic masses for isotropic and anisotropic stellar objects. We use the exact solutions obtained in the work by Sunzu and Danford in order to compute the relativistic stellar masses. We have used a model that is charged and adopted a linear equation of state consistent for a quark matter. It is indicated that the relativistic masses generated are in the acceptable range when compared with stellar masses previously found by other researchers. Our model indicates that the masses for the isotropic case are higher than that of anisotropic object. Our results are therefore significant for the study of effects of the anisotropy on the charged stellar objects.**

***Key Words*:-- Einstein-Maxwell field equation, charged matter, anisotropy, isotropy, stellar masses.**

I. INTRODUCTION

It has been indicated that applying the Einstein-Maxwell field equations, different models which describe behaviors of the stellar objects are found. Using the spacetime that is static and spherically symmetry, models with astrophysical significance have been generated. In this direction models found have revealed properties and structure for the relativistic objects such as quark stars, dark energy stars, neutron stars, gravastars, and black holes. It is for this reason that mathematicians and physicists are attracted to generate models that produce results in line with astrophysical and astronomical findings. Mak and Harko [1] presented a model for an object with mass 2.86MΘ and the radius 9.46 km. In the work by Gangopadhyay et al [2] an astronomical object with mass 1.60 MΘ and the radius 9.40 km is found. Sunzu et al [3] has obtained stellar masses within the ranges 1.28 MΘ - 1.73 MΘ and the radii 5.77 km - 7:61 km. Other works with astronomical and astrophysical signi_cance include the performance by Dey et al [4] who found an object with mass 1:433 MΘ and radius 7:07 km, Thirukkanesh and Maharaj [5], Mafa Takisa and Maharaj [6], Sunzu et al [7], Negreiros et al [8], Sunzu and Danford [9] and Guver et al [10, 11].

When modeling the stellar objects it is important that the ingredient of pressure anisotropy be considered. This is due to the fact that the pressure anisotropy do affect variability of the relativistic stellar objects. It is indicated in several studies that the pressure anisotropy have impact on the physical behaviour and properties, stability and structure of stellar objects. In the work performed by Sharma and Mukherjee [12] it is indicated the presence of anisotropy is essential in describing properties of dense objects with quark materials. The results found by Gleiser and Dev [13] have shown that the physical structures of relativistic matter is affected by this ingredient. It is also shown in their paper that the presence of pressure anisotropy may lead observational effects. It is highlighted that relativistic objects are more stable when the pressure anisotropy exists near the core of the matter. Several findings indicate that when the anisotropy is present in a stellar object with electromagnetic field distribution, the stability under radial adiabatic perturbations is improved than when compared with objects with isotropic pressure Dev and Gleiser [14]. There are several research work with both electromagnetic field and anisotropy present. These include the models developed by Ngubelanga et al [15], Maharaj and Mafa Takisa [16], Feroze and Siddiqui [17], Mafa and Maharaj [18], Maharaj et al [19], Kileba Matondo and Maharaj [20] and others are performed in [3,7,21{25]. However most of models with charge have the anisotropy always present and cannot draw comparison with isotropic case. A model that compares the effect of anisotropy on matter variables is necessary.

The objective of this paper is to generate and compare relativistic stellar masses and radii for isotropic and anisotropic models using the exact model found by Sunzu and Daniford [9]. The results obtained by using Python programing language are presented through graphs and table.

II. BASIC EQUATIONS

The spacetime geometry which is static and spherically symmetric is represented by the line element.

$$ds^2 = -e^{2\upsilon(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2). \qquad (1)$$

In the above, $\upsilon(r) and\ \lambda(r)$ define the graviatational potentials. The exterior spacetime is given by Reisser Nordstrom line element

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \qquad (2)$$

where M and Q stands for the total mass and charge measured by an observer at infinity. The energy momentum tensor for a charged anisotropic matter is given by

$$\tau_{ij} = diag\left(-\rho - \frac{1}{2}E^2, p_r - \frac{1}{2}E^2, p_t + \frac{1}{2}E^2, p_t + \frac{1}{2}E^2\right), \qquad (3)$$

Where ρ is the energy density, p_r is the radial pressure, p_t is the tangential pressure and E is the electric filed inside the charged stellar objects.

