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Almost Contra-Ω*GA-Continuous Functions

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***Abstract*—The notion of contra continuous functions was introduced by Dontchev. In this paper we apply the notion of Ω* - open sets in topological space to present and study a new class of functions called almost contra-Ω*gα-continuous functions as a new generalization of contra continuity. Furthermore, we obtain basic properties and preservation theorems of almost contra-Ω*gα-continuity and investigate the relationship between almost contra-Ω*gα-continuity and Ω*gα-regular graph.**

***Index Terms*— M-Ω*gα-closed map, Almost contra-Ω*gα –continuity, Ω*gα-regular graph**

1. INTRODUCTION

Dontchev[**3**] introduced the notions of contra-continuity in topological spaces.
He defined a function f : X→Y is contra continuous if the preimage of every open set of Y is closed in X. Recently Ganster and Reilly[**6**] introduced a new class of functions called regular set connected functions(in 1999). Jafari and Noiri[**7**] introduced contra-pre-continous functions. Almost contra-pre-continuous functions were introduced by Ekici[**4**]. J.Mercy and I.Arockiarani[**12**]introduced On Ω*-closed sets and Ωp-closed sets in topological spaces. In this paper we introduce and study a new class of functions called almost contra-Ω*gα -continuous functions which generalize classes of regular set connected [6] contra continuous [**3**] and perfectly continuous[**13**] functions. Moreover, the relationship between almost contra-Ω*gα -continuity and Ω*gα -regular graphs are also investigated.

2. PRELIMINARIES

Throughout this paper,spaces (X,τ) and (Y,σ)or(Simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicity stated. For a subset A of (X,τ),cl(A)and int (A) represent the closure of A and interior of A with respect to τ respectively.

DEFINITION 2.1. A subset A of a topological space (X,τ) is said to be preopen[**11**]
(resp. preclosed) if A⊂ Int(cl(A))(resp.cl(int(A) ⊂ A)).

DEFINITION 2.2. A subset A of a topological space (X,τ) is said to be regular open[**15**]
(resp. regular closed) if A=int(cl(A))(resp.A=cl(int(A))).

DEFINITION 2.3. A subset A of a topological space (X,τ) is said to be α-closed[**14**]
(resp. α-closed) if Cl(Int(Cl(A))) ⊂ A(resp. A ⊂ Int(Cl(Int(A))).

DEFINITION 2.4. The intersection of all α-closed sets containing A is called α-closure of A and is denoted by α-cl(A).

DEFINITION 2.5. The α-interior of A is defined by the union of α-open sets contained in A and is denoted by α-int(A).

DEFINITION 2.6. A subset A of a topological space (X,τ) is said to be generalized α-closed set[**10**](briefly gα-closed) if α-cl(A)⊂ U whenever A⊂ U and U is α-open.

DEFINITION 2.7. A function f : (X,τ)→ (Y,σ) is called

1. Contra-continuous [**3**] if f^{-1}(V) is closed in (X,τ) for every open set V of (Y,σ).
2. Regular set connected[**6**] if f^{-1}(V) is clopen in X for every Vε RO(Y).
3. Perfectly-continuous[**13**] if f^{-1}(V) is both open and closed in (X,τ) for every open set V of (Y,σ).
4. Almost-continuous[16] if f -1(V) is open in X for every regular open set V of (Y,σ).

DEFINITION 2.8. A subset A of a topological space (X,τ) is said to be πgα –closed[**1**]
if α-cl(A)⊂ U whenever A⊂ U and U is π - open.

DEFINITION 2.9. A function f : (X,τ)→ (Y,σ) is called πgα –continuous[**2**] if f^{-1}(V)
is πgα -open in (X,τ) for every open set V of (Y,σ).

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DEFINITION 2.10. A function f: (X,τ)→ (Y,σ) is said to be almost contra-πgα –continuous[**8**] if f^{-1}(V) ε πGαC(X,τ) for every Vε RO(Y,σ).

DEFINITION 2.11. A subset A of a topological space (X,τ) is said to be **Ω*-closed**[**12**]
if pcl(A)⊂ Int(U), whenever A⊂ U and U is pre-open in (X,τ).

3. ALMOST CONTRA-Ω*gα -CONTINUOUS FUNCTIONS

DEFINITION 3.1.
A subset A of a topological space (X,τ) is said to be
(a) Ω*gα –closed if α-cl(A)⊂ U whenever A⊂ U and U is Ω*-open.
(b) Ω*gα – open if X-A is Ω*gα –closed.
The family of all Ω*gα –closed sets of X (resp. Ω*gα –open sets) are denoted by
Ω*GαC(X,τ) (resp. Ω*GαO(X,τ)).

DEFINITION 3.2.
A function f : (X,τ)→ (Y,σ) is called
1. Ω*gα -continuous if f^{-1}(V) is Ω*gα -open in (X,τ) for every open set V of (Y,σ).
2. Almost-**Ω***gα -continuous if f^{-1}(V) is **Ω***gα -open in X for every regular open set V of (Y,σ).
3. Contra- Ω*gα -continuous if f^{-1}(V) is Ω*gα -closed in (X,τ) for every open set V of (Y,σ).
4. M- **Ω***gα-open (resp. M- **Ω***gα –closed) if image of each **Ω***gα -open set
(resp. **Ω***gα –closed) is **Ω***gα-open(resp. Ω*gα –closed).

DEFINITION 3.3 :
A function f: (X,τ)→ (Y,σ) is said to be almost contra-Ω*gα -continuous if f^{-1}(V) ε **Ω***GαC(X,τ) for every Vε RO(Y,σ).

THEOREM 3.4 :
Let (X,τ) and (Y,σ) be topological spaces. The following statements are equivalent for a function f : X→Y.
1. f is almost contra-Ω*gα -continuous.
2. f^{-1}(F) ε **Ω***GαO(X,τ) for every F ε RC(Y,σ).
3. for each x ε X and each regular closed set F in Y containing f(x), there exists a **Ω***gα -open set U in X containing x such that f(U)⊂ F.
4. for each x ε X and each regular open set V in Y not containing f(x), there exists a **Ω***gα -closed set K in X not containing x such that f^{-1}(V)⊂ K.
5. f^{-1}(int(cl(G)) ε **Ω***GαC(X,τ) for every open subset G of Y.
6. f^{-1}(cl(int(F)) ε **Ω***GαO(X,τ) for every closed subset F of Y.

PROOF:
(1) ⇒ (2) : Let Fε RC(Y).Then Y-Fε RO(Y,σ).
By (1), f^{-1}(Y-F) =X- f^{-1}(F) ε **Ω***GαC(X,τ).
This implies f^{-1}(F) ε **Ω***GαO(X,τ).
(2) ⇒ (1) : Let Vε RO(Y,σ).
Then Y-Vε RC(Y,σ).By(2) f^{-1}(Y-V)=X- f^{-1}(V) ε Ω*GαO(X,τ).
This implies f^{-1}(V) ε **Ω***GαC(X,τ).
(2) ⇒ (3) : Let F be any regular closed set in Y containing f(x).By (2), f^{-1}(F) ε **Ω***GαO (X,τ) and x ε f^{-1}(F).
Take U= f^{-1}(F).Then f(U)⊂ F.
(3) ⇒ (2) : Let F ε RC(Y,σ) and x ε f^{-1}(F).From (3), there exists a **Ω***gα -open set Ux in X containing x such that
Ux ⊂ f^{-1}(F).We have f^{-1}(F)= ∪ Ux .
Thus , f^{-1}(F x ε f^{-1}(F) is Ω*gα -open.
(3) ⇒ (4): Let V be a regular open set in Y not containing f(x). Then Y-V is a regular closed set containing f(x).
By (3) there exists a **Ω***gα -open set U in X containing x such that f(U)⊂Y-V.
Hence U⊂ f^{-1}(Y-V) ⊂ X- f^{-1}(V) and then f^{-1}(V) ⊂ X-U.
Take K=X-U.
We obtain a **Ω***gα -closed set K in X not Containing x.
(4) ⇒ (3): Let F be regular closed set in Y containing f(x).
Then Y-F is a regular open set in Y not containing f(x).
By (4) there exist a **Ω***gα -closed set K in X not containing x such that f^{-1}(Y-F) ⊂ K.
This implies X- f^{-1}(F) ⊂ K ⇒ X-K⊂ f^{-1}(F) ⇒ f (X-K) ⊂ F. Take U=X-K.
Then U is a **Ω***gα -open set in X containing x such that f(U) ⊂ F.
(1) ⇒ (5): Let G be an open subset of Y.Since int(cl(G)) is regular open, then by(1)
f^{-1} (int(cl(G))) ε **Ω***GαC(X,τ).
(5) ⇒ (1): Let Vε RO(Y,σ).Then Vis open in Y.By (5) f^{-1} (int(cl(V))) ε **Ω***GαC(X,τ) ⇒ f^{-1}(V) ε **Ω***gα -closed in (X,τ).
(2) ⇔ (6) The proof is obvious from the definitions.

