

# Viscous and Joule's dissipation effects on Bio-convection MHD Casson radiative fluid flow over a stretching sheet with slip condition

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**Abstract:** --- The present work deals with an investigation of steady two dimensional flow of analyzed slip effects of viscous, joules dissipation, inclined magnetic field Casson fluid flow containing both nanoparticles and gyrotactic microorganism with analyzed on heat, mass, concentration and motile microorganism. The governing partial differential equations (PDEs) are complex and highly non-linearized. These equations are transfigured to system of ordinary differential equations (ODEs) using suitable transformations. The governing mathematical expressions are converted into non-dimensional form via nonlinear type similarity variables. The resulting mathematical model is numerically solved with the help of MATLAB solver bvp4c. Further interesting aspects of viscous dissipation, magnetic parameter, Radiation parameter, Peclet number and Joule heating on the non-dimension velocity, temperature, concentration, the distribution of motile microorganisms are examined. The results are obtained from the skin friction coefficient, local Nusselt number and local Sherwood number are computed and explicated through table as well as graphs.

**Key Words:-** Joule's heating, viscous dissipation, MHD, Casson fluid, thermal slip parameter, Eckert number.

## 1.INTRODUCTION

The flow over a stretching/shrinking surface is an important problem in many researchers doing with applications in industries such as extrusion, drawing wire, fiber-glass productions, rubber sheets and plastic manufacture, cooling of large metallic plats in a bath and glass, and polymer industries. Joule heating and viscous dissipations changes the temperature by playing a role an energy source, which leads to affected heat transfer rates. Crane [1] was first one to study the boundary layer flow of viscous fluid over a stretching sheet. Cortell [2] investigate the effects of suction and heat absorption through porous medium heat and mass transfer over a stretching sheet. An analysis of thermal boundary layer in an electrically conducting fluid over a linear stretching sheet in the magnetic field carried out by Chaim [3]. Viscous and Joules dissipation and internal heat generations was taken into the energy equation. Sakiadis [4] examined the boundary layer behavior on a continuous solid surface. Makinde and Animasaun [5] analyzed the bio-convection in MHD nanofluid flow with nonlinear thermal radiation and quartic autocatalysis chemical reaction past an upper surface of a paraboloid of revolution. Yin [6] proposed the viscous and Joule heating effects on Non-Darcy MHD natural convection flow over a permeable sphere in porous media with internal heat generation. Chen [7] discussed the combined heat and mass transfer in magnetohydrodynamic (MHD) free convection from a vertical surface with Ohmic heating and viscous dissipation. Sadeghi and Saidi [8] portrayed the viscous dissipation effects on thermal transport

characteristics of combined pressure and electrostatically driven flow in micro channels. Subhas Abel et al. [9] studied the viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and Ohmic dissipations.

Non-Newtonian fluids are imperative in boundary layer flows because of their engineering and technology related applications. Well known examples of non-Newtonian fluids are polymeric-liquids, apple-sauce, tomato-sauce, cosmetic-products, human-blood, wall-paints, sops, glues, jells, honey etc. As there are diversity of non-Newtonian fluids so it is reasonably difficult to form an equation expressing the elastic and viscous properties of these fluids. In comparison to viscous fluids, mathematical modeling of non-Newtonian fluids is considerably complex and challenging. A mathematical analysis has been carried out on momentum and heat transfer is an incompressible, electrically conducting viscous boundary layer fluid flow over a linear stretching sheet with variable viscosity has been studied by Pantokratoras [10]. Effects of viscous dissipation and joule heating on magnetohydrodynamic (MHD) flow of a fluid with variable properties past a stretching vertical plate analyzed by Jaber [11]. Partial slip and dissipation on magnetohydrodynamic (MHD) radiative ferro-fluid over a non-linear permeable convectively heated stretching sheet discussed by Durga Prasad et al. [12].