The Einstein-Maxwell field equations for anisotropic matter with charge in general relativity is given as

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2, \qquad (4a)$$

$$\frac{-1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2, \qquad (4b)$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2, \qquad (4c)$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}\left(r^2 E\right)', \qquad (4d)$$

where σ is a proper charged density. In the system (4), prime denotes the differentiation of the variables with respect to radial coordinate r. we consider a linear relationship between the radial pressure and the energy density as

$$p_r = \frac{1}{3}\left(\rho - 4B\right), \qquad (5)$$

where B is a bag constant. Equation (5) is the Bag equation and is consistent with quark matter. The mass function contained within the charged sphere is given by

$$m(r) = \frac{1}{2}\int_0^r \omega^2 \rho d\omega, \qquad (6)$$

III. TRANSFORMATIONS

The fundamental line element (1) and the field equations (4) can be transformed to a simple form by introducing the following transformations.

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2y^2(x) = e^{2\upsilon(r)}, \qquad (7)$$

where C and A are the arbitrary constants. The transformation in the system (7) was suggested by Durgapal and Bannerji [26]. Therefore the Einstein-Maxwell field equations (4) and the equation of state (5) can be written in the following form:

$$\rho = 3p_r + 4B, \qquad (8a)$$

$$\frac{p_r}{C} = Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, \qquad (8b)$$

$$p_t = p_r + \Delta, \qquad (8c)$$

$$\Delta = 4CxZ\frac{\ddot{y}}{y} + C\left(6Z + 2x\dot{Z}\right)\frac{\dot{y}}{y} + C\left(2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right), \qquad (8d)$$

$$\frac{E^2}{2C} = \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, \qquad (8e)$$

$$\sigma = 2\sqrt{\frac{CZ}{x}}(x\dot{E} + E). \qquad (8f)$$

The mass function in Equation (6) becomes

$$M(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{\omega}\rho d\omega. \qquad (9)$$

The system (8) has six equations in eight variables (ρ, p_r, p_t, E, Z, y, σ; Δ). The nonsingular exact solutions to the system (8) were obtained in Sunzu and Danford [9] after specifying the metric function y and the measure of anisotropy Δ in the forms

$$y = \left(a + x^m\right)^n, \qquad (10)$$

$$\Delta = \alpha_0 x^3 + \alpha_1 x^4, \qquad (11)$$

where a, m, n, α_0, α_1 are arbitrary constants. The choice of metric function (10) is very crucial in modeling relativistic matter. It observed to be finite, regular and continuous throughout the interior of stellar objects. This choice of metric function was also adopted by Komathiraj and Maharaj [27] and Sunzu et al [3]. We note that when $\alpha_0 = \alpha_1 = 0$, we have $\Delta = 0$ and the model becomes isotropic. The exact solution obtained in Sunzu and Danford [9] was a generalization of the results in Mak and Harko [1], Komathiraj and Maharaj [27], and Misner and Zapolsky [28]. In our work we establish the comparison between stellar masses and radii for isotropic and anisotropic models an aspect missing in most of the investigations.

Substituting Eq. (10) and Eq. (11) in Eq. (8d) after partial decomposition we obtain

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$$\frac{\left(1-\frac{2xB}{C}+\frac{x^3\left(\alpha_0+\alpha_1 x\right)}{C}\right)\left(a+x^m\right)}{2x(nmx^m+x^m+a)} =$$
$$\dot{Z}+\left(\frac{1}{2x}+\frac{m\left(4\left(mn+1\right)-3m\right)x^{m-1}}{2\left(a+\left(1+nm\right)x^m\right)}+\frac{2m\left(n-1\right)x^{m-1}}{a+x^m}\right)Z. \qquad (12)$$

Equation (12) is a general master differential equation that governs the model for stellar body with charge and anisotropy present.

The nonsingular exact solution for this model was obtained by Sunzu and Danford [9] by choosing m = 1 and n = 1. They obtained the following gravitational potential and matter variables:

$$e^{2\nu} = A^2\left(a+x\right)^4, \qquad (13a)$$
$$e^{2\lambda} = 315\left(a+x\right)^2\left(a+3x\right)\left[9\left(35a^3+35a^2x+21ax^2+5x^3\right) - \frac{2B}{C}\left(105a^3x+189a^2x^2+35ax^3+35x^4\right)+\frac{315H(x)}{C}\right]^{-1}, \qquad (13b)$$
$$p_r = \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35\left(a+x\right)^3\left(a+3x\right)^2} - \frac{B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105\left(a+x\right)^3\left(a+3x\right)^2} - \frac{\Omega(x)}{105\left(a+x\right)^3\left(a+3x\right)^2}, \qquad (13c)$$

$$\rho = \frac{3C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35\left(a+x\right)^3\left(a+3x\right)^2} - \frac{3\Omega(x)}{105\left(a+x\right)^3\left(a+3x\right)^2} - \frac{3B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105\left(a+x\right)^3\left(a+3x\right)^2}+4B \qquad (13d)$$

$$p_t = \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35\left(a+x\right)^3\left(a+3x\right)^2}+\frac{\Gamma(x)}{105(a+x)^3(a+3x)^2} - \frac{B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105\left(a+x\right)^3\left(a+3x\right)^2}, \qquad (13e)$$
$$\Delta = x^3\left(\alpha_0+\alpha_1 x\right), \qquad (13f)$$
$$E^2 = -\frac{B\left(168a^4x+1296a^3x^2+6528a^2x^3+7280ax^4+2520x^5\right)+630\xi(x)}{315\left(a+x\right)^3\left(a+3x\right)^2} + \frac{C\left(196a^3x+1452a^2x2+1356ax^3+420x^4\right)}{35\left(a+x\right)^3\left(a+3x\right)^2}. \qquad (13g)$$

Here

$$\Omega(x) = \alpha_o\left(\frac{70}{3}a^5x^3+\frac{3710}{33}a^4x^4+\frac{2310}{13}a^3x^5+\frac{17206}{143}a^2x^6+\frac{1701}{26}ax^7\right) +\alpha_1\left(\frac{525}{22}a^5x^4+\frac{3255}{26}a^4x^5+\frac{32403}{143}a^3x^6+\frac{41517}{221}a^2x^7+\frac{2415}{34}ax^8+\frac{315}{34}x^9\right),$$
$$\Gamma(x) = \alpha_o\left(\frac{245}{3}a^5x^3+\frac{27475}{33}a^4x^4+\frac{38640}{13}a^3x^5+\frac{673484}{143}a^2x^6+\frac{88389}{26}ax^7+945x^8\right) +\alpha_1\left(\frac{1785}{22}a^5x^4+\frac{21315}{26}a^4x^5+\frac{418047}{143}a^3x^6\frac{1025913}{221}a^2x^7+\frac{115395}{34}ax^8+\frac{31815}{34}x^9\right)$$
$$\xi(x) = \alpha_o\left(\frac{1}{3}a^5x^3+\frac{265}{99}a^4x^4+\frac{1150}{143}a^3x^5+\frac{7012}{715}a^2x^6+\frac{115}{26}ax^7+\frac{9}{5}x^8\right) +\alpha_1\left(\frac{7}{22}a^5x^4+\frac{719}{286}a^4x^5+\frac{417}{55}a^3x^6+\frac{150527}{14365}a^2x^7+\frac{17677}{2465}ax^8+\frac{57}{34}x^9\right)$$

$$H(x) = \alpha_0\left(\frac{a^3x^4}{9}+\frac{3a^2x^5}{11}+\frac{ax^6}{13}+\frac{x^7}{15}\right)+\alpha_1\left(\frac{a^3x^5}{11}+\frac{3a^2x^6}{13}+\frac{3ax^7}{15}+\frac{x^8}{17}\right).$$

The mass function (9) corresponding to this model becomes

$$M(x) = \frac{1}{2042040C^{\frac{3}{2}}}\left[-\frac{40040}{3}ax^{\frac{9}{2}}\alpha_1-2730x^{\frac{11}{2}}\alpha_1-\frac{22}{3}ax^{\frac{7}{2}}\left(4131\alpha_0+292a\alpha_1\right)\right.$$
$$+\frac{14}{45}a^2x^{\frac{5}{2}}\left(249135\alpha_0+22204a\alpha_1\right)+\frac{154}{27}x^{\frac{3}{2}}\left(39780B-53363a^3\alpha_0-1484a^4\alpha_1\right)$$
$$+\frac{2}{81}\sqrt{x}\left[-437580\left(22aB-27C\right)+69243363a^4\alpha_0+512036a^5\alpha_1\right]$$
$$-\frac{17a\sqrt{x}}{16\left(a+x\right)}\left[-522053a^4\alpha_0+64\left(-2431\left(2aB+9C\right)+63a^5\alpha_1\right)\right]$$
$$-\frac{3a^2\sqrt{x}}{8\left(a+x\right)^2}\left[163863a^4\alpha_0+64\left(-2431\left(2aB+9C\right)+63a^5\alpha_1\right)\right]$$
$$-\frac{1}{16}\sqrt{a}\arctan\left(\sqrt{\frac{x}{a}}\right)\left[35260057a^4\alpha_0+1984\left(-2431\left(2aB+9C\right)+63a^5\alpha_1\right)\right]$$
$$+\frac{\sqrt{a}}{108\sqrt{3}}\arctan\left(\sqrt{\frac{3x}{a}}\right)\left[215509a^4\alpha_0+16\left(-21879\left(188aB+1161C\right)+4886a^5\alpha_1\right)\right]$$
$$\left.+\frac{a\sqrt{x}}{648\left(a+3x\right)}\left[-769335a^4\alpha_0+32\left(-21879\left(188aB+1161C\right)+4886a^5\alpha_1\right)\right]\right]. \qquad (14)$$