REMARK 3.5: The following diagram holds.

Perfectly continuous ⇒	Contra continuous ⇒	Contra- Ω*gα -continuous
⇓	⇓	
Regular set connected ⇒	Almost Contra **Ω***gα -continuous	

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None of the implications is reversible for almost Contra Ω*gα -continuity as shown by the following examples.

EXAMPLE 3.6 : Let X = {a,b,c}, τ ={Φ, X,{a}} and σ ={ Φ, X,{b},{c},{b,c}}.
Then the identity function f: (X,τ)→ (X,σ) is almost contra-Ω*gα -continuous but not regular set connected.

EXAMPLE 3.7 : Let X = {a,b,c,d}, τ ={X,Φ, {a},{a,c},{a,d},{a,c,d}} and σ ={X, Φ,{a}{a,b},{a,c,d}}.
Then the identity function f: (X,τ)→ (X,σ) is almost Contra-Ω*gα -continuous but not contra-Ω*gα -continuous.

EXAMPLE 3.8: Let X = {a,b,c}, τ ={X,Φ,{a,b}} and σ ={X, Φ,{a},{a,b}}. Then the identity function f: (X,τ)→ (X,σ) is contra-Ω*gα -continuous but not contra-continuous.

THEOREM 3.9: Suppose that Ω*gα -closed sets are closed under finite intersection.
If f: X→Y is almost contra-Ω*gα -continuous function and A is Ω*gα -open subset of X, Then the restriction f/A: A→Y is almost contra-Ω*gα -continuous.
PROOF: Let Fε RC(Y). Since f is almost contra-Ω*gα -continuous then f^{-1}(V) ε Ω*GαO(X,τ). Since A is Ω*gα -open in X if follow that $(f/A)^{-1}$(F) = A∩ f^{-1}(F)) ε Ω*GαO(A,τ). Therefore, f/A is almost contra-Ω*gα -continuous function.

REMARK 3.10: Every restriction of an almost contra-Ω*gα -continuous function is not necessarily almost contra-Ω*gα -continuous.

EXAMPLE 3.11 : Let X = {a,b,c,d}, τ ={ Φ, X,{a},{d},{a,d},{c,d},{a,c,d}} and
σ ={Φ, X, {b},{c},{b,c}}.
Then the identity function f: (X,τ) → (X,σ) is almost contra-Ω*gα -continuous but if A = {a,b,c },where A is not Ω*gα -open in (X,τ) and τ_A ={Φ, {a,b,c},{a},{c},{a,c}} is the relative topology on A induced by τ , then
f/A : (A,τ_A) → (X,σ) is not almost contra-Ω*gα -continuous. Note that {a,b,d} is regular closed in (X,τ) but that $(f/A)^{-1}${a,b,d} = A∩{a,b,d}={a,b,c}∩{a,b,d}={a,b} is not
Ω*gα -open in (A,τ_A).

DEFINITION 3.12: A cover ∑ = {Uα : α ε I } of subsets of X is called a Ω*gα -cover if Uα is Ω*gα -open for each α ε I .

THEOREM 3.13: Suppose that Ω*GαO(X,τ) sets are closed under finite intersection.
Let f: X→Y be a function and ∑ = {Uα : α ε I } be a Ω*gα -cover of X .
If for each α ε I, f/Uα is almost contra-Ω*gα -continuous, then f: X→Y is almost contra-Ω*gα –continuous.
PROOF: Let Vε RC(Y).Since f/Uα is almost contra-Ω*gα -continuous function,
$(f/U\alpha)^{-1}$(V) ε Ω*GαO(Uα). Since Uα ε Ω*GαO(X), by the result if U is Ω*gα -open in X and V is Ω*gα -open in X, it follows $(f/U\alpha)^{-1}$(V)) ε Ω*GαO(X) for each α ε I. Then
f^{-1}(V) = ∪ $(f/U\alpha)^{-1}$(V) ε Ω*GαO(X). This gives f is almost contra-Ω*gα continuous α ε I function.

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THEOREM 3.14: Let f: X→ Y and let g : X→ X ×Y be the graph function of f defined by g(x)=(x,f(x)) for every x ε∈X. If g is almost contra-Ω*gα -continuous then f is almost contra-Ω*gα -continuous.
PROOF: Let Vε RC(Y), then X × V = X× cl(int(V) = cl(int(X)×cl(int(V) = cl(int(X ×V)).
Therefore X × Vε RC(X ×Y). Since g is almost contra-Ω*gα -continuous, g^{-1}(X ×V) ε Ω*gα -open in X.
This implies f^{-1}(V) = g^{-1}(X ×V) ε Ω*gα -open in X. Thus, f is almost contra-Ω*gα -continuous.

THEOREM 3.15: Let f: X→ Y and g: Y→ Z be function. Then, the following properties hold:
1) If f is almost contra-Ω*gα -continuous and g is regular set connected, then gof : X→ Z is almost contra-Ω*gα -continuous and almost Ω*gα -continuous.
2) If f is almost contra-Ω*gα -continuous and g is perfectly continuous then gof : X→ Z is Ω*gα -continuous and contra-Ω*gα -continuous.
3) If f is almost contra-Ω*gα -continuous and g is regular set-connected then gof : X→ Z is almost contra-Ω*gα -continuous almost Ω*gα -continuous.
PROOF: Let Vε RO(Z) Since g is regular set connected g^{-1}(V) is clopen in Y. Since f is almost contra-Ω*gα -continuous, $f^{-1}(g^{-1}(V))= (gof)^{-1}(V)$ is Ω*gα -open and Ω*gα -closed. Therefore gof is almost contra-Ω*gα -continuous and almost Ω*gα -continuous. (2) and (3) can be obtained similarly.

THEOREM 3.16: If f: X→ Y is a surjective M-Ω*gα -open and g: X→ Z is a function such that gof: X→ Z is almost contra-Ω*gα -continuous,then g is almost contra-Ω*gα -continuous.
PROOF : Let V be any regular closed set in Z. Since gof is almost contra-Ω*gα -continuous,$(gof)^{-1}$(V)) ε Ω*gα -open in (X,τ).

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Since f is surjective , M-Ω*gα -open map,$f((gof)^{-1}(V)) = f(f^{-1}(g^{-1}(V)) = g^{-1}(V)$ is Ω*gα -open.Therefore g is almost contra-Ω*gα -continuous.

THEOREM 3.17: If f: X→ Y is a surjective M-Ω*gα -closed map and g: X→ Z is a function such that gof: X→ Z is almost contra-Ω*gα -continuous,then g is almost contra-Ω*gα -continuous.
PROOF: Similarly as the previous theorem.

THEOREM 3.18: If a function f: X→ Y is almost contra-Ω*gα -continuous and almost continuous then f is regular set-connected.
PROOF: Let Vε RO(Y). Since f is almost contra-Ω*gα -continuous and almost continuous $f^{-1}(V)$ is Ω*gα -closed and open. Hence $f^{-1}(V)$ is clopen. Hence f is regular set-connected.

DEFINITION 3.19: A filter base Λ is said to be Ω*gα -convergent (resp. rc-convergent) to a point x in X if for any Uε Ω*gα -open in X containing x (resp.Uε RC(X)) there exist a B ε Λ Such that B⊂ U.

THEOREM 3.20: If a function f: X→ Y is almost contra-Ω*gα -continuous, then for each point x ε X and each filter base Λ in X Ω*gα -converging to x, the filter base f(Λ) is rc-convergent to f(x).
PROOF: Let x ε X and Λ be any filter base in X Ω*gα -converging to x. Since f is almost contra-Ω*gα -continuous then for any Vε RC(Y) containing f(x) there exist U ε Ω*gα -open in X containing x such that f(U)⊂ V. Since Λ is Ω*gα -converging to x, there exist a B ε Λ such that B⊂ U.This means that f(B)⊂ V and therefore the filter base f(Λ) is rc-convergent to f(x).

Note that a function f: X→ Y is almost contra-Ω*gα -continuous at x if each regular closed set F in Y containing f(x), there exist Ω*gα -open set U in X containing x such that f(U)⊂ F.

THEOREM 3.21 : Let f: X→ Y be a function and x ε X. If there exist Uε Ω*gα -open in X such that x ε U and the restriction of f to U is almost contra-Ω*gα -continuous at x then f is almost contra-Ω*gα -continuous at x.
PROOF: Suppose that F ε RC(Y) containing f(x). Since f / U is almost contra-Ω*gα -continuous at x, there exists Vε Ω*gα -open set U in X containing x such that f(V)=(f/U)(V)⊂ F. Since Uε Ω*gα -open in X containing x it follows that Vε Ω*gα -open in X containing x. This shows clear that f is almost contra-Ω*gα -continuous at x.