The main purpose of current study is to investigate the effect of viscous and Joule's dissipation effects on Bio-convection magnetohydrodynamic casson radiative fluid flow over a stretching sheet with slip condition. The governed partial

differential relations of flow are reduced into non-linear coupled ordinary differential systems by using similarity variables. These relations are numerically solved using MATLAB bvp4c.

## 2. MATHEMATICAL FORMULATION

We consider a steady two dimensional, electrically conducting viscous and incompressible Casson fluid over an inclined permeable stretching sheet with porous medium and buoyancy effects are taken into consideration. The velocity of the stretching sheet is assumed in the form  $\lambda u_w(x)$ , with  $\lambda > 0$  for a stretching surface, where  $x$ - and  $y$ -axes are measured along the stretching surface and normal to it, respectively, and the flow being confined to  $\lambda > 0$ . It is assumed that the surface is permeable and the mass flux velocity is  $v_0$  with  $v_0 < 0$  for suction and  $v_0 > 0$  for injection. It is also assumed that the constant temperature, concentration and motile microorganism at the surface of the sheet are  $T_w$ ,  $C_w$  and  $N_w$  while those of the ambient (inviscid) fluid are  $T_\infty$ ,  $C_\infty$  and  $N_\infty$  the rheological equation of state for an isotropic and incompressible flow of a Casson fluid.

The system of equations, which models the flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} + g \beta_T (T - T_\infty) \cos \alpha + g \beta_c (C - C_\infty) \cos \alpha - u \frac{\nu}{k'} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2 u^2}{\rho C_p} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty) \quad (4)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_n \frac{\partial^2 N}{\partial y^2} - \frac{b W_c}{\Delta C} \left( \frac{\partial N}{\partial y} \frac{\partial C}{\partial y} + \frac{\partial^2 C}{\partial y^2} \right) \quad (5)$$

where  $u$  and  $v$  are the velocity component along the  $x$  and  $y$  axes,  $T$  is the fluid temperature,  $C$  is the concentration,  $N$  is the motile microorganism,  $\nu$  is the kinematic viscosity of the fluid,  $\alpha$  is the thermal diffusivity of the fluid,  $\rho$  is the density of the fluid,  $D_B$  is the mass diffusion coefficient,  $C_p$  is the specific heat at constant pressure,  $\sigma$  is the electric conductivity of the fluid,  $B_0$  is the applied uniform magnetic field normal to the surface of the sheet,  $q_r$  is the radiative heat flux,  $Q_0$  is the volumetric rate of heat generation or absorption,  $g$  is gravitational acceleration,  $\beta_T$  is the thermal expansion coefficient,  $\beta_c$  is the solutal expansion coefficient,  $k'$  is the permeability coefficient of porous medium,  $\alpha$  is the inclination of the stretching sheet parameter and  $k_0$  is respectively the constant chemical reaction rate,  $D_n$  the diffusion coefficient of the microorganism,

We assume that Eqs. (1) - (5) are subjected to the boundary conditions

$$v = v_0, u = \lambda u_w(x) + L \frac{\partial u}{\partial y}, T = T_w + S' \frac{\partial T}{\partial y},$$

$$SC = C_w + K' \frac{\partial C}{\partial y}$$

$$N \rightarrow N_w \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, N \rightarrow N_\infty \text{ as } y \rightarrow \infty \quad (6)$$

where we assume that  $u_w(x) = ax$ , with  $a > 0$

By using the Rosseland approximation the energy Eq. (4) can be written as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( 1 + \frac{16\sigma^* T_\infty^3}{3kk^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) + \frac{v}{C_p} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2 u^2}{\rho C_p} \quad (7)$$

where  $k$  is the thermal conductivity,  $\sigma^*$  is the Stefan-Boltzmann constant and  $k^*$  is the mean absorption coefficient.

