

Second Redefined Zagreb Index of Generalized Transformation Graph

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Abstract— *The topological indices are useful part in the investigations of quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) in mathematical chemistry. During this paper, the expressions for the Second Redefined Zagreb Index of the Generalized Transformation Graphs G_{xy} and its supplement graphs are acquired.*

Keywords: *Second Redefined Zagreb index; Redefined Zagreb index; generalized transformation graphs*

Mathematics Subject Classification: *05C76, 05C07, 92E10..*

I. INTRODUCTION

In the context of new technologies for molecular innovation, such as combinatorial chemistry and high-throughput screening, topological indices play a crucial role for the examination of molecular diverseness and lead to optimization through well demonstrate structure-property relationships [2]. The study of topological indices play a prominent role in Quantitative structure-activity relationships (QSAR) and Quantitative structure property relationships (QSPR) study. Topological indices correlate the precise the physico-chemical properties (stability, boiling point, enthalpy of vaporization, strain energy etc) of chemical compounds specially organic family. For more details, see [4, 5, 6, 7, 10, 11].

A Graph consists of set of vertices and set of edges. It can be represented as $G = (V,E)$ where $V=v_1,v_1,v_1\dots$ are vertices and $E=e_1 e_2, e_3,\dots$ are edges

Let G be a simple, undirected graph with p vertices and r edges.

Let $V_s(G)$ and $E_s(G)$ be the vertex set and edge set of G respectively.

If p and q are adjacent vertices of G , then the edge connecting them will be represented as pq . The degree of a vertex p in G is that the amount of edges incident thereto and is represented

by $d_G(p)$. Ranjini et al. [13] defined the Second Redefined Zagreb $ReZG_2$ index, that is,

$$ReZG_2 = \sum_{pq \in E(G)} \frac{d_G(p) \cdot d_G(q)}{d_G(p) + d_G(q)} \quad 1$$

Recently, H. S. Ramane et al. [8,9] acquired some topological indices of generalized transformation graphs and its supplements.

W.Nazeer et. al [12] acquired the Second Redefined Zagreb index of line graph of subdivision of star and friendship graphs and M. Ahmad et. al [15] computed Second Redefined Zagreb index of dominating David derived networks.

During this paper we acquired the articulations for the Second Redefined Zagreb ($ReZG_2$) index of generalized transformation graphs G_{xy} and their supplements $\overline{G^{xy}}$

II. GENERALIZED TRANSFORMATION GRAPHS (G_{XY})

The semi total – point graph $T_2(G)$ of a graph G is a graph its vertex set is $V(T_2(G)) = V(G) \cup E(G)$ and two vertices are adjacent in $T_2(G)$ if and only if (i) they're adjacent vertices of G or (ii) one is a vertex of G and other is an edge of G incident with it .It was established by Sampathkumar and Chikkodimath [14]. Basavanagoud et al. [1] defined

some new graphical transformation which generalizes the concept of semiotal-point graph. Lately, R.B Jummannaver et al.[3] defined k^{th} Generalized transformation graphs and obtained Zagreb index and co-index transformations.

The generalized transformation graph G^{xy} is a graph its vertex set is $V_s(T_2(G)) = V_s(G) \cup E_s(G)$ and $\lambda, \eta \in V_s(G^{x,y})$. The vertices λ and η are adjacent in G if and only if (a) and (b) holds:

(a) $\lambda, \eta \in V_s(G)$, λ, η are adjacent in G if $x = +$ and λ, η aren't adjacent in G if $x = -$

(b) $\lambda \in V_s(G)$ and $\eta \in E_s(G)$, λ, η are incident in G if $y = +$ and λ, η aren't incident in G if $y = -$

Proposition 2.1: [1]. Let G be a graph with i vertices and j edges. Let $p \in V_s(G)$ and $r \in E_s(G)$. Then the degrees of point and line vertices in G^{xy} are

- [1] $d_{G^{++}}(p) = 2d_G(p)$ and $d_{G^{++}}(r) = 2$.
- [2] $d_{G^{+-}}(p) = j$ and $d_{G^{+-}}(r) = i - 2$.
- [3] $d_{G^{-+}}(p) = i - 1$ and $d_{G^{-+}}(r) = 2$.
- [4] $d_{G^{--}}(p) = i + j - 1 - 2d_G(p)$ and $d_{G^{--}}(r) = i - 2$.

The supplement of G will be represented by \bar{G} . If G has i vertices and j edges then the number of G^{xy} vertices of G is $i + j$ By Proposition 2.1 and taking into account that

$d_{\bar{G}}(p) = i - 1 - d_G(p)$. We have following results for the degrees of point and line vertices in \bar{G}^{xy} are

- [5] $d_{\bar{G}^{++}}(p) = i + j - 1 - 2d_G(p)$ and $d_{\bar{G}^{++}}(r) = i + j - 3$
- [6] $d_{\bar{G}^{+-}}(p) = i - 1$ and $d_{\bar{G}^{+-}}(r) = j + 1$.
- [7] $d_{\bar{G}^{-+}}(p) = j$ and $d_{\bar{G}^{-+}}(r) = i + j - 3$
- [8] $d_{\bar{G}^{--}}(p) = 2d_G(p)$ and $d_{\bar{G}^{--}}(r) = j + 1$.