IV. RESUTLS AND DISCUSSION

In this section we generate and discuss relativistic stellar masses and radii by considering the isotropic and anisotropic models. For isotropic model we have Δ=0 which is satisfied when $\alpha_0 = \alpha_1 = 0$ and for anisotropic model the measure of anisotropy Δ≠0. We are comparing the masses and radii for charged stellar objects for isotropic and anisotropic models. We have transformed the parameters in mass equation (14) using the following transformation.

$$\tilde{\alpha}_0 = \alpha_0 R^2, \tilde{\alpha}_1 = \alpha_1 R^2, \,\tilde{a} = aR^2, \tilde{B} = BR^2, \tilde{C} = CR^2.$$

Table 1: comparison between the relativistic masses for the isotropic and anisotropic stellar objects

Name	$\tilde{B}$	$\tilde{C}$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{a}$	r(km)	$\frac{M}{M_\odot}(\Delta=0)$	$\frac{M}{M_\odot}(\Delta\neq 0)$
R1	2.0	1.0	10.0	5.0	200	8.71	2.778	2.653
R2	1.0	1.0	4.0	10.0	220.0	10.00	2.837	2.480
R3	1.0	1.0	2.0	10.0	100	9.090	2.559	2.404
R4	1.5	1.0	5.0	1.0	300.0	6.67	1.342	1.285
R5	0.5	1.0	0.2	0.2	350	7.52	1.246	1.220

The masses and radii are generated for two cases namely isotropic and anisotropic at various choices of parameters are presented in Table (1). We see that the masses for the isotropic models ranges from 1.246 M_Θ - 2.778 M_Θ with radii in the range 6:67 km - 10:00 km while the masses for the anisotropic case range from 1.220 M_Θ - 2.653 M_Θ. In each case we observed that the stellar masses for the isotropic matter is greater than that of anisotropic model. However, it is interesting that the stellar masses and radii generated in each case of our models are in acceptable ranges according to Sunzu et. al [3], Guver et. al [10, 11], Mak and Hark [1], Gangopadhyay et. al [2] and many others. We have plotted the graphs for the mass against the radial distance using the values indicated in the Table 1 by python programing language.

We observe from these figures the variability of the masses with the distance from the centre to the surface. From the

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Figs. 1-5, we see that in general case the isotropic plot lies above the anisotropic plot indicating the stellar masses for isotropic case is larger. From Fig. (1) and Fig. (3), it can be seen that the difference in masses occur in region away the centre of the stellar interior.

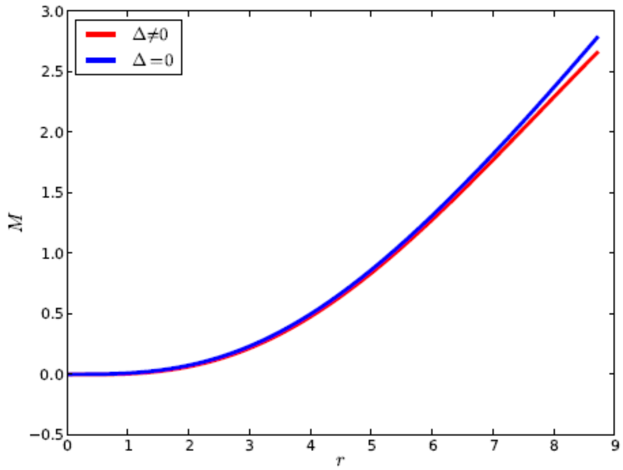

***Fig 1:* Comparison between the masses using data tabulated in R1.**

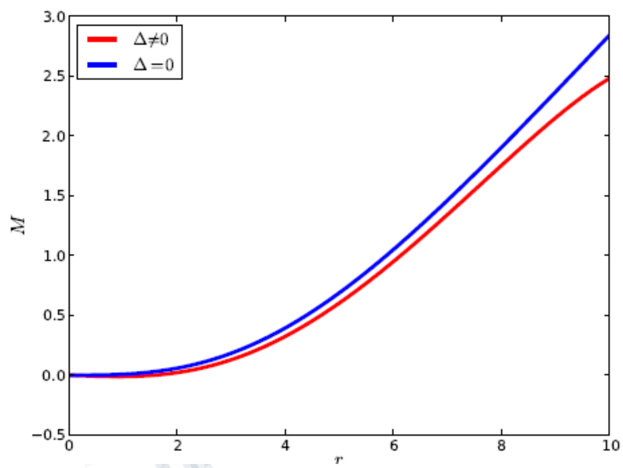

***Fig 2:* Comparison between the masses using data tabulated in R2.**

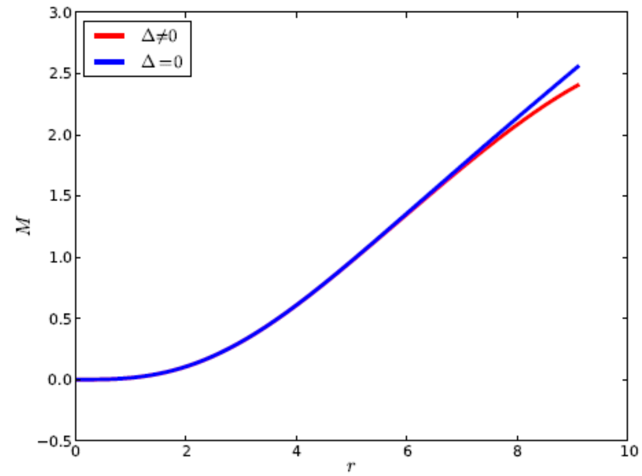

***Fig 3:* Comparison between the masses using data tabulated in R3.**

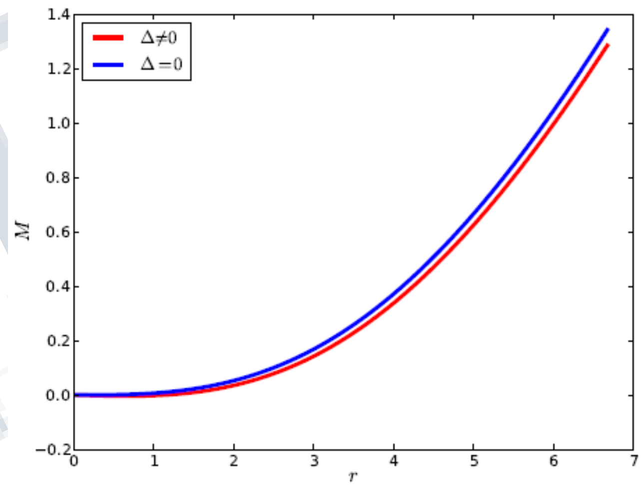

***Fig 4:* Comparison between the masses using data tabulated in R4.**

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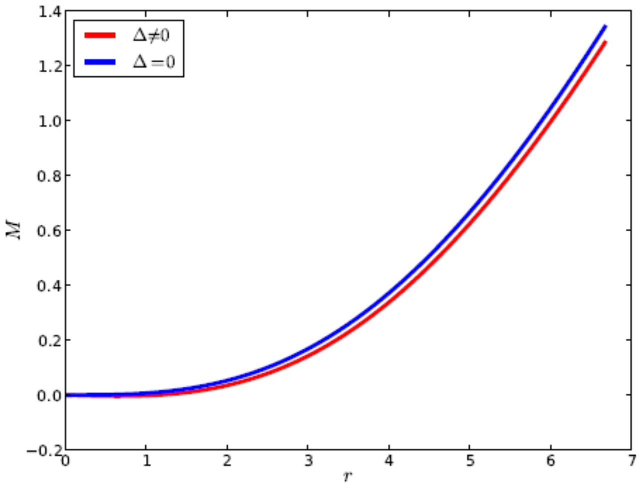

Fig 5: Comparison between the masses using data tabulated in R5.

CONCLUSION

In our work we have obtained the exact models with quark strange equation of state generated by Sunzu and Danford [9]. We have generated masses and radii for charged stellar object. These quantities are found for both anisotropic and isotropic cases. Masses obtained for isotropic models are ranging from 1.246 MΘ- 2.778 MΘ with radii in the range 6.67 km – 10.00 km while the masses for the anisotropic case range from 1.220 MΘ - 2.653 MΘ and for the case of anisotropic models the stellar masses ranging from1.220 MΘ - 2.653 MΘ. We found that the anisotropic masses are less than the isotropic one. These results are in good agreement with reported studies.

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