We look for a similarity solution of Eqs. (2)-(5) and (7) of the following form:

$$\psi = \sqrt{av} x f(\eta), \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \phi(\eta) = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \chi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}, \eta = y \sqrt{a/v} \quad (8)$$

Where  $\psi(x, y)$  is the stream function, which is defined in the usual way as  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$ . Thus, we have

$$u = axf'(\eta), v = -\sqrt{av} f(\eta) \quad (9)$$

Where primes denote differentiation with respect to  $\eta$ . Thus, we take the dimensionless parameter  $S$  and  $\delta, \gamma$  and  $\chi$  are defined as

$$S = -v_0/\sqrt{av}, \delta = \sqrt{\frac{a}{v}}L, \gamma = \sqrt{\frac{a}{v}}S', X = \sqrt{\frac{a}{v}}K' \quad (10)$$

Where  $L$  is the velocity slip length,  $S > 0$  corresponds to suction,  $S < 0$  for injection,  $\delta$  is the velocity slip parameter,  $\gamma$  is the thermal slip/jump parameter and  $\beta$  is the Casson parameter.

Substituting (8) into Eqs. (2), (7) and (4), (5) we obtain the following ordinary differential equations

$$\left( 1 + \frac{1}{\beta} \right) \frac{\partial^3 f}{\partial \eta^3} + \frac{\partial^2 f}{\partial \eta^2} f - \left( \frac{\partial f}{\partial \eta} \right)^2 + Gr \cos \alpha \theta + Gc \cos \alpha \phi - (M + K) \frac{\partial f}{\partial \eta} = 0 \quad (11)$$

$$\frac{1}{Pr} \left( 1 + \frac{4R}{3} \right) \frac{\partial^2 \theta}{\partial \eta^2} + f \frac{\partial \theta}{\partial \eta} + Q\theta + Ec \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial^2 f}{\partial \eta^2} \right)^2 + J \left( \frac{\partial f}{\partial \eta} \right)^2 = 0 \quad (12)$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + Scf \frac{\partial \phi}{\partial \eta} - Sc\Gamma \phi = 0, \quad (13)$$

$$\frac{\partial^2 \chi}{\partial \eta^2} + Pef \frac{\partial \chi}{\partial \eta} - PeLe \left( \frac{\partial \phi}{\partial \eta} \frac{\partial \chi}{\partial \eta} + \frac{\partial^2 \phi}{\partial \eta^2} \right) = 0 \quad (14)$$

Subject to the boundary conditions

$$f(0) = S, \frac{\partial f(0)}{\partial \eta} = \lambda + \delta \frac{\partial^2}{\partial \eta^2} (f(0)), \theta(0) = 1 + \gamma \frac{\partial}{\partial \eta} (\theta(0)), \phi(0) = 1 + X \frac{\partial}{\partial \eta} (\phi(0)), \chi(0) = 1, f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0, N \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (15)$$

The dimensionless constants  $Pr$  denotes Prandtl number,  $Sc$  denotes Schmidt number,  $M$  denotes magnetic parameter,  $R$  denotes radiation parameter,  $Gr$  denotes thermal Grashof number,  $Gc$  denotes solutal Grashof number,  $Q$  denotes heat source/sink parameter,  $K$  denotes Permeability parameter,  $\Gamma$

denotes Chemical reaction parameter,  $Ec$  denotes Eckert number,  $J$  denotes Joule heating parameter,  $Pe$  denotes Peclet number,  $Le$  denotes Lewis number which are defined as

$$\begin{aligned}
 Pr &= \frac{\nu}{\alpha}, Sc = \frac{\nu}{D_B}, M = \frac{\sigma B_0^2}{\rho \alpha}, \\
 R &= \frac{4\sigma^* T_\infty^3}{kk^*}, Gr = \frac{g\beta_T(T_w - T_\infty)}{a^2 x}, \\
 Gc &= \frac{g\beta_C(C_w - C_\infty)}{a^2 x}, \\
 Q &= \frac{Q_0}{a\rho C_p}, K = \frac{\nu}{k'a}, \Gamma = \frac{k_0}{a}, Ec = \frac{u_w^2}{C_p[T_w - T_\infty]}, \\
 J &= \frac{\sigma B_0^2 a^2 x}{\rho C_p (T_w - T_\infty)}, \\
 Pe &= \frac{\nu}{D_n}, Le = \frac{bW_c}{\nu} \quad (16)
 \end{aligned}$$

