ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 3, Issue 3, March 2018

Viscous Dissipation Effects on Unsteady MHD Parabolic Flow past an Infinite Isothermal Vertical Plate In The Presence Of Radiation and Chemical Reaction

[1] B. Prabhakar Reddy, [2] Jason M. Mkenyeleye
[1] Department of Mathematics, College of Natural and Mathematical Sciences,
The University of Dodoma, P. Box No. 338, Dodoma, Tanzania
[1][2] Department of Mathematics, Geethanjali College of Engineering &Technology,
Cheeryal (V), Keesara (M), Medchal (Dist) – 501301, Telangana, India

Abstract- **The effects of viscous dissipation on unsteady MHD free convection flow with a parabolic starting motion of an infinite isothermal vertical porous plate in the presence of radiation and chemical reaction has been presented. The fluid is considered a grey, absorbing emitting radiation but a non-scattering medium. The dimensionless governing equations for this investigation are solved numerically by applying finite element method. The numerical results for the velocity field, temperature fields and concentration field are presented through graphs for different values of the physical parameters involved. We found that there is a fall in the temperature and velocity in the boundary layer as the radiation parameter increased. The velocity field increases with increase in the thermal and mass Grashof numbers and decreases with increase in the magnetic parameter. Further, the concentration and velocity fields decrease with increasing Schmidt number and chemical reaction parameter.**

***Key words:* - MHD, isothermal vertical plate, magnetic field, Radiation parameter, chemical reaction parameter.**

I. INTRODUCTION

The study of magneto-hydrodynamics (MHD) with heat and mass transfer in the presence of radiation has attracted the attention of many researchers during the past decades due to its varied and wide applications. In astrophysics and geophysics, it is functional to study the stellar structures, radio propagation through the ionosphere, etc. Radiative flow are encountered in many industrial and environmental processes e.g., heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry etc. Muthucumaraswamy and Janakiraman [1] analyzed MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Shanker and Gnaneshwar [2] investigated the effects of radiation on MHD flow past an impulsively started infinite vertical plate through a porous medium with variable temperature and mass diffusion. Thermal radiation effects on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate was investigated by Ahmed and Sarmah [3]. The effects of thermal radiation on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity was presented by Mahmoud [4]. Mukhopadhyay [5] analyzed the effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. Sharma and Deka [6] presented the thermal radiation and oscillating plate temperature effects on unsteady MHD flow past a semi-infinite vertical porous plate in the presence of chemical reaction. The effect of radiation on linearly accelerated plate with variable mass diffusion in the presence of magnetic field was investigated by Muralidharan and Muthucumaraswamy [7].

The growing need for chemical reaction in chemical and hydrometallurgical industries require the study of heat and mass transfer in the presence of chemical reaction. There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of the chemical reaction. These processes are observed in nuclear reactor safety and combustion systems, solar collectors as well as metallurgical and chemical engineering. Muthucumaraswamy and Ganesan [8] analyzed the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation /absorption and a chemical reaction was presented by Chamkha [9]. The effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection was investigated by Kandasamy *et. al* [10]. Ibrahim *et. al* [11] presented the effects of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Muthucumaraswamy and Janakiraman [12] investigated mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction. Mahapatra *et. al* [13] presented the effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Unsteady MHD double-diffusive convection boundary layer flow past a radiative hot vertical surface in a porous media in the presence of chemical reaction and heat sink was reported by Mohamed *et. al* [14]. MHD free convection parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction was reported by Muthucumaraswamy and Sivakaumar [15].

The dissipation of energy is significant when considering unsteady magneto-hydrodynamic natural convection flows. Gebhart [16] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal vertical plate with constant heat flux. Gebhart and Mollendorf [17] considered the effects of viscous dissipation for external natural convection flow over a surface. The viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate was reported by Soundalgekar [18]. Gupta *et. al* [19] presented free convection effects on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method. The computational analysis of coupled radiation-convection dissipation non-gray gas flow in a non-Darcy porous medium with the Keller-Box implicit difference scheme was reported by Takhar [20]. Zueco [21] investigated radiation and viscous dissipation effects on unsteady MHD free convection flow over a vertical porous plate by Network simulation method. Recently, Reddy [22] studied mass transfer effects on an unsteady MHD free convective flow of an incompressible viscous dissipative fluid past an infinite vertical porous plate by finite element method.

The aim of this paper is to investigate the viscous dissipation effects on unsteady MHD free convection with a parabolic starting motion of an infinite isothermal vertical porous plate taking into account viscous dissipation effect. The problem is governed by the system of coupled non-linear partial differential equations, whose exact solutions are difficult to obtain, if possible. So, Ritz finite element method has been adopted for its solution, which is more economical from computational point of view. The influence of material parameters encountered in the problem under the investigation on the flow has been presented through graphs and then discussed.

II. BASIC EQUATIONS

We consider a two-dimensional unsteady MHD flow of viscous incompressible fluid with a parabolic started motion of an infinite isothermal vertical porous plate in the presence of radiation and chemical reaction of first order. The $x'-$axis is taken along the plate in the upward direction and $y'-$axis is taken normal to the plane of the plate in the fluid. A magnetic field of strength B_0 applied transversely to the direction of the flow. The fluid is considered a gray, absorbing emitting radiation but a non-scattering medium. At time $t' \leq 0$, the plate and fluid are at same temperature $T_\infty^{'}$ and concentration level $C_\infty^{'}$ at all points. At time $t' > 0$, the plate started with velocity $u' = u_0 t'^2$ in its own plane against the gravitational field and the temperature from the plate rises to $T_w^{'}$ and the concentration level near the plate rises to $C_w^{'}$. Since the plate is of infinite extent along x and z directions, so all physical quantities depends on y' and t' only. Under the usual Boussinesq's and boundary layer approximations, the unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T_\infty^{'}) + g\beta^*(C' - C_\infty^{'}) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu u'}{K'} - \frac{\sigma B_0^2}{\rho}u' \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - k_l(C' - C_\infty^{'}) \quad (3)$$

u' is the velocity components in the x' direction, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of concentration expansion, t' is the time, $T^{'}$ is the temperature of the fluid, T'_∞ is the temperature of the fluid far away from the plate, T'_w is the temperature at the plate, C' is the species concentration in the fluid, C'_∞ is the concentration in the fluid far away from the plate, C'_w is the species concentration at the plate, k is the thermal conductivity, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity, c_p is the specific heat at constant pressure, D is the mass diffusion.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} &t'\le 0; && u'=0, T'=T'_\infty, C'=C'_\infty && \text{for all } y'=0\\ &t'>0; && u'=u_0 t'^2, T'=T'_\infty, C'=C'_\infty && \text{at } y'=0\\ & && u'\to 0, T'\to T'_\infty, C'\to C'_\infty && \text{at } y'\to\infty \end{aligned} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed as:

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma(T'_\infty - T'^4) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 can be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, one obtain

$$T'^4 \cong 4T'_\infty T' - 3T'^4_\infty \tag{6}$$

Using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - 16a^*\sigma T'^3_\infty (T' - T'_\infty) + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{7}$$

Let us introduce the following non-dimensional quantities.

$$u = u'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, y = y'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, t = t'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, P_r = \frac{\mu c_p}{k},$$

$$S_c = \frac{\nu}{D}, M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}}, K = K'\left(\frac{u_0}{\nu^2}\right)^{\frac{2}{3}}, \delta = k_l\left(\frac{\nu}{u_o}\right)^{\frac{1}{3}},$$

$$\theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \phi = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)}, R = \frac{16a^*\sigma T'^3_\infty}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}},$$

$$E_c = \frac{\nu}{C_p(T'_w - T'_\infty)}, G_r = \frac{g\beta(T'_w - T'_\infty)}{(\nu u_0)^{1/3}}, H = M + \frac{1}{K},$$

$$G_m = \frac{g\beta^*(C'_w - C'_\infty)}{(\nu u_0)^{1/3}}.$$

The governing system of equations (1), (3), (4) and (7) with the use of above non-dimensional quantities is reduced to the following non-dimensional form:

$$\frac{\partial u}{\partial t} = G_r\theta + G_m\phi + \frac{\partial^2 u}{\partial y^2} - Hu \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} - \frac{R}{P_r}\theta + E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2} - \delta\phi \tag{10}$$

The corresponding initial and boundary conditions in non-dimensional form are:

$$\begin{aligned} &t\le 0; && u=0, \theta=0, \phi=0 && \text{for all } y\\ &t>0; && u=t^2, \theta=1, \phi=1 && \text{at } y=0\\ & && u\to 0, \theta\to 0, \phi\to 0 && \text{as } y\to\infty \end{aligned} \tag{11}$$

III. SOLUTIONS OF THE EQUATIONS

Equations $(8)-(10)$ are non-linear system of partial differential equations are to be solved under subject to the boundary conditions given in equation (11). However, whose exact or approximate solutions are difficult to obtain, whenever possible. So that, the Ritz finite element method has been adopted for its solution, which is more economical from a computational point of view. The algorithm for Ritz finite element method can be summarized by the following steps.

1. Division of the whole domain into smaller elements of finite dimensions called “finite elements”.
2. Generation of the element equations using variational formulations.
3. Assembly of element equations as obtained in step (2).
4. Imposition of boundary conditions to the equations obtained in step (3).
5. Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration method. The numerical solutions for the velocity profiles (u), temperature profiles (θ) and concentration profiles (C) are computed by using $C-$ program. To prove the convergence and stability of the Ritz finite element method, the same program was run making with small changes in t and $y-$directions. For these slightly changed values, no significant change was observed in the values of velocity, temperature and concentration profiles. Hence, we conclude that the Ritz finite element method is convergent and stable.

IV. NUMERICAL RESULTS AND DISCUSSION

In order to get the physical insight of the problem, we have computed numerical results for the velocity profiles, the temperature profiles and the concentration profiles and presented them through graphs. The obtained results have been discussed by assigning various values to the material parameters encountered in the problem under the investigation. The value of P_r is taken to be 0.71, which corresponds to air and the value of S_c is chosen to be 0.22, which represents hydrogen at 25^0C and one atmosphere pressure. During the numerical computations of the results, the values of the other material parameters are considered as $G_r = 5.0$, $G_m = 5.0$, $\lambda = 1.0$, $\delta = 0.5$, $E_c = 0.1$, $M = 0.5$, $K = 1.0$ and $t = 0.2$.

Velocity profiles: The effects of the radiation parameter λ on the velocity profiles are shown in Fig. 1. It is observed that the fluid velocity decreases with increasing values of radiation parameter. The velocity of the fluid increases quickly near the plate and then decreases gradually to zero as $y \to \infty$. Figure 2 shows the effects of Eckert number E_c on the velocity profiles. Here, the positive values of Eckert number indicates cooling of the plate i.e., loss of the heat to the fluid from the plate. It is seen that an increase in the Eckert number causes a rise in the fluid velocity. Figure 3 depicts the effects of Schmidt number S_c on the velocity profiles for $S_c = 0.22, 0.60$ and 0.78, corresponds to hydrogen, water-vapour and ammonia respectively. It is observed that an increase in the Schmidt number leads to decrease in the velocity of the fluid. Figure 4 presents the effects of the chemical reaction parameter δ on the velocity profiles. It can be seen that the fluid velocity decreases with increasing chemical reaction parameter. Figure 5 illustrates the effects of magnetic parameter M on the velocity profiles. The velocity curves show that the rate of transport is remarkably reduced with increasing values of magnetic parameter. The variation of the velocity profiles with dimensionless permeability parameter K is presented in Fig. 6. This figure clearly indicates that the fluid velocity increases with increasing values of permeability parameter. Physically, this result can be achieved when the holes of the porous medium are very large so that the resistance of the medium may be neglected. Figure 7 shows the effect of thermal Grashof number G_r on the velocity profiles. Here, the positive values of thermal Grashof number correspond to cooling of the plate. It is observed that an increase in the value of thermal Grashof number leads to increase in the fluid velocity. The effects of mass Grashof number G_m on the velocity profiles are presented in Fig.8. As seen from this figure, the effects of G_m on the fluid velocity is same as that of G_r. this fact is achieved by comparing Figs. 7 and 8. Figure 9 plotted to show the effects of time parameter t on the velocity profiles. It is observed that an increase in the value of time parameter t increase the fluid velocity.

Temperature profiles: Figure 10 represents the effects of the radiation parameter λ on the temperature profiles. It is seen from this figure that there is a fall in the temperature profiles as the radiation parameter increased. The effects of the Eckert number E_c on the temperature profiles are presented in Fig. 11. It is observed that an increase in the Eckert number leads to increase in the temperature profiles.

Concentration profiles: Figure 12 plotted to show the effects of Schmidt number S_c on the concentration profiles for different realistic values of $S_c = 0.22, 0.60$ and 0.78 that are physically corresponds to hydrogen, water-vapour and ammonia, respectively. It is observed that the concentration decreases with increasing Schmidt number. The influence of chemical reaction parameter δ on the concentration profiles is shown in Fig. 13. It is noticed from this figure that there is a marked effect of increasing values of chemical reaction parameter on the concentration distribution in the boundary layer. It is clear that the increasing values of δ decreases the concentration of species in the boundary layer.

FIGURES

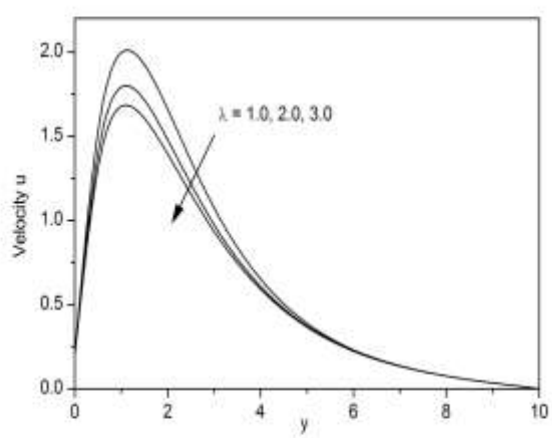

Fig. 1: Effect of radiation parameter (λ) ***on the velocity profiles.***

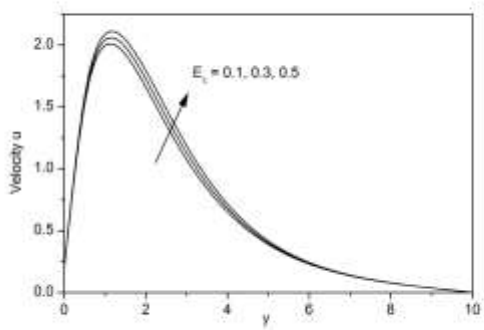

Fig. 2: Effect of Eckert number (E_c) *on the velocity profiles.*

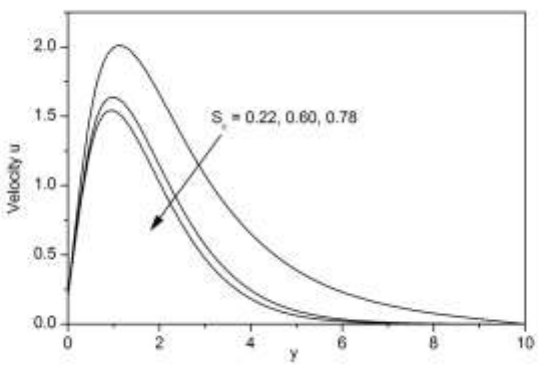

Fig. 3: Effect of Schmidt number (S_c) *on the velocity profiles.*

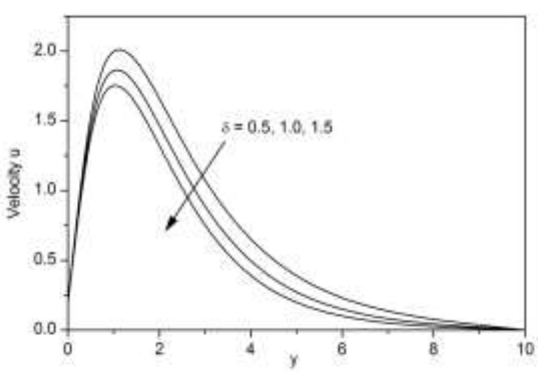

Fig. 4: Effect of chemical reaction parameter (δ) *on the velocity profiles.*

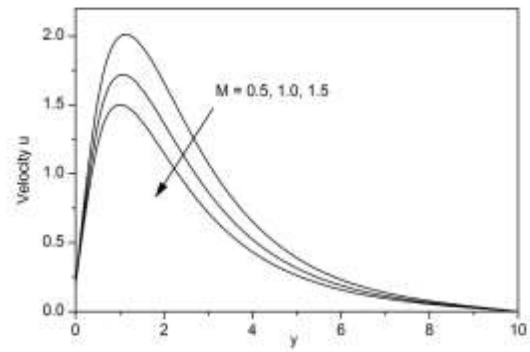

Fig. 5: Effect of magnetic parameter (M) *on the velocity profiles.*

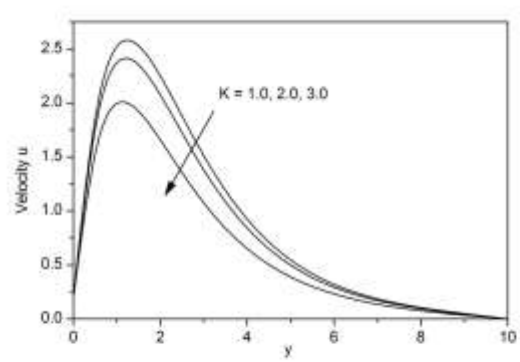

Fig. 6: Effect of permeability parameter (K) *on the velocity profiles.*

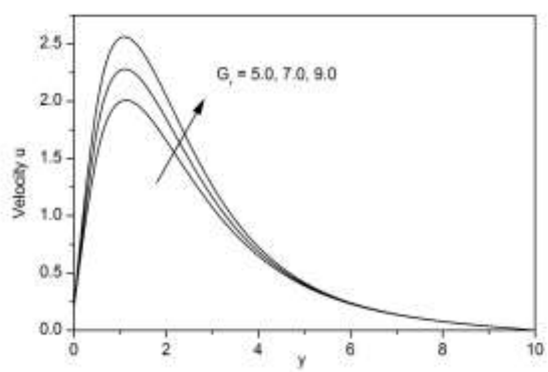

Fig. 7: Effect of thermal Grashof number (G_r) *on the velocity profiles.*

International Journal of Science, Engineering and Management (IJSEM)
Vol 3, Issue 3, March 2018

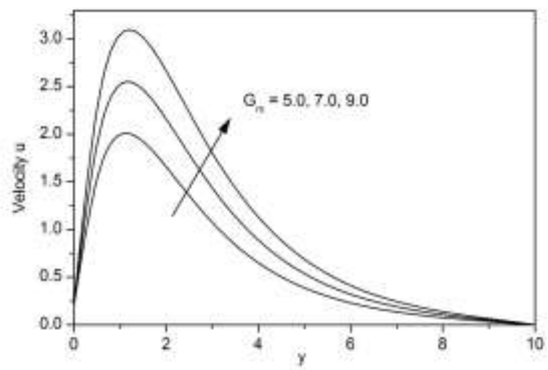

Fig. 8: Effect of mass Grashof number (G_m) ***on the velocity profiles.***

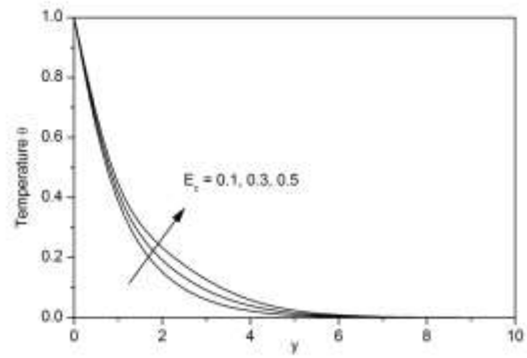

Fig. 11: Effect of Eckert number (E_c) ***on the temperature profiles.***

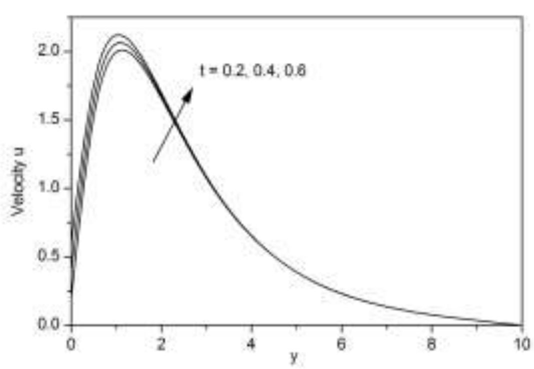

Fig. 9: Effect of time parameter (t) ***on the velocity profiles.***

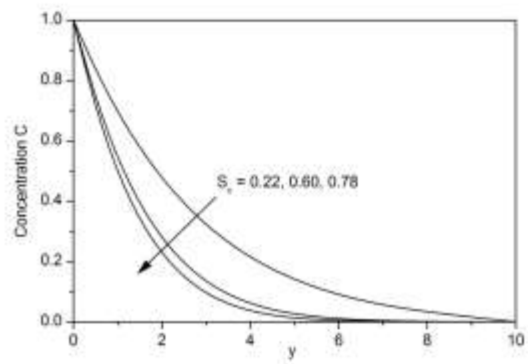

Fig. 12: Effect of Schmidt number (S_c) ***on the concentration profiles.***

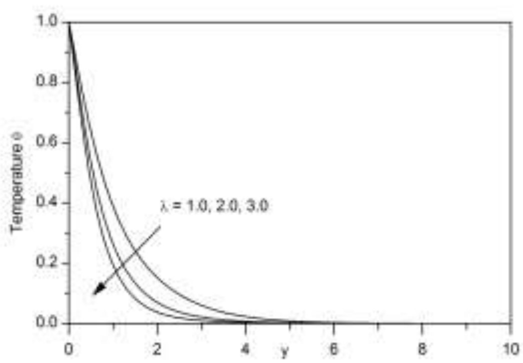

Fig. 10: Effect of radiation parameter (λ) ***on the temperature profiles.***

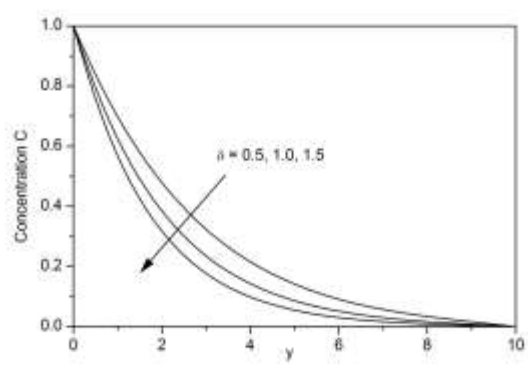

Fig. 13: Effect of chemical reaction parameter (δ) ***on the concentration profiles.***

V. CONCLUSIONS

In this work, the problem of unsteady MHD free convection with a parabolic starting motion of an infinite isothermal vertical porous plate in the presence of radiation and chemical reaction is provided. The governing equations of the problem solved numerically by applying Ritz finite element method. We conclude that

- The velocity of the fluid increases with increasing Eckert number or permeability parameter or thermal Grashof number or mass Grashof number or time parameter.
- An increase in the radiation parameter or Schmidt number or chemical reaction parameter or magnetic parameter decreases the fluid velocity
- The fluid temperature decreases with increasing radiation parameter and increases with increasing Eckert number.
- The fluid concentration decreases with increasing values of Schmidt number and chemical reaction parameter.

REFERENCES

[1] R. Muthucumaraswamy and b. Janakiraman B, “MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion” Theoret. Appl. Mech, vol. 33, pp.17 – 29, (2006).

[2] R. t. Shanker and R. M. Gnaneshwar, “Radiation effects on MHD flow past an impulsively started infinite vertical plate through a porous medium with variable temperature and mass diffusion’ J. Pure and Appl. Phys, vol. 19, No. 3, pp.191 – 200, (2007).