III. SECOND REDEFINED ZAGREB (ReZG₂)INDEX OF G^{XY}

Assume the following abbreviations in the theorems below (3.1 -4.4).

Theorem= Thrm, Graph = Gph, Vertices= vrtxs, Edges= eds ,& Incident= incdnt.

Thrm 3.1: Let G be a Gph with 'i' vrtxs and 'j' eds, then

$$\text{ReZG}_2(G^{++}) = 2\text{ReZG}_2(G) + \sum_{p \in V(G)} \frac{2d_G(p) \cdot d_G(p)}{d_G(p) + 1}$$

Proof: Divide the edg set $E_s(G^{++})$ into subsets E_{s_1} and E_{s_2} , where $E_{s_1} = \{pq : pq \in E_s(G)\}$ and $E_{s_2} = \{pr \mid \text{the vrtx } p \text{ is incdnt to the edg } r \text{ in } G\}$.

It is simple to inspect that $|E_{s_1}| = j$ and $|E_{s_2}| = 2j$. By Proposition 2.1[1], if $p \in V_s(G)$ then $d_{G^{++}}(p) = 2d_G(p)$ and if $r \in E_s(G)$ then $d_{G^{++}}(r) = 2$.

$$\begin{aligned} \text{ReZG}_2(G^{++}) &= \sum_{pq \in E_s(G^{++})} \frac{d_{G^{++}}(p) \cdot d_{G^{++}}(q)}{d_{G^{++}}(p) + d_{G^{++}}(q)} \\ &= \sum_{pq \in E_{s_1}} \frac{d_{G^{++}}(p) \cdot d_{G^{++}}(q)}{d_{G^{++}}(p) + d_{G^{++}}(q)} + \sum_{pr \in E_{s_2}} \frac{d_{G^{++}}(p) \cdot d_{G^{++}}(r)}{d_{G^{++}}(p) + d_{G^{++}}(r)} \\ &= \sum_{pq \in E_s(G)} \frac{2d_G(u) \cdot 2d_G(v)}{2d_G(u) + 2d_G(v)} + \sum_{pr \in E_{s_2}} \frac{2d_G(u) \cdot 2}{d_G(u) + 1} \end{aligned}$$

$$\text{ReZG}_2(G^{++}) = 2\text{ReZG}_2(G) + \sum_{pr \in E_{s_2}} \frac{2d_G(p)}{d_G(p) + 1}$$

The quantity $\sum_{pr \in E_{s_2}} \frac{2d_G(p)}{d_G(u) + 1}$ appears

$d_G(u)$ times hence the expression can be written as

$$\text{ReZG}_2(G^{++}) = 2\text{ReZG}_2(G) + \sum_{p \in V_s(G)} \frac{2d_G(p)d_G(p)}{d_G(p) + 1}$$

Thrm 3.2: Let G be a Gph with 'i' vrtxs and 'j' eds, then

$$\text{ReZG}_2(G^{+-}) = \frac{j^2}{2} + \frac{j(i-2)^2}{j+i-2}$$

Proof: Divide the edge set $E_s(G^{+-})$ into subsets E_{s1} and E_{s2} , where $E_{s1} = \{pq \mid pq \in E_s(G)\}$ and $E_{s2} = \{pr \mid \text{the vertex } p \text{ isn't incident to the edge } r \text{ in } G\}$. It is simple to examine that $|E_{s1}| = j$ and $|E_{s2}| = j(i-2)$. By Proposition 2.1[2], if $p \in V_s(G)$ as well $d_{G^{+-}}(p) = j$ and if $r \in E_s(G)$ as well $d_{G^{+-}}(r) = i-2$.

$$\begin{aligned} \text{ReZG}_2(G^{+-}) &= \sum_{pq \in E_s(G^{+-})} \frac{d_{G^{+-}}(p) \cdot d_{G^{+-}}(q)}{d_{G^{+-}}(p) + d_{G^{+-}}(q)} \\ &= \sum_{pq \in E_{s1}} \frac{d_{G^{+-}}(p) \cdot d_{G^{+-}}(q)}{d_{G^{+-}}(p) + d_{G^{+-}}(q)} + \sum_{pr \in E_{s2}} \frac{d_{G^{+-}}(p) \cdot d_{G^{+-}}(r)}{d_{G^{+-}}(p) + d_{G^{+-}}(r)} \\ &= \sum_{pq \in E_{s1}(G)} \frac{m \cdot m}{m + m} + \sum_{pr \in E_{s2}} \frac{m(n-2)}{m + (n-2)} \\ &= \frac{m^2}{2} + \frac{m(n-2)^2}{m+n-2} \end{aligned}$$