The quantities of physical interest in this problem are the skin friction coefficient  $C_f$ , the local Nusselt number  $Nu_x$  and the local Sherwood number  $Sh_x$ , which are defined as

$$\begin{aligned}
 C_f &= \frac{\tau_w}{\rho u_w^2(x)}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \\
 Sh_x &= \frac{xq_m}{D_B(C_w - C_\infty)}, \\
 Sh_x &= \frac{xq_n}{D_B(N_w - N_\infty)} \quad (17)
 \end{aligned}$$

Where  $\tau_w$ ,  $q_w$  and  $q_m$  are the skin friction or shear stress, heat flux and mass flux from the sheet, which are given by

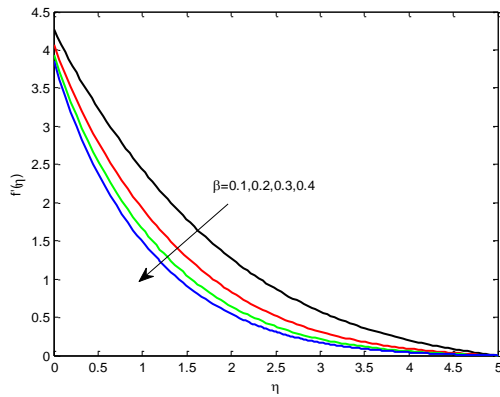
$$\begin{aligned}
 \tau_w &= \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, q_w = - \left( \frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_{y=0}, \\
 q_m &= -D_B \left( \frac{\partial C}{\partial y} \right)_{y=0}, q_n = -D_B \left( \frac{\partial N}{\partial y} \right)_{y=0} \quad (18)
 \end{aligned}$$

Where  $\mu$  is the dynamic viscosity of the fluid. Using (8), (17) and (18), we get

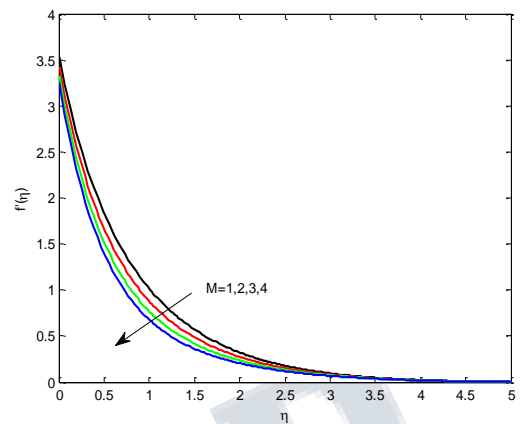
$$\begin{aligned}
 Re_x^{1/2} C_f &= \left( 1 + \frac{1}{\beta} \right) f''(0), Re_x^{-1/2} Nu_x = -\theta'(0), \\
 Re_x^{-1/2} Sh_x &= -\phi'(0), Re_x^{-1/2} Sh_x = -\chi'(0) \quad (19)
 \end{aligned}$$