4. THE PRESERVATION THEOREMS

In this section, we investigate the relationships among almost contra-Ω*gα -continuous functions,separation axioms,connectedness and compactness.

DEFINITION 4.1: A space X is said to be weakly Hausdorff [**19**] if each element of X is an intersection of regular closed sets.

DEFINITION 4.2 : A space X is said to be Ω*gα -To if for each pair of distinct points in X there exists a Ω*gα -open set of X containing one point but not the other.

DEFINITION 4.3: A space X is said to be Ω*gα -T_l if for each pair of distinct points x and y in X there exists a Ω*gα -open sets U and V containing x and y respectively such that y∉ U and x ∉V.

DEFINITION 4.4: A space X is said to be Ω*gα -Hausdorff if for each pair of distinct points x and y in X there exists U ε Ω*gα -open in (X,x) and V ε Ω*gα -open in (Y,y) such that U∩V=ϕ.

THEOREM 4.5: If f: X→ Y is an almost contra-Ω*gα -continuous injection and Y is weakly Hausdorff then X is Ω*gα -T_l.
PROOF : Suppose that Y is weakly Hausdorff .For any distinct points x and y in X there exist V, Wε RC(Y) such that f(x) ε V , f(y) ε W , f(x) ∉ W, f(y) ∉ V .Since f is almost
Ω*gα -continuous, f^{-1}(V) and f^{-1}(W) are Ω*gα -open subsets of X such that x ε f^{-1}(V) and y ε f^{-1}(W) ,y∉ f^{-1}(V) , x ∉ f^{-1}(W),This shows that X is Ω*gα -T_l.

DEFINITION 4.6 : A topological space X is called Ω *gα-ultra connected if every two non-void Ω*gα -closed subsets of X intersect.

DEFINITION 4.7 : A topological space X is called hyper connected [**20**] if every open set is dense.

THEOREM 4.8 : If X is Ω*gα -ultra connected and f: X→ Y is almost contra-Ω*gα -continuous and surjective, then Y is hyper connected.
PROOF : Assume that Y is hyper connected. Then there exist an open set V such that V is not dense in Y.Then there exist disjoint non-empty regular open subsets B_1 and B_2 in Y
namely B_1=int cl(V) and B_2 =Y-cl(V). Since f is almost contra-Ω*gα -continuous and surjective, $A_1 = f^{-1}(B_1)$ and $A_2 = f^{-1}(B_2)$ are disjoint non-empty Ω*gα -closed subsets

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of X which is a contradiction to the fact that X is Ω*gα -ultra connected.Hence Y is
hyper connected.

DEFINITION 4.9 : A space X is called Ω*gα - connected provided that X is not the union of two disjoint non-empty Ω*gα -open sets.

THEOREM 4.10 : If f: X→ Y is almost contra-Ω*gα -continuous surjection and X is Ω*gα - connected then Y is connected.

PROOF: Suppose that Y is not connected.Then there exist non-empty disjoint open sets V_1 and V_2 such that $Y=V_1\cup V_2$. Therefore V_1 and V_2 are clopen in Y.Since f is almost contra-Ω*gα -continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$ which is a contradiction to the fact that X is Ω*gα - connected. Hence Y is connected.

DEFINITION 4.11 : A space X is said to be

a) Ω*gα -closed if every Ω*gα -closed cover of X has a finite subcover.

b) Countable Ω*gα -closed if every countable cover of X by Ω*gα -closed sets has a finite subcover.

c) Ω*gα -Lindelof if every cover of X by Ω*gα -closed sets has a countable cover.

d) Nearly compact if every regular open cover of X has a finite subcover.[**17**]

e) Nearly countably compact if every countably cover of X by regular open sets has a finite subcover.[**5 , 18**]

f) Nearly Lindelof [**4**] if every cover of X by regular open sets has a countable subcover.

THEOREM 4.12: Let f: X→ Y be an almost contra-Ω-continuous surjection. Then the following statements hold.

a) If X is Ω*gα -closed then Y is nearly compact.

b) If X is Ω*gα -lindelof then Y is nearly lindelof.

c) If X is countably-Ω*gα -closed,then Y is nearly countably compact.

PROOF: Let {Vα : α ε I }be any regular open cover of Y. Since f is almost contra-Ω*gα -continuous, then { f^{-1}(Vα) : α ε I } is a Ω*gα -closed cover of X.Since X is Ω*gα -closed there exist a finite Io of I such that X= ∪{ f^{-1}(Vα) : α ε Io}.Thus we have Y=∪{ Vα : α ε Io}and Y is nearly compact.