Thrm 3.3: Let G be a Gph with ‘ i ’ vrtxs and ‘ j ’ edges, then

$$\text{ReZG}_2(G^{-+}) = (i-1) \left[\frac{i(j-1)}{4} - \frac{j}{2} + \frac{4j}{j+1} \right]$$

Proof: Divide the edge set $E_s(G^{-+})$ into subsets E_{s1} and E_{s2} , where $E_{s1} = \{pq \mid pq \notin E_s(G)\}$ and $E_{s2} = \{pr \mid \text{the vertex } p \text{ is incident to the edge } r \text{ in } G\}$. It is simple to examine that $|E_{s1}| = \binom{i}{2} - j$ and $|E_{s2}| = 2j$. By Proposition 2.1[3], if $p \in V_s(G)$ as well $d_{G^{-+}}(p) = i-1$ and if $r \in E_s(G)$ as well $d_{G^{-+}}(r) = 2$.

Therefore

$$\begin{aligned} \text{ReZG}_2(G^{-+}) &= \sum_{pq \in E_s(G^{-+})} \frac{d_{G^{-+}}(p) \cdot d_{G^{-+}}(q)}{d_{G^{-+}}(p) + d_{G^{-+}}(q)} \\ &= \sum_{pq \in E_{s1}} \frac{d_{G^{-+}}(p) \cdot d_{G^{-+}}(q)}{d_{G^{-+}}(p) + d_{G^{-+}}(q)} + \sum_{pr \in E_{s2}} \frac{d_{G^{-+}}(p) \cdot d_{G^{-+}}(r)}{d_{G^{-+}}(p) + d_{G^{-+}}(r)} \\ &= \sum_{pq \in E_{s1}(G)} \frac{(i-1)}{2} + \sum_{pr \in E_{s2}} \frac{(i-1) \cdot 2}{i+1} \\ &= (i-1) \left[\frac{i(i-1)}{4} - \frac{j}{2} + \frac{4j}{i+1} \right] \end{aligned}$$

Thrm 3.4: Let G be a Gph with ‘ i ’ vrtxs and ‘ j ’ edges, then

$$\begin{aligned} \text{ReZG}_2(G^{--}) &= \sum_{pq \in E_s(G)} \frac{[i+j-1-2d_G(p)] \cdot [i+j-1-2d_G(q)]}{2[i+j-1-2d_G(p)-2d_G(q)]} \\ &+ \sum_{p \in V_s(G)} \frac{(j-d_G(p))[i+j-1-2d_G(p)] \cdot (i-2)}{2i+j-3-2d_G(p)} \end{aligned}$$

Proof: Divide the edge set $E_s(G^{--})$ into subsets E_{s1} and E_{s2} , where $E_{s1} = \{pq \mid pq \notin E_s(G)\}$ and $E_{s2} = \{pr \mid \text{the vertex } p \text{ is not incident to the edge } r \text{ in } G\}$. It is simple to inspect that $|E_{s1}| = \binom{i}{2} - j$ and $|E_{s2}| = j(i-2)$. By Proposition 2.1[4], if $p \in V_s(G)$ then $d_{G^{--}}(p) = i+j-1-2d_G(p)$ and if $r \in E_s(G)$ then $d_{G^{--}}(r) = i-2$.

Therefore

$$\begin{aligned} \text{ReZG}_2(G^{--}) &= \sum_{pq \in E_s(G^{--})} \frac{d_{G^{--}}(p) \cdot d_{G^{--}}(q)}{d_{G^{--}}(p) + d_{G^{--}}(q)} \\ &= \sum_{pq \in E_{s1}} \frac{d_{G^{--}}(p) \cdot d_{G^{--}}(q)}{d_{G^{--}}(p) + d_{G^{--}}(q)} + \sum_{pr \in E_{s2}} \frac{d_{G^{--}}(p) \cdot d_{G^{--}}(r)}{d_{G^{--}}(p) + d_{G^{--}}(r)} \\ &= \sum_{pq \in E_{s1}(G)} \frac{[i+j-1-2d_G(p)] \cdot [i+j-1-2d_G(q)]}{2[i+j-1-d_G(p)-d_G(q)]} \\ &+ \sum_{pr \in E_{s2}} \frac{[i+j-1-2d_G(p)] \cdot (i-2)}{2i+j-3-2d_G(p)} \\ &= \sum_{pr \in E_{s2}(G)} \frac{[i+j-1-2d_G(p)] \cdot [i+j-1-2d_G(q)]}{2[i+j-1-d_G(p)-d_G(q)]} \\ &+ \sum_{p \in V_s(G)} \frac{(j-d_G(p))[i+j-1-2d_G(p)] \cdot (i-2)}{2i+j-3-2d_G(p)} \end{aligned}$$

IV. REDEFINED ZAGREB (REZG₂(G)) INDEX OF

$\overline{G^{xy}}$

Thrm 4.1: Let G be a Gph with ‘i’ vