ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 4, Issue 2, February 2019

A Study on Restrained Triple Connected Two Domination Number of a Graph

[1] Dr. A.Punitha Tharani, [2] A.Robina Tony
[1] Associate Professor, [2] Research Scholar
[1][2] Department of Mathematics, St.Mary's College (Autonomous), Thoothukudi, Tamil Nadu, India

***Abstract:* -- The concept of triple connected graphs with real life application was introduced by considering the existence of a path containing any three vertices of a graph G. Mahadevan et. al., introduced the concept of triple connected domination number of a graph. In this paper, we introduce a new domination parameter, called restrained triple connected two domination number of a graph. A subset S of V of a non – trivial graph G is said to be a restrained triple connected two dominating set, if S is a restrained two dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all restrained triple connected two dominating sets is called the restrained triple connected two domination number of G and is denoted by γ2rtc(G). Any restrained triple connected two dominating set with γ2rtc vertices is called a γ2rtc- set of G. We determine this number for some standard and special graphs and obtain bounds for general graph. Its relationship with other graph theoretical parameters is also investigated.**

***Keywords:* -- Triple connected graphs, restrained triple connected, restrained triple connected two domination**

I. INTRODUCTION

All graphs considered here are finite, undirected without loops and multiple edges. Unless and otherwise stated, the graph G = (V, E) considered here have p = $|V|$ vertices and q = $|E|$ edges.

A subset S of V of a nontrivial graph G is called a *dominating set* of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The *domination number* $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A subset S of V of a nontrivial graph G is called a *restrained dominating set* of G if every vertex in $V - S$ is adjacent to at least one vertex in S as well as another vertex in V - S. The *restrained domination number* $\gamma_r(G)$ of G is the minimum cardinality taken over all restrained dominating sets in G. A subset S of V is said to be two dominating set if every vertex in V – S is adjacent to atleast two vertices in S. The minimum cardinality taken over all two dominating sets is called the two domination number and is denoted by $\gamma_2(G)$. A subset S of V is said to be a *restrained 2-dominating set* of G if every vertex of V - S is adjacent to at least two vertices in S and every vertex of V - S is adjacent to a vertex in V - S. The minimum cardinality taken over all restrained two dominating sets is called the *restrained two domination number* and is denoted by $\gamma_{r2}(G)$.

A subset S of V of a nontrivial graph G is said to be triple connected dominating set, if S is a dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number and is denoted by γ_{tc}. A subset S of V of a nontrivial graph G is said to be *restrained triple connected dominating set*, if S is a restrained dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all restrained triple connected dominating sets is called the *restrained triple connected domination number* and is denoted by γ_{rtc}. A subset S of V of a non – trivial graph G is said to be a restrained triple connected two dominating set, if S is a restrained two dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all restrained triple connected two dominating sets is called the restrained triple connected two domination number of G and is denoted by $\gamma_{2rtc}(G)$. Any restrained triple connected two dominating set with γ_{2rtc} vertices is called a γ_{2rtc}- set of G.

Theorem 1.1: A tree is triple connected iff T ≅ P_p, p ≥ 3.

Theorem 1.2: If the induced subgraph of each connected dominating set of G has more than two pendant vertices, then G does not contain a triple connected dominating set.

Theorem 1.3: For any graph G, $\left\lceil \frac{p}{\Delta + 1} \right\rceil \leq \gamma$ (G)

II. RESTRAINED TRIPLE CONNECTED TWO DOMINATION NUMBER

Definition 2.1: A subset S of V of a non – trivial graph G is said to be a restrained triple connected two dominating set, if S is a restrained two dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all restrained triple connected two dominating sets is called the restrained triple connected two domination number of G and is denoted by $\gamma_{2rtc}(G)$. Any restrained triple connected two dominating set with γ_{2rtc} vertices is called a γ_{2rtc}- set of G.

Example 2.2: For the graph G_1 in Figure 2.1, S = {v_3, v_4, v_6, v_7} forms a γ_{2rtc}- set. Hence $\gamma_{2rtc}(G)$ =4.

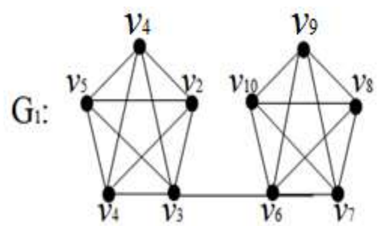

Figure 2.1

Observation 2.3: γ_{2rtc}- set does not exists for all graphs if exists $\gamma_{2rtc}(G) \geq 3$.

Observation 2.4: Every γ_{2rtc}- set is a dominating set but not conversely.

Example 2.5: For the graph G_2, in Figure 2.2 S = {v_1} is a dominating set but not a γ_{2rtc}- set.

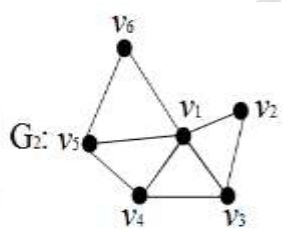

Figure 2.2

Observation 2.6: Every γ_{2rtc}- set is a connected dominating set but not conversely.

Example 2.7: For the graph G_2, in Figure 2.2 S = {v_1, v_2} is a connected dominating set but not a γ_{2rtc}- set.

Observation 2.8: Every γ_{2rtc}- set is a triple connected dominating set but not conversely

Example 2.9: For the graph G_2, in Figure 2.2 S = {v_1, v_5, v_6} is a triple connected dominating set but not a γ_{2rtc}- set.

Observation 2.10: Every γ_{2rtc}- set is a restrained triple connected dominating set but not conversely.

Example 2.11: For the graph G_2, in Figure 2.2 S = {v_1, v_5, v_6} is a triple connected dominating set but not a γ_{2rtc}- set.

Observation 2.12: The complement of the γ_{2rtc}- set need not be a γ_{2rtc}- set.

Example 2.13: For the graph G_2, in Figure 2.2 S = {v_1, v_2, v_5, v_6} is a triple connected dominating set but the complement V-S = {v_3, v_4} is not a γ_{2rtc}- set.

Theorem 2.14: For any connected graph G, $\gamma_c(G) \leq \gamma_{tc}(G) \leq \gamma_{2rtc}(G)$.

2.15 Exact value for some standard graphs:

i. For any path of order $p \geq 3$, $\gamma_{2rtc}(P_p) = p$
ii. For any cycle of order $p \geq 3$, $\gamma_{2rtc}(C_p) = p$
iii. For the complete graph of order $p \geq 3$, $\gamma_{2rtc}(K_p)$ $= \begin{cases} p, & p = 3,4 \\ 3, & p \geq 5 \end{cases}$
iv. For the complete bipartite graph $K_{m,n}$,

$$\gamma_{2rtc}(K_{m,n}) = \begin{cases} m+n, & m \text{ or } n = 2 \\ & m \text{ or } n \geq 2 \\ 4, & m \text{ or } n \geq 3 \text{ and} \\ & m \text{ or } n \geq 3 \end{cases}$$

Theorem 2.16: If the induced subgraph of each connected dominating set of G has more than two pendant vertices, then G does not contain a restrained triple connected two dominating set.

The proof follows from Theorem 1.2

Theorem 2.17: For any connected graph G with $p \geq 3$ we have $3 \leq \gamma_{2rtc} \leq p$ and the bounds are sharp.

Proof: The lower bound follows from the definition of restrained triple connected two dominating set and the upper bound is obvious. For K_p the equality of the lower bound is attained and for C_p and P_p the equality of the upper bound is attained.

Theorem 2.18: For any connected graph G with five vertices $\gamma_{2rtc}(G) = p - 2$ iff G is isomorphic to any of the following graphs, K_5, K_4 (1), K_4 (2), K_4 (3), C_4 (3), C_4 (4), $K_4 - e$ (3).

Proof: If G is isomorphic to K_5, K_4 (1), K_4 (2), K_4 (3), C_4 (3), C_4 (4) and $K_4 - e$ (3) then it can be verified that $\gamma_{2rtc}(G) = p - 2$. Conversely let G be a connected graph with five vertices and $\gamma_{2rtc}(G) = 3$. Let S = {v_1, v_2, v_3} be the γ_{2rtc} – set of G. Take V – S = {v_4, v_5} and hence < V – S> = K_2. Also <S> = P_3 or C_3.

Case (i) <S> = P_3 and < V – S> = K_2

Let v_1, v_2, v_3 be the vertices of P_3 and v_4, v_5 be the vertices of K_2. Since G is connected v_1 (or equivalently v_3) is adjacent to v_4 (or) v_2 is adjacent to v_4 (or equivalently v_5). If v_1 is adjacent to v_4 and $d(v_4) = 3$ then we can find new graphs by increasing the degrees of v_5. If $d(v_4) = 3$, then v_4 is adjacent to (v_1 and v_2) or (v_1 and v_3) or (v_2 and v_3).If v_4 is adjacent to v_1 and v_2 then we can find new graphs by increasing the degrees of v_5 and we observe that G is isomorphic to K_4 (1), K_4 (2), C_4 (3), C_4 (4). If $d(v_4) = 4$, then v_4 is adjacent to v_1,v_2 and v_3. We can find new graphs by increasing the degrees of v_5 and we observe that G is isomorphic to K_4 (2), $K_4 - e$ (3), K_4 (3). Suppose v_2 is adjacent to v_4 then we can find new graphs by increasing the degrees of v_5. We observe that G is isomorphic to K_4 (1), C_4 (3), K_4 (2), K_4 (1) and K_4 (2), $K_4 - e$ (3).and K_4 (3).

Case (ii) $<S> = C_3$ and $<V - S> = K_2$.

Let v_1, v_2, v_3 be the vertices of C_3 and v_4, v_5 be the vertices of K_2. Since G is connected v_4 or v_5 is adjacent to C_3. If $d(v_4) = 3$, then v_4 is adjacent to (v_1 and v_2) or (v_1 and v_3) or (v_2 and v_3). If v_4 is adjacent to v_1 and v_2 then we can find new graphs by increasing the degrees of v_5. We observe that G is isomorphic to K_4 (2), C_4 (4) or $K_4 - e$ (3) and K_4 (3), $K_4 - e$ (3), K_4 (2) and K_4 (3). If $d(v_4) = 4$, then v_4 is adjacent to v_1,v_2 and v_3. By increasing the degree of v_5 G is isomorphic to K_4 (3) and K_5.