Proof of b) and c) are analogue to a).

DEFINITION 4.13 : A space X is said to be Mildly Ω*gα -compact if every Ω*gα -clopen cover of X has a finite subcover.

a) Mildly countably-Ω*gα -compact if every Ω*gα -clopen countable cover of X has a countable subcover.

b) Mildly Ω*gα -Lindelof if every Ω*gα -clopen cover of X has a countable subcover.

THEOREM 4.14: If f: X→ Y is an **almost** contra-Ω*gα -continuous and almost contra-Ω*gα -continuous surjection.Then

a) If X is mildly Ω*gα - compact then Y is nearly compact.

b) If X is mildly countably- Ω*gα -compact then Y is nearly countably compact.

c) If X is mildly Ω*gα - lindelof then Y is nearly Lindelof.

PROOF: (a) Vε RO(Y). Then since f is almost contra-Ω*gα -continuous almost Ω*gα -continuous, f^{-1}(V) is clopen.Let {Vα : α ε I }be any regular open cover of Y. Then { f^{-1}(Vα) : α ε I } is a clopen cover of X. Since X is mildly Ω*gα - compact,there exist a finite subset Io of I such that X= ∪{ f^{-1}(Vα) : α ε Io} Hence Y is nearly compact.

Proof of b) and c) are similar to a).

5. Ω*gα -REGULAR GRAPHS

In this we define Ω*gα -regular graphs and investigate the relationships between Ω*gα -regular graphs and almost contra-Ω*gα -continuous functions.

DEFINITION 5.1: For a function f: X→ Y the subset {(x,f(x) / x∈X}⊂ X × Y is called the graph of f and is denoted by G(f)[**4**]

DEFINITION 5.2 : A graph G(f) of a function f: X→ Y is said to be Ω*gα -regular if for each (x,y) ε X × Y-G(f), there exist a Ω*gα -closed set U in X containing x and Vε RO(Y) containing y such that (U × V) ∩ G(f) = ϕ.

LEMMA 5.3: The following properties are equivalent for a graph G(f) of a function

1. G(f) is Ω*gα -regular.
2. for each point (x,y) ε X × Y-G(f) there exist a Ω*gα -closed set U in X containing x and Vε RO(Y) containing y such that f(U) ∩V = ϕ.

PROOF : It follows from definition and the fact that for any subsets U⊂ X,V⊂Y(U × V)∩ G(f) = ϕ iff f(U) ∩V = ϕ.

THEOREM 5.4: If f: X→ Y is almost contra-Ω*gα -continuous and Y is T_2 , then G(f) is
Ω*gα -regular graph in X × Y.

PROOF : Let (x,y) ε X × Y-G(f). It follows that f(x) ≠ y. Since Y is T_2 , there exist open sets V and W containing f(x) and y respectively such that V∩W= ϕ.We have int(cl(V)) ∩

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int(cl(W)) = ϕ. Since f is almost contra-Ω*gα -continuous, f^{-1} (int(cl(V))) is Ω*gα -closed in X containing x . Take U = f^{-1} (int(cl(V))). Then f(U) ⊂ int(cl(V)) Therefore
f(U) ∩ int(cl(W)) = ϕ. Hence G(f) is Ω*gα -regular.

THEOREM 5.5 : Let f: X→ Y have Ω*gα -regular graph G(f). If f is injective, then X is Ω*gα -T_l.
PROOF : Let x and y be any two distinct points of X.Then we have (x,f(y)) ε X × Y-G(f). By definition of Ω*gα -regular graph,there exist a Ω*gα -closed set U of X and
V ε RO(Y) such that (x ,f(y)) ε U × V and U ∩ f^{-1} (V) = ϕ. Therefore we have
Y ∉ U.Thus y ε X-U. x∉ X-U. X-U ε Ω*gα -open in (X,τ) implies X is Ω*gα -T_l.

THEOREM 5.6: Let f: X→ Y have Ω*gα -regular graph G(f) If f is surjective, then Y is weakly T_2.
PROOF: Let y_1 and y_2 be any two distinct points of Y. Since f is surjective f(x) = y_1 for some x ε X and (x , y_2) ε X × Y-G(f).By lemma 5.3,there exist a Ω*gα -closed set U of X and F ε RO(Y) such that (x, y_2) ε U × F and f(U) ∩F =ϕ. Hence y_1∉ F. Then y_2∉Y- F ε RC(Y) and y_1 ε Y-F. This implies that Y is weakly T_2.

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