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Fractal Geometry: A fascinating world

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Abstract— ***Fractal geometry is a concept that has evolved recently, however it has the potential to change the world for the better. This research paper is an attempt to help establish the basic idea of fractals and open up a conversation on fractals, a lesser known topic. This is also a celebration of the irregularities and the chaos of the world.***

Index Terms— ***Fractal, Fractal geometry, Mandelbrot, Fractal Dimension, Iteration, Nature***

I. INTRODUCTION

The term "Fractal" was coined by a Polish-French-American mathematician Benoit B. Mandelbrot in 1975. It was derived from the Latin word 'fractus' which means 'broken' or an irregular surface like that of a broken stone. Up until 1975, everything was measured and represented by Euclidean Geometry, however it was not enough for measuring most aspects of nature. Euclidean Geometry is all about smooth surfaces and regular structures but natural elements are made up of rough irregularities [1]. The fronds of ferns couldn't be measured by a simple scale or by methods used to measure smooth surfaces. Similarly the length of the coastlines nor the perimeter of the snowflakes could be calculated (figure 1 and 2).

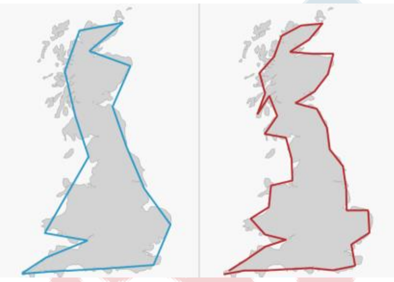

Fig 1. Coastline of Great Britain | Business Insider

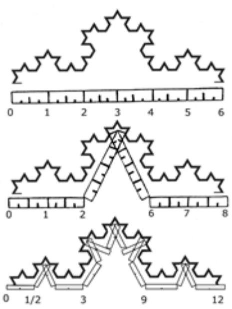

Fig 2. Snowflake | ResearchGate

It was possible to mathematically explore the rough irregularities of nature due to the fractal theory. Mandelbrot stated *"In the whole of science, the whole of mathematics, smoothness was everything. What I did was to open up roughness for investigation."* Mandelbrot's discovery introduced a new way of viewing nature, not as something disorganized and ruled by chance, but as something meticulously constructed. The resulting discipline of fractal geometry allows us to define and measure these strange structures, as well as recreate them when applied to the subject of biomimicry. Thus emerged a world of its own-Fractal Geometry.

Fractals (figure 3,4,5) are infinitely complex patterns that are self-similar if not identical, across different scales. They embody the property to look like themselves regardless of which part of it is observed and regardless of how many times it is enlarged. Thus, a part of the fractal can represent the whole.

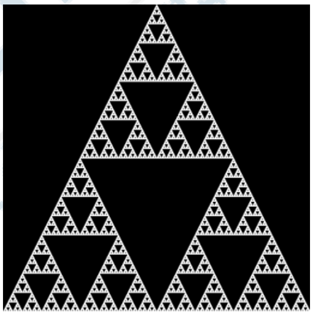

Fig 3. The Sierpinski triangle- a fractal

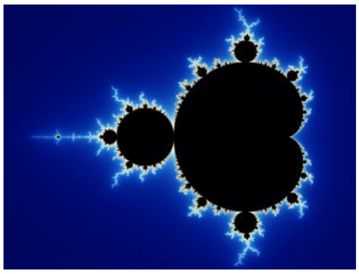

Figure 4. The Mandelbrot Set- a fractal | Wikipedia

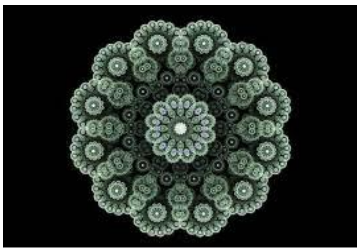

Figure 5: A fractal | Champs UNC Asheville

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Figure 6 shows how a part of a fractal called the Mandelbrot fractal represents the whole. Each highlighted square is enlarged.

