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A modest perspective on equivalent solutions of some integral calculus problems

[1] SelinaV.P, [2] Jacob J, [3] Linty Antony
[1][2][3] A.P-S&H, IES College of Engineering
[1] vpselin1984@gmail.com, [2] jjacobsep10@gmail.com, [3] lintyies@gmail.com

***Abstract*—This paper presents a gentle approach to demonstrate that equivalent solutions exist for some specific integral Calculus problems. In order to prove the idea, an experimental study was conducted among 40 students, including both boys and girls, from higher secondary section. The 40 students were randomly selected and divided into 4 groups of 10, each under the guidance of a subject expert. The 4 groups were named as A, B, C and D. A common problem on integration was given to each group. The 4 groups were suggested to use any method of their choice to solve the problem. The solutions to the given integral calculus problem, the students came up with, were different from each other. In order to draw the conclusion on equivalent solutions, each of the results obtained by the individual group was evaluated on a specified interval to obtain the numerical value. Graphical methods were also used to verify whether the solutions from each group are equivalent or not. The numerical value and the graphical representation of the solutions had shown that the results obtained by the respective groups are equivalent. This shows the existence of equivalent solutions to some problems on integral calculus.**

***Index Terms*—Integral Calculus, Graphical Representation of solutions, Equivalent Solutions, Methods of Integration**

1. INTRODUCTION

Mathematics is passionately regarded as the heart of science and technology. It is also described as the mirror of civilization in all the centuries of painstaking calculation. Mathematics is the most basic discipline for any person who would be truly educated in any science and in many other endeavours. Calculus is a branch of mathematics with a wide range of applications in physical sciences and engineering. This study is restricted to integral calculus focusing on determining an equivalent set of solutions to a given integrable function over specified intervals.

2. FOCUS OF THE PAPER

This paper earnestly presents a simple approach to determine if there is an equivalent solution set or difference in the integral of a given function.

PRELIMINARIES

1. Any function F such that $\mathbf{F'(x) = f(x)}$ is called primitive, anti derivative, or indefinite integral of f(x).
2. ***Fundamental Theorem Of Calculus**:* Let f(x) be a differentiable function everywhere on [a,b], then $\int_a^b \mathbf{f(x)dx = F(b) - F(a)}$.
3. Any two anti derivatives F and G of f(x) differ at most by a constant that is, F(x)-G(x) = C.

SOME AXIOMS OF DEFINITE INTEGRALS

Let f(x) and g(x) be arbitrary integrable functions in an interval [a,b], then

1. $\int_a^b \mathbf{f(x) \pm g(x)dx} = \int_a^b \mathbf{f(x)dx} \pm \int_a^b \mathbf{g(x)dx}$.
2. $\int_a^b \mathbf{f(x)dx} = \int_a^c \mathbf{f(x)dx} + \int_c^b \mathbf{f(x)dx}$, such that f(x) is integrable in [a,c] and [c,b].
3. $\int_a^a \mathbf{f(x)dx = 0}$.
4. $\int_a^b \mathbf{f(x)dx} = \int_a^b \mathbf{f(t)dt}$ if x= t.
5. $\int_a^b \mathbf{f(x)dx} = -\int_b^a \mathbf{f(x)dx}$.
6. $\int_a^b \mathbf{f(x)dx} = \int_a^b \mathbf{f(a+b-x)dx}$.
7. $\int_0^a \mathbf{f(x)dx} = \int_0^a \mathbf{f(a-x)dx}$.
8. $\int_{-a}^a \mathbf{f(x)dx} = \begin{cases} \mathbf{2\int_0^a f(x)dx}, & \textbf{if f(x) is even} \\ \mathbf{0}, & \textbf{if f(x) is odd} \end{cases}$

For even function f(-x)=f(x) and for odd function f(-x) = -f(x).

3. METHODOLOGY

An experimental study was carried out among forty students from higher secondary section. The students were grouped into four of 10 students each. Some problems on integration which has to do with indefinite integrals were given to students to solve as a group work. The students in their respective group were subjected to the same problems on integration where they were free to use any method of their choice. The students came up with various solutions to a given integral calculus problem. The question now is how to determine if the solutions are correct in the actual sense or not equivalent or differ from each other.

4. RESULTS

For illustration, we consider a simple problem on integration among many other problems.

PROBLEM 1
Use any technique of integration to solve $\int \boldsymbol{sin(x)cos(x)dx}$.

SOLUTION OF GROUP A

Using integration by parts method – *If f(x) and g(x) are two continuous functions of x, then* $\int \boldsymbol{f(x).g(x)dx}$ $=f(x)\int \boldsymbol{g(x)dx}-\int[\frac{d}{dx}(\boldsymbol{f(x)}.\int \boldsymbol{g(x)dx}]\boldsymbol{dx}$.

Let $I=\int \mathbf{sin(x)cos(x)dx}$

→ $I=\sin(x)\int\cos(x)dx-\int[\frac{d}{dx}(\mathbf{sin(x)}.\int\mathbf{cos(x)dx}]\mathbf{dx}$.

→ $I = \sin(x)\sin(x)-\int\cos(x)\sin(x)dx$.

→ $I=\mathbf{sin^2(x) - I}$.

→ $2I=\mathbf{sin^2(x)}$.

→ $I=\frac{\sin^2(x)}{2}$ + C, where C is the constant of integration.

SOLUTION OF GROUP B

Using the method of substitution

Let u = sin(x)

→ du = cos(x)dx.

→ $dx = \frac{du}{\cos(x)}$.

→ $\int \mathbf{sin(x)cos(x)dx}$

$=\int u.\cos(x).\ \frac{du}{\cos(x)}$

$= \int u.du = \frac{u^2}{2} + C$

$= \frac{\sin^2(x)}{2}$ +C, where C is the constant of integration.

It is clear from the above two methods that the solution of group A and group B coincides

SOLUTION OF GROUP C

Using the method of substitution

Let u = cos(x)

→ du = -sin(x)dx.

→ $dx = \frac{-du}{\sin(x)}$.

→ $\int \mathbf{sin(x)cos(x)dx}$

$=\int\sin(x).\boldsymbol{u}\ \frac{-du}{\sin(x)}$

$= \int -u.du$

$= \frac{-u^2}{2} + C$

$= \frac{-\cos^2(x)}{2}$ +C, where C is the constant of integration.

SOLUTION OF GROUP D

Using trigonometric identity

We have sin2x = 2sin(x)cos(x).

→ $\sin(x).\cos(x) = \frac{\sin(2x)}{2}$.

→ $\int \mathbf{sin(x)cos(x)dx} = \int \frac{\sin 2x}{2}\mathbf{dx}$

$=\frac{1}{2}\int \mathbf{sin2x\ dx}$

$=\frac{1}{2}.(\frac{-\cos(2x)}{2}) + C$

$= \frac{-\cos(2X)}{4}$ +C, where C is the constant of integration.

STEP-1

The entire problem was changed to definite integral form and each of the solution was evaluated to obtain the numerical value.

STEP-2

The solution to the integration problem was graphed on the same interval to determine the behaviour of the solution.

STEP-1 EXPLANATION- *Assuming that the solution exists for the interval* $[0,\frac{\pi}{2}]$

For the solution $\boldsymbol{y_1} = \int \mathbf{sin(x)cos(x)dx}$

$= \frac{\sin^2(x)}{2} + C$,

we have $\int_0^{\frac{\pi}{2}} \mathbf{sin(x)cos(x)dx} = [\frac{\sin^2(\frac{\pi}{2})}{2} + C] - [\frac{\sin^2(0)}{2} + C]$

$= [\frac{1}{2} + C] - [0+C]$

$=\frac{1}{2}$.