## RESULTS AND DISCUSSIONS

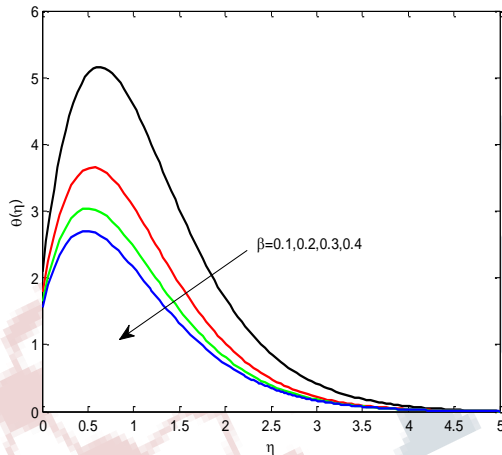
The governing boundary layer equations are converted into non-linear partial differential equations. To analyze the physical insight into the problem the numerical computations were carried out for governing parameters, suction/injection parameters  $S$ , velocity slip parameters  $\delta$ , thermal slip/jump parameters  $\gamma$ , Casson parameter  $\beta$ , Prandtl numbers  $Pr$ , Schmidt numbers  $Sc$ , magnetic parameters  $M$ , radiation parameters  $R$ , thermal Grashof numbers  $Gr$ , modified Grashof numbers  $Gc$ , heat source/sink parameters  $Q$ , permeability parameters  $K$ , chemical reaction parameters  $\Gamma$ , Eckert numbers  $Ec$ , Joule heating parameters  $J$ , Peclet numbers  $Pe$ , Lewis numbers  $Le$  and mass slip parameter  $X$ .



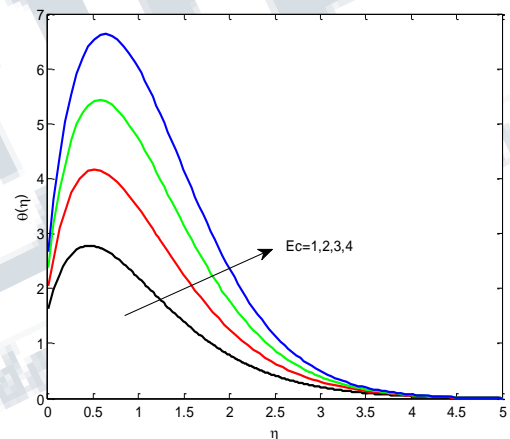
*Fig.1 The effect of  $\beta$  on Velocity profile.*



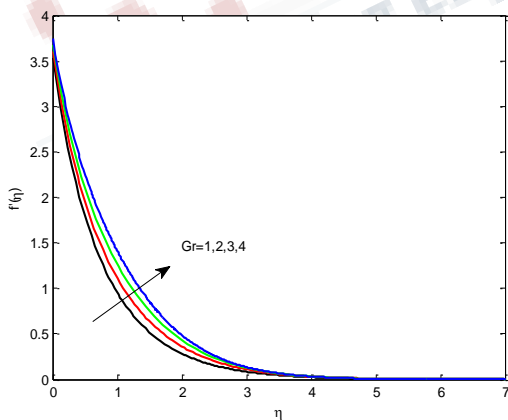
*Fig.4 The effect of  $M$  on Velocity profile.*



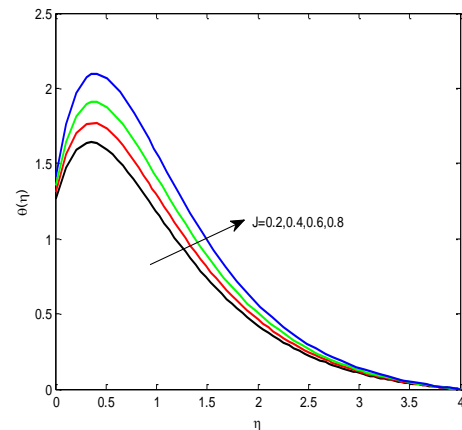
*Fig.2 The effect of  $\beta$  on Temperature profile.*



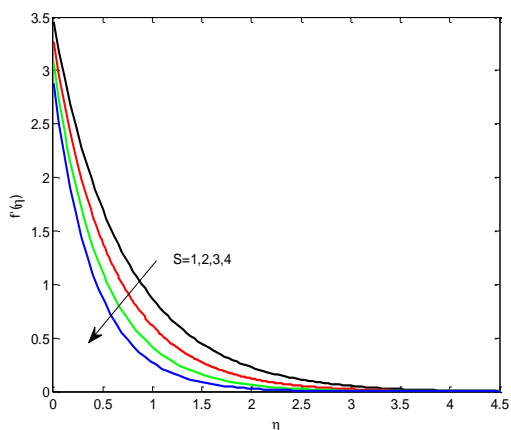
*Fig.5 The effect of  $Ec$  on Temperature profile.*