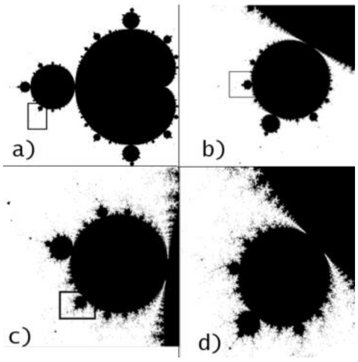


Figure 6. Enlargement of Mandelbrot fractal

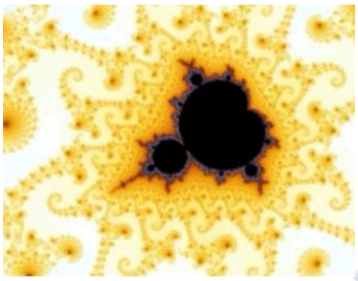


Figure 7. Zoomed version of the Mandelbrot Fractal | Wikipedia

For instance ,if you take a broccoli and cut a small piece of the curd or the head, the small piece would be similar to the broccoli it was cut from. Furthermore, the small piece can be cut into smaller parts, which would be similar to the small cut piece and the whole broccoli [3]. This can be done repeatedly and you would get the same results. Thus, a broccoli can be considered to be like a fractal (figure 8).

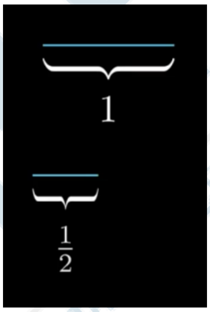
Figure 8. Broccoli-fractal like structure | greatist.com

Fractal geometry is the method to measure fractals. It gives us a quantitative way to describe roughness. Contrary to common conception, fractals are much more than just beautiful patterns and have more character than simply repeating surfaces [4]. Fractals are the main differentiators of objects that occur naturally and objects that are man-made. The basic properties of fractals other than self-similarity are "fractal dimension" and "formation by iteration." Fractal dimension ,in simple words, is the degree of irregularity. On the other hand, formation by iteration means that the fractals are constructed when we apply a certain process repeatedly.

II. FRACTAL DIMENSION:

The fractal dimension is a number that indicates the space occupied by the fractal. In other words, the fractal dimension is used to indicate the density of the object in space, and the number of parts that arise when the resolution is increased [2]. When we look at a line, we can easily say that it is 1-Dimensional. Similarly a square is 2-Dimensional and a Cube is 3-Dimensional. So, what would the dimension of a fractal be? The interesting aspect of fractals is that they are shapes with a non-integer (no whole number) dimension. The non-integer dimension simply correlates to the rough irregularities. It is important to understand what dimension truly means and for the same, we need to understand terms like "Scaling Factor" and "Mass scaling factor".

Example 1: Let's take a line as shown in the figure below.



Here the scaling factor is the number by which we have scaled the smaller line with respect to the original line. In this case, the smaller line is half the original line, and thus its scaling factor is ½.

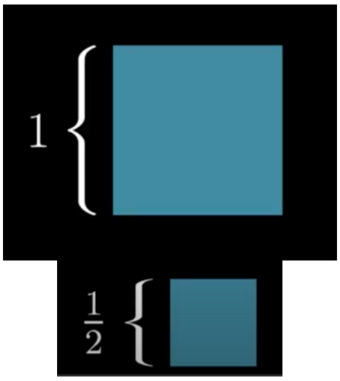
The mass scaling factor is the scaling factor raised to the dimension of the object.

$$Mass\ scaling\ factor\ = (\,scaling\ factor)^D$$
$$Where\ D\ is\ the\ dimension\ of\ the\ object$$

Thus, the mass scaling factor of the line is $(1/2)^1 = 1/2$ as the line is 1-Dimensional.

Lines are not fractals as they have a whole number dimension.

Example 2: Let's take a square.



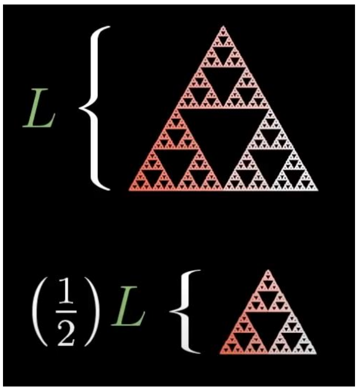
Scaling Factor= ½

Mass Scaling factor=$(1/2)^2 = 1/4$

Squares are not fractals as they have a whole number dimension.

Thus, we can say that what it means for a shape to be '2-Dimensional' is that when you scale it by some factor, its mass is scaled by that factor raised to the second power. Furthermore, for a shape to be 'x-Dimensional', when you scale it by some factor, its mass is scaled by that factor raised to the power x [5].

Example 3: For a Sierpinski triangle, let the dimension be D. A Sierpinski triangle is a fractal.



It can be seen that the scaling factor is ½

Thus, the Mass factor = $(Scaling\ Factor)^{D} = (\frac{1}{2})^{D}$

We can see that the length of the smaller Sierpinski triangle is ½ that of the original triangle and 3 of such smaller triangles can form 1 original Sierpinski triangle.

Therefore, we can say that $(\frac{1}{2})^{D} = \frac{1}{3} \Rightarrow log\ _{2}3 = D \Rightarrow D = 1.585$

Thus, a Sierpinski triangle is 1.585-Dimensional

Similarly the dimension of the Von Koch curve (also known as the Koch snowflake shown in figure 8) is approximately 1.262 [7].

Another interesting thing about fractals is that if you try describing its mass with length and area, none of them seem fitting. This is because the length of the Sierpinski will be infinite and its area would be zero. Figure 9 shows the length and area of the Von Koch curve/ Koch snowflake [8].

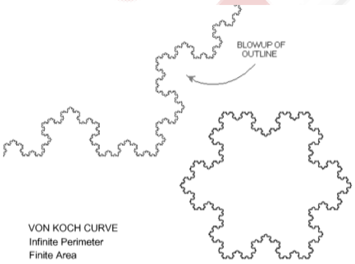
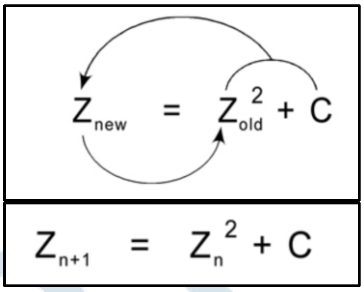


Figure 9. Von Koch curve | stsci.edu

So, a Von Koch curve encloses a finite area with an infinite perimeter. Thus, depending on the type of fractal, the area of the fractal can be zero, infinite, or finite, however the perimeter of the fractal will always be infinite [9].

III. ITERATION

The property of iteration means that a simple process is repeated several times using the output from one iteration as the input to the next to get a specific outcome. In the case of fractals, Mandelbrot developed a formula, which when repeated gave the Mandelbrot fractal [6].



Formula 1| Mandelbrot's formula Formula 2 | Mandelbrot's formula

It can be inferred that the above formulae are different versions of the same formula.

In the iterated equation 1(formula 1)- the new value of a variable Z equals the old value of Z, squared, C, a constant.

In the iterated equation 2 (formula 2)- n is zero or a positive integer/natural number and

c and z are complex numbers.

Mandelbrot said, *"A formula can be very simple, and create a universe of bottomless complexity."* He stated this because the whole complex world of fractals depends on such a simple formula. There can be several types of outputs of the formula as shown in the below example.

For eg: 1. $0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0..$

2. $-1 \rightarrow 0 \rightarrow -1 \rightarrow 0..$

3. $1 \rightarrow 2 \rightarrow 5 \rightarrow 25 \rightarrow 671 \ldots \rightarrow \infty$

From the above outcomes, eg 1,2 are not fractals, whereas eg 3 is a fractal.