Similarly for the solution $\boldsymbol{y_2} = \int \mathbf{sin(x)cos(x)dx}$

$= \frac{-\cos^2(x)}{2} + C$,

we have $\int_0^{\frac{\pi}{2}} \mathbf{sin(x)cos(x)dx} = [\frac{-\cos^2(\frac{\pi}{2})}{2} + C] - [\frac{-\cos^2(0)}{2} + C] = [0+C] - [\frac{-1}{2} + C]$

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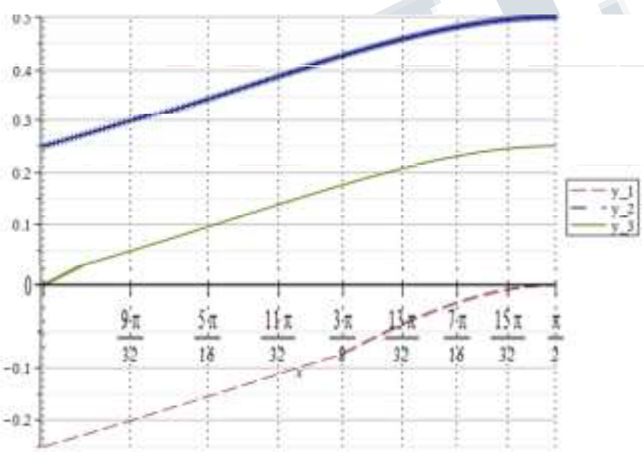
= $\frac{1}{2}$.

Considering the solution y_3= $\int \sin(x)\cos(x)dx$
\= $\frac{-\cos(2x)}{4}$ + C,
we have $\int_0^{\frac{\pi}{2}} \sin(x)\cos(x)dx$ = [$\frac{-\cos(2*\frac{\pi}{2})}{4}$ + C]- [$\frac{-\cos(0)}{4}$ + C]
=[$\frac{1}{4}$ + C] - [$\frac{-1}{4}$ + C]
= $\frac{2}{4}$
= $\frac{1}{2}$.

From the numerical result of the three solution set $\{y_1, y_2, y_3\}$ of the given integration problem, we can see that $y_1 = y_2 = y_3 = \frac{1}{2}$. Thus without loss of generality, we can clearly see the existence of an equivalent solution to the given integral problem.

STEP-2 EXPLANATION- ***Graphical method***

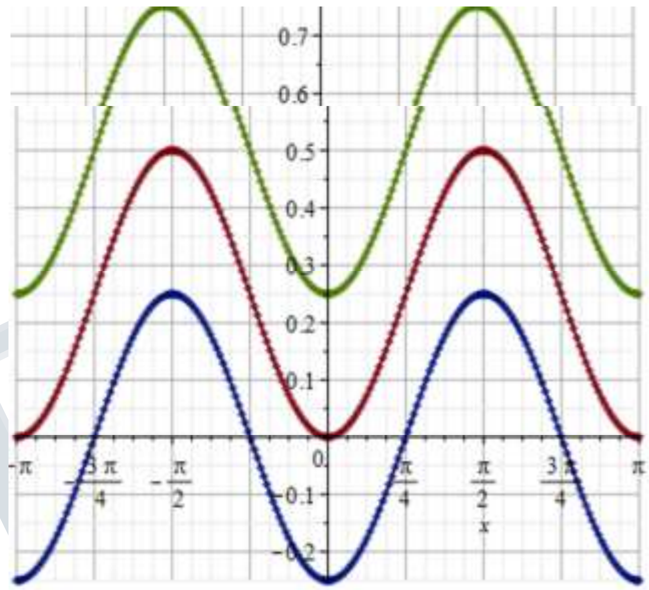
Figure 1.The graph of the solution set {y1, y2, y3} in [0, π/2].



From the above figure we can see that the curves representing the three solutions are in the same direction, at same distance apart intersecting the vertical axis at different values which denotes the constant of integral of the function. Here 0.25, -0.25and 0 are the constant C of integration for y1, y2 and y3 respectively. Let us consider graph of the same solution set in the interval [-π/2, π/2] and [-π, π].

Figure 2.The graph of the solution set {y1, y2, y3} in [-π, π]

Here also we can see that the curves are having the same crest and troughs .The only observable difference is the y intercept of the curves which shows the difference in the constant of integration



CONCLUSION

We evaluated the solution of the definite integral of the given integrand over a specified interval of integration carried out through various integration techniques and under different assumptions. The results shows the evidence of the presence of striking similarity in the solution set as we obtained the same numerical value. We further demonstrated the graphical representation of the solution set to see the behavior of the system of solutions obtained. The graphs also show that solutions obtained in integral calculus are not always unique, but possible to have equivalent solution set with only observable feature of difference in constant of integration.

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