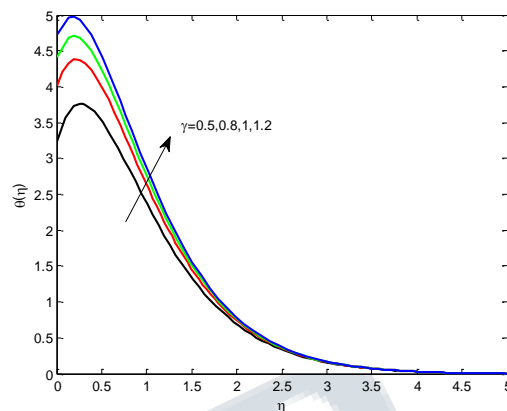
*Fig.3 The effect of  $Gr$  on Velocity profile.*



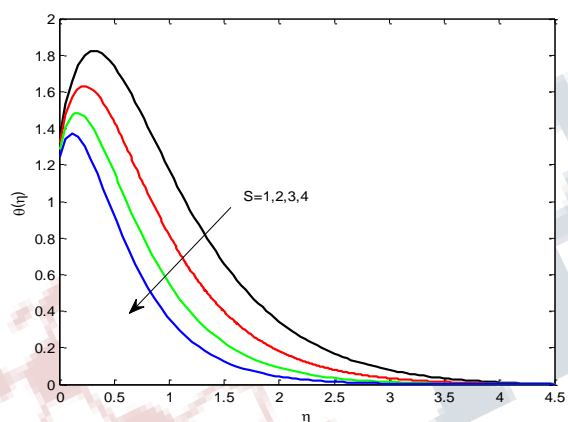
*Fig.6 The effect of  $J$  on Temperature profile.*



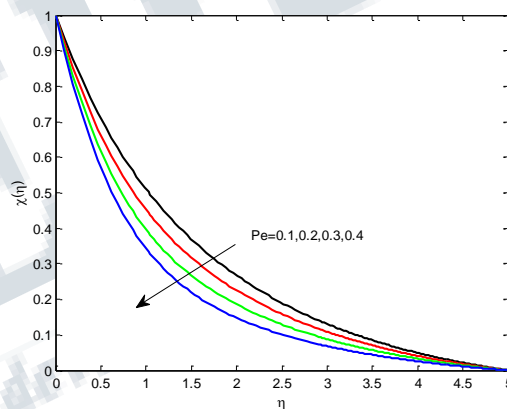
**Fig.7 The effect of  $S$  on Velocity profile.**



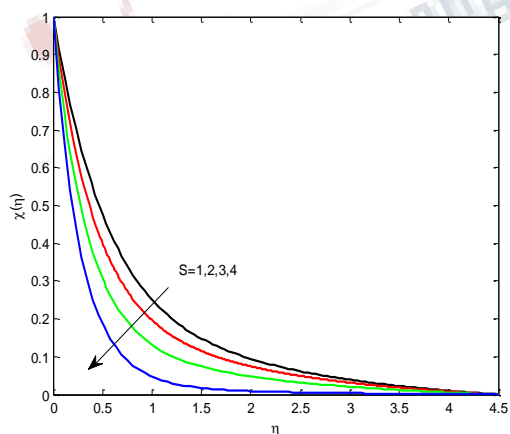
**Fig.10 The effect of  $\gamma$  on temperature profile.**