The basis of fractals lies on principles of "self-similarity", "fractal dimension", and "formation by iteration".

IV. APPLICATION OF THE FRACTAL THEORY:

Fractals can be found everywhere, from trees to human lungs, in solar panels and antennas, in music and art, in hydrolysis, in crystal growth, in architecture and design, in movies and video games, in cosmology, telecommunication, genetics, and so much more.

Nature has the best fractals. Most of the things you see in nature are fractals or are like fractals. The lightning we see is also a fractal. A small part of the lightning bolt represents the whole. When you zoom in a tree branch, it represents the whole tree. Further, if you zoom in the shoot of the branch, it will also resemble the tree. You could do this till the tiniest offshoot and still find that the smallest part would represent the whole. Also every foot of the shoreline has the same underlying wavy texture and form as a mile or 1000 miles of

the coastline. Thus the coastline is also like a fractal. Despite their seemingly random appearance, natural elements such as crystals, clouds, river systems, mountains, snowflakes, trees, lightning, growth spirals, human lungs, and even galaxies are highly ordered, with bigger shapes made of microscopic clones of those same forms [10]. And those replicas, in turn, contain even smaller replicas. Fractals have made it possible to gain deeper knowledge on natural phenomena like seismic patterns, flow of rivers, and mineral fragmentation. Moreover, fractals have been used to model soil erosion, find out how forest fires spread and estimate how much CO2 forests can produce. Fractals have provided us a way to understand the complexities of a "system".
Figure 10,11,12 are natural fractals [11].

Figure 10. Fern

Figure 11. Lightning | bbvaopenmind

Figure 12. Coastline | bbvaopenmind

Fractal geometry can describe the unpredictable. Fractal patterns have surfaced in all of our bodies' physiological activities [13]. The variation in a person's heart and the prevalence of diseases are two cases in which fractals play a major role. The human heart has long been thought to beat in a linear manner, but new research has revealed that the real rhythm of a healthy heart swings wildly in a fractal pattern. Blood is also dispersed in a fractal pattern throughout the body [14]. Ultrasound imaging is being used by researchers in Toronto to discover the fractal patterns of blood flows in both healthy and damaged kidneys. The goal is to employ mathematical models to discover cancerous cells earlier than ever before by analysing the fractal dimensions of these blood flows [15]. If we succeed in doing so, the early detection will come from mathematics rather than microscopy. Additionally, according to Harvard Medical School researchers, variations in the fractal scale are the core cause of pathophysiological changes, including the syndrome of sudden cardiac death. Fractals can explain how gas exchange surfaces in our lungs are ventilated. Lungs have a mechanism that optimises energy utilisation by following a specific fractal pattern. One of the most promising uses of fractals will be to identify when optimized programming fails, thereby turning it into a strong diagnostic tool that can outperform any machine.

Fractals are now used to solve one of the world's biggest concerns- Depletion of Energy. Fractals are used to generate renewable sources of energy. Fractal solar panels instead of the normal solar panels are being developed. These fractal solar panels are both aesthetic and efficient. By incorporating nature's fractals in the solar cell, we are able to increase the surface area of the photoelectric material and minimize the electrical losses. Researchers from the University of Oregon (UO) have developed a hybrid electrode pattern that combines the busbar and fractal designs effectively. The hybrid design is based on the fractal tree structure known as the 'H tree'. H-tree is a geometric shape that consists of a pattern that repeats the letter "H" as shown in figure 13. Several studies indicate that the fractal patterns observed on rooftop solar panels might help alleviate stress in people passing by [12].

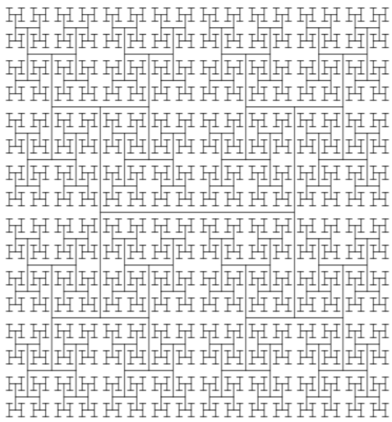


Figure 13. H-tree | Mathematica Stack Exchange

Fractals substantially increase surface area. Based on this concept, Cell phone manufacturers have already developed new ways to maximize signal reception by bending antennas into fractal patterns, adding length without increasing the amount of space the antennas take up. Fractals are also used to study star formations. It's not surprising, then, that the creation of technologies that rely on physical phenomena, such as telecommunications, uses fractal notions to improve their performance. Antennas, for example, are commonly used in wireless devices and are developed using this mathematical foundation to reach a larger variety of frequencies [16].

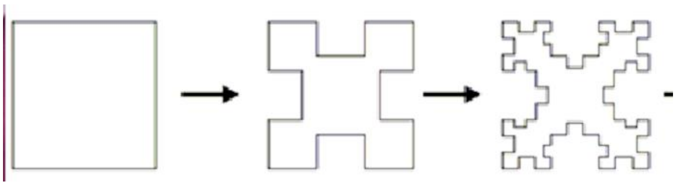


Figure 14. Formation of Antenna Fractals | Semantic Scholar

Fractals are now used in the creative industries to make music, graphics, and art. As we now know, any fractal can be created by iterating a simple algebraic formula. This process of iteration is used to create music and graphics with the help of computers. By means of special programs, Music can be generated by fractal images. Each fractal image is made up of a large number of points that are plotted in the plane of a graphic environment.When a distinct music note is associated with each point, we get a fractal tune. Below is the Mandelbrot fractal and the music notes generated by the Mandelbrot fractal [17]. (figure 15,16)

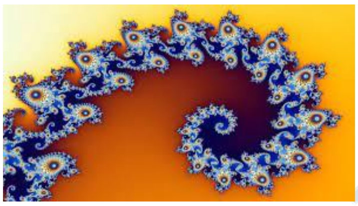


Figure 15. Mandelbrot fractal

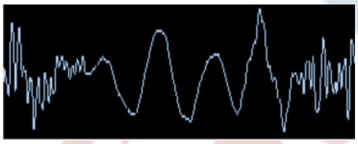


Figure 16. Music notes generated by the fractal

Fractals are also used in acoustics. Acousticians and concert-hall designers are well aware that sound bounces off and echoes when it hits a smooth, flat wall. A wall with a rough surface, on the other hand, that simulates the fractal patterns absorbs sound far better.

It's fascinating to see how using fractal art and architecture can reduce psychological stress. Several studies suggest that as humans increase spending time indoors, surrounded by Euclidean design and architecture, they tend to experience more headaches. The additional effort required by the viewer to process the spatial frequencies causes discomfort. Adding several fractal elements into a space minimises the stress levels of the occupants. The benefits of viewing fractals suggest the inherent need of humans to interact with nature.

Figure 17. A fractal designed wall | pinterest

Fractal art can be created using many technologies and manual methods. One such method to create fractal art is the chaotic pendulum method (figure 18). The chaotic pendulum creates poured paintings.

Figure 18. left- The chaotic pendulum, top right- non-fractal painting, bottom right- fractal painting | Richard Taylor

Fractal designs in modern architecture are the new way to bring nature closer to us [18]. In architecture, fractal geometry serves as a means of expressing the beauty in complexity. It gives shape to an architect's visions and depicts the universe's energy flow via structures and monuments. Long before the invention of the fractal theory, Hindu temples were constructed using fractal phenomena. Now, Modern architecture contains several examples of fractal geometry consciously being used in design and building.

The core concept of the Fractal theory-each part being geometrically similar to the whole- is infact one of the basic principles of Hindu temple architecture. The Hindu Temple is created and built as tiny representations of the Cosmos as conceived in Hindu philosophy and beliefs [19].