Theorem 2.19: Let G be a graph such that G and $\bar{G}$ have no isolates of order $p \geq 3$.Then

i. $\gamma_{2rtc}(G) + \gamma_{2rtc}(\bar{G}) \leq 2p$

ii. $\gamma_{2rtc}(G) \cdot \gamma_{2rtc}(\bar{G}) \leq p^2$ and the bound is sharp

Proof: The bound directly follows from Theorem 2.17. For cycle C_p and path P_p equality of both the bounds are attained.

Relation with other graph parameters:

Theorem 2.20: For any connected graph G, γ_{2rtc} (G) + χ (G) $\leq$ 2p and the inequality holds if and only if G is isomorphic to K_3 or K_4 or C_3.

Proof: It is clear that, γ_{2rtc} (G) $\leq$ p and χ (G) $\leq$ p. Thus, γ_{2rtc} (G) + χ (G) $\leq p + p = 2p$. Suppose G is isomorphic to K_3 or K_4. Then clearly $\gamma_{2rtc}(G) + \chi(G) = 2p$. Conversely, let γ_{2rtc} (G) + χ (G) = 2p, the only possible case is γ_{2rtc} (G) = p and χ (G) = p. If χ (G) = p then G is isomorphic to K_p. In K_p, γ_{2rtc} (G) = 3, $p \neq 4$ and $\gamma_{2rtc}(K_4) = 4$, so that G is isomorphic to K_3 or K_4. Also if G is isomorphic to C_3, then γ_{2rtc} (G) = p and χ (G) = p is possible.

Theorem 2.21: For any connected graph G $\gamma_{2rtc}(G) + \kappa(G) \leq 2p - 1$ and the equality holds if and only if G is isomorphic to K_3 or K_4.

Proof: It is clear that $\gamma_{2rtc}(G) \leq p$ and $\kappa(G) \leq p - 1$. Thus, $\gamma_{2rtc}(G) + \kappa(G) \leq p + p - 1 = 2p - 1$. Suppose G is isomorphic to K_3 or K_4. Then clearly $\gamma_{2rtc}(G) + \kappa(G) = 2p - 1$. Conversely, let $\gamma_{2rtc}(G) + \kappa(G) = 2p - 1$, then the only possible case is $\gamma_{2rtc}(G) = p$ and $\kappa(G) = p - 1$. Since $\kappa(G) = p - 1$, G is a complete graph. In K_p, $\gamma_{2rtc}(G) = 3$, $p \neq 4$ and $\gamma_{2rtc}(K_4) = 4$. Hence G is isomorphic to K_3 or K_4.

Theorem 2.22: For any connected graph G with $p \geq 3$ vertices, $\gamma_{2rtc}(G) + \Delta(G) \leq 2p - 1$ and the bound is sharp.

Proof: Let G be a connected graph with $p \geq 3$ vertices. We know that, $\Delta(G) \leq p - 1$ and by Theorem 2.17 $\gamma_{2rtc}(G) \leq p$. Hence $\gamma_{2rtc}(G) + \Delta(G) \leq 2p - 1$. For K_3 and K_4 the bound is sharp.

.

REFERENCES

1. John Adrian Bondy, Murty U.S.R. (2009): Graph Theory, Springer, 2008.
2. J. Jagan Mohana , Indrani Kelkarb, Restrained 2-Domination Number of Complete Grid Graphs, International Journal of Applied Mathematics and Computation Journal, ISSN: 0974 - 4665 (Print) 0974 - 4673 (Online) Volume 4(4) 2012 352–358.
3. V.R.Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
4. V.R.Kulli, Theory of Domination in Graphs, Vishwa International Publications, Gulbarga, India (2010).
5. V.R.Kulli and S.C.Sigarakanti, The connected edge domination number of a graph, Technical Report 8801, Dept Math. Gulbarga University, (1988).
6. G. Mahadevan, A. Selvam, J. Paulraj Joseph and, T. Subramanian, Triple connected domination number of a graph, International Journal of Mathematical Combinatorics, Vol.3 (2012), 93 - 104.
7. G. Mahadevan, A. Selvam, V. G. Bhagavathi Ammal and, T. Subramanian, Restrained triple connected domination number of a graph, International Journal of Engineering Research and Application, Vol. 2, Issue 6 (2012), 225- 229.
8. S. L. Mitchell and S. T. Hedetniemi, Edge domination in trees. Congr. Numer.19 (1997) 489- 509.
9. Paulraj Joseph J., Angel Jebitha M.K., Chithra Devi P. and Sudhana G. (2012): Triple connected graphs, Indian Journal of Mathematics and Mathematical Sciences, ISSN 0973-3329 Vol. 8, No. 1: 61-75.
10. Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater (1998): Fundamentals of domination in graphs, Marcel Dekker, New York.