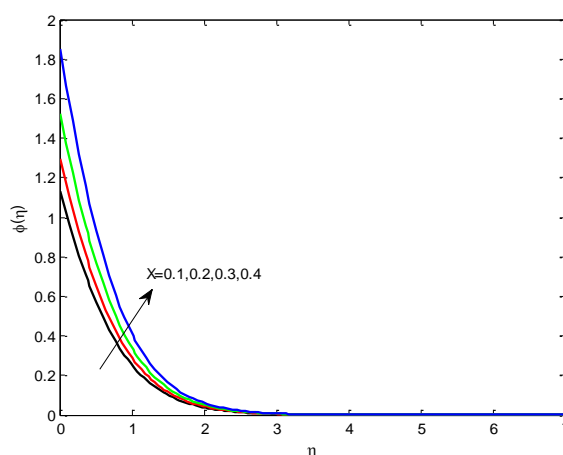
**Fig.8 The effect of  $S$  on Temperature profile.**



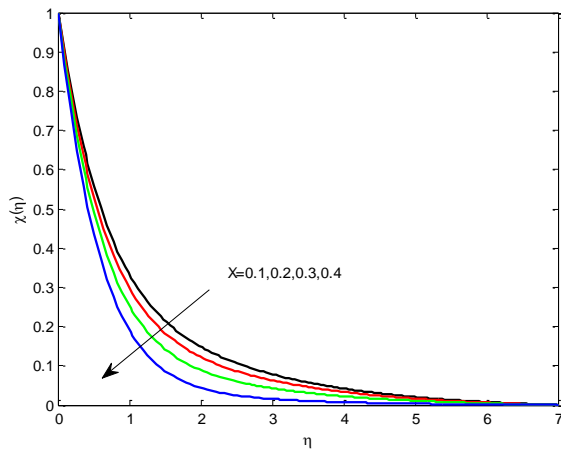
**Fig.11 The effect of  $Pe$  on motile microorganism profile.**



**Fig.9 The effect of  $S$  on motile microorganism profile.**



**Fig.12 The effect of  $X$  on concentration profile.**



**Fig.13 The effect of  $X$  on motile microorganism profile.**

The effect of fluid parameter on velocity and temperature profiles which is presented in displayed in Figs. 1-2. Here it is visualized that the curves decay with a rise in fluid number. In Fig.3, the effect of thermal Grashof number on velocity is presented. As increase in  $Gr$  enhance the velocity distribution. This is due to the buoyancy which is acting on the fluid particles due to gravitational force that enhances the fluid velocity.

Fig. 4 shows the effects of the magnetic parameter  $M$  on the velocity profiles within the boundary layer. It is noticed that the increasing of the magnetic field parameter  $M$  decreases the velocity. In general, applying of a transverse magnetic field normal to the flow direction has a tendency to induce a flow-resistive force in the x-direction. This force tends to slow down the motion of the fluid upwards along the plate. Accordingly, increases the temperature in the boundary layer.

Fig.5 shows the variations of field temperature  $\theta(\eta)$  for different values of Eckert number are noteworthy on temperature profile i.e. when effects of Eckert number are assumed there is noticeable change in temperature is observed. For increasing the values of Eckert number hence it enhances the temperature. The effect of Joule heating parameter ( $J$ ) on velocity and temperature profile is plotted in Fig.6. It is seen that for small values of Joule heating parameter intensifies (improves) the momentum and thermal boundary layer thickness. Generally, the rising values joule heating encourages the diffusion of particles due to this cause we saw improvement in momentum and thermal boundary layers.

Figs. 7-9 represent the suction parameter on velocity, temperature and density of motile organism fields. It is seen

that velocity, temperature and density of motile organism fields decelerates with increasing suction parameter. The suction at the stretching sheet surface has a tendency to reduce the momentum boundary layer thickness. The influence of thermal slip parameter  $\gamma$  on temperature distribution  $\theta(\eta)$  is shown in Fig.10. It can be seen from figure that thermal slip parameter enhances temperature fields.