The universe is described as holonomic (whole to part relation) in Hindu philosophy. Each component of the cosmos is thought to be entire in itself and to have knowledge that is similar to the whole. The fractal designs can be seen in the Vimana (tower) part of the temples. The Vimana tower is surrounded by smaller towers, encircled by even smaller towers, and so on, for eight or more levels.

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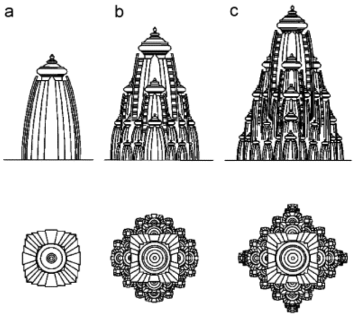

Figure 19. Vimana of different temples | Science direct

Figure 20. Fractal design of the temple | Pinterest

Figure 21. Fractal like patterns of the Borobudur Temple | Richard Taylor

The Eiffel Tower is made up of fractal-like structural members as shown in figure 22.

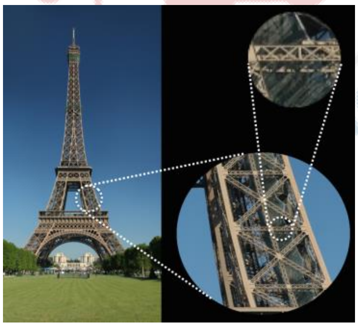

Figure 22. Fractal-like structural members | Wired

Modern architecture is adapting several fractal like designs as shown in the images below. The Grand Egyptian Museum has adopted the Sierpinski triangle design while several homes, airports and cafes incorporate the Arboreal architecture. Arboreal architecture is tree-inspired architecture that has dendriforms and fractal-like branching.

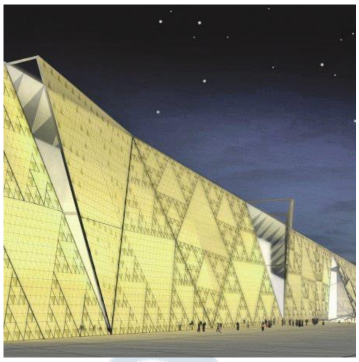

Figure 23. Grand Egyptian Museum | Researchgate

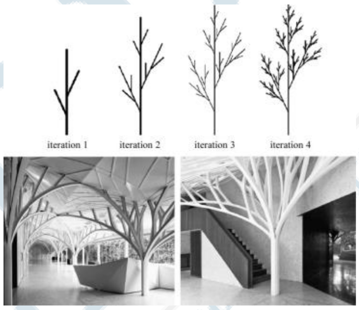

Figure 24: Arboreal Architecture | Science Direct

The principle of design and architecture has been greatly impacted by fractal geometry.

Fractal geometry is being used in a variety of fields, notably biomimicry and healthcare. The Mandelbrot set has generated as many different developments as the fascinating designs it produces.The Wyss Institute for Biologically Inspired Engineering at Harvard University has developed shrilk, a biodegradable alternative to plastic- inspired by the cuticle of an insect's fractal layers- that has outstanding strength. Many wireless gadgets now employ fractal-based antennas to pick up the largest spectrum of known frequencies. Fractals are used in graphic design and picture editing software to generate stunningly intricate landscapes and lifelike special effects. Fractals are used everywhere- from compressing data to building street systems. This phenomena is used in hydrology, meteorology, fluid dynamics, cosmology, genetics, and energy flow. We have just begun to scrape the surface of what fractal geometry has to offer. Galaxy clusters, weather patterns,and stock market price fluctuations have all been discovered to be fractal, but what will we do with this knowledge? The possibilities are limitless, just like the Mandelbrot set.

We can say that fractals are everywhere around us, they are the future, they are the answers to many of our problems, and someday they will be everywhere across everything we do.

"The nature of fractals is meant to be gradually discovered by the reader, not revealed in a flash by the author," said Mandelbrot. It takes time, patience, and absorption to see fractals. However, once you've found them, they're everywhere [20].

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