[3] N. Ahmed and H. K. Sarmah, “Thermal radiation effects on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate” Int. J. Appl. Math and Mech, vol. 5, No. 5, pp.87 – 98, (2009).

[4] M. A. A. Mahmoud, “Thermal radiation effects on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity” Canadian J. Chem. Engg, vol. 87, pp.47 – 52, (2009).

[5] S. Mukhopadhyay, “The effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium” Int. J. heat and Mass Transfer, 52, pp.3261 – 3265, (2009).

[6] S. Sharma and R. K. Deka, “Thermal radiation and oscillating plate temperature effects on MHD unsteady flow past a semi-infinite porous vertical plate under and chemical reaction” Int. J. Phys and Math. Sci, vol.2 No.2, pp. 33 – 52, (2012).

[7] M. Muralidharan and R. Muthucumaraswamy, “Radiation effects on linearly accelerated plate with variable mass diffusion in the presence of magnetic field” Applied Mathematical Science, vol.7, No.113, pp.5645 – 5656, (2013).

[8] R. Muthucumaraswamy and P. Ganesan, “Effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate” J. Appl. Mech. Tech. Phys, vol.42, No.4, pp.665 – 671, (2001).

[9] J. Chamkha, “MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction” Int. Commun. Heat and Mass Transfer, vol. 30, No.3, pp. 413 – 422, (2003).

[10] R. Kandasamy, K. Periasamy and K. K. Prabhu, “Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection” Int. J. Heat and Mass Transfer, vol. 48, No.7, pp.1388 – 1394, (2005).

[11] F. S. Ibrahim, A. M. Elaiw and A. A. Bakr, “Effects of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction” Commun. in Non-Linear Sci. Numerical Simulation, vol.13, No.6, pp.1056 – 1066, (2008).

[12] R. Muthucumaraswamy and B. Janakiraman, “Mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction” Int. J. Appl. Math and Mech, vol.4, No.1, pp. 66 – 74, (2008).

[13] N. Mahapatra, G. C. Dash, S. Panda and M. Acharya, “Effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface” J. Eng. Phys and Thermo-Phys, vol.83, No.1, pp. 130 – 140, (2010).

[14] R. A. Mohamed, N. A. Osman and S. M. Abo-Dahadb, "Unsteady MHD double-diffusive convection boundary layer flow past a radiative hot vertical surface in a porous media in the presence of chemical reaction and heat sink" Meccanica, vol.48, pp.931 – 941, (2013).

[15] R. Muthucumaraswamy and P. Sivakaumar, "MHD flow past a parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction" Int. J. Appl. Mech. and Engg, vol.21, No.1, pp. 95 – 105, (2016).

[16] B. Gebhart, "Effects of viscous dissipation in natural convection" Journal of Fluid Mechanics, vol.14, pp.225-232, (1962).

[17] B. Gebhart and J. Mollendorf, "Viscous dissipation in external natural flows" Journal of Fluid Mechanics, vol.38, pp.79-107, (1969).

[18] V. M. Soundalgekar, "Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction" Int. J. Heat and Mass Transfer, vol.15, pp. 1253-1261, (1972).

[19] S. Gupta, I. Pop and V. M. Soundalgekar, "Free convection effects of the flow past an accelerated vertical plate in an incompressible dissipative fluid" Rev. Rom. Sci. Techn-Mec. Apl, vol.24, pp. 561 – 568, (1979).

[20] H. S. Takhar, O. A. Beg and M. Kumari, "Computational analysis of coupled radiation-convection dissipation non-gray gas flow in a non-Darcy porous medium using the Keller-Box implicit difference scheme" Int. J. Energy Res, vol.22, pp. 141-159, (1998).

[21] J. Zueco Jordan, "Network simulation method applied to radiation and viscous dissipation effects on MHD unsteady free convection over vertical porous plate" Appl. Mathematical Modeling, vol.31, No.20, pp.2019-2033, (2007).

[22] B. Prabhakar Reddy, "Mass transfer effects on an unsteady MHD free convective flow of an incompressible viscous dissipative fluid past an infinite vertical porous plate" Int. J. Appl. Mech. and Eng, vol.21, No.1, pp.143 – 153, (2016).