It is observed from Figs. 11 there exists a solution for different values of Peclet numbers ( $Pe$ ) that evident the microorganism profiles decrease with the increase of Peclet number. The thermal mass slip parameter increases the solutal boundary thickness and decelerates the density of motile organism thickness (Figs. 12-13).

Table 1 shows the variations of skin friction, Nusselt and Sherwood number with modifying values of  $\alpha, J, X, \gamma, \delta, Ec, \beta, Pe, Gr, \Gamma$ . It is clear that the skin friction coefficient is encouraged with boosting values of the inclination angle and Peclet number the rest of the parameters are opposite behaviour respectively. It is interesting to mention that the local Nusselt and Sherwood numbers are decreases with increase in values of  $\alpha, Ec, Pe, \Gamma$ . On the other hand, the local Sherwood numbers improves the Joule heating parameter, inclination angle, thermal slip parameter, thermal Grashof number and chemical reaction parameter while rest of parameters.

### CONCLUSIONS REMARKS

In the present analysis Viscous and Joule's dissipation effects on Bio-convection MHD Casson radiative fluid flow over a stretching sheet with slip condition are performed numerically. Effects of various parameters are studied graphically. The following main conclusions are drawn from the analysis.

- The skin friction coefficient is encouraged with boosting values of the inclination angle and Peclet number.
- The local Nusselt and Sherwood numbers are decreases with increase in values of  $\alpha, Ec, Pe, \Gamma$ .
- The velocity distribution decelerates with increase in casson fluid parameter.
- The Joule's heating improves the temperature distributions.

**Table 1:** Numerical values of skin friction coefficient, Nusselt number and Sherwood numbers with  $\alpha, J, X, \gamma, \delta, Ec, \beta, Pe, Gr, \Gamma$ .

$\alpha$	$J$	$X$	$\gamma$	$\delta$	$Ec$	$\beta$	$Pe$	$Gr$	$\Gamma$	$f''(0)\left(1+\frac{1}{\beta}\right)$	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
$\pi/6$	0.2	0.4	0.1	0.3	0.5	0.1	0.5	0.3	0.5	30.379209	-8.906367	2.788553	1.894815
$\pi/4$										30.609943	-8.961717	2.780368	1.890748
$\pi/3$										30.911633	-9.035105	2.769637	1.885433
	0.5									30.314571	-9.963665	2.791148	1.896106
	1.0									30.206088	-11.73762	2.795487	1.898283
	1.5									30.096622	-13.52634	2.799902	1.900477
		0.2								30.578299	-8.926694	1.787677	1.404680
		0.4								30.379209	-8.906367	2.788553	1.894815
		0.6								29.665085	-8.836731	6.416146	3.682730
			0.1							30.379209	-8.906367	2.788553	1.894815
			0.2							30.306232	-8.195836	2.790883	1.895989
			0.3							30.244120	-7.589778	2.792894	1.896988
				0.3						30.379209	-8.906367	2.788553	1.894815
				0.5						26.527730	-7.089627	2.584112	1.793325
				0.7						23.612817	-5.818411	2.428853	1.716233
					0.5					30.379209	-7.906367	2.788553	1.894815
					1.5					29.278303	-8.723848	2.833462	1.917130
					2.5					28.235576	-9.004496	2.876228	1.938379
						1				10.021361	-2.742273	1.976194	1.490481
						2				8.191201	-2.131426	1.847010	1.425966
						3				7.532224	-1.910771	1.796867	1.400900
							1			30.379216	-8.906439	2.788503	3.267756
							3			30.379217	-8.906483	2.788504	4.811211
							5			30.379218	-8.906492	2.788520	6.378371
								0.2		16.957065	-4.990474	2.340552	1.672057
								0.4		16.756708	-4.903448	2.359735	1.681606
								0.6		16.560856	-4.821082	2.378420	1.690907
									0.2	16.869546	-4.944121	2.063684	1.534843
									0.4	16.860862	-4.945646	2.253010	1.628685
									0.6	16.851552	-4.946988	2.449255	1.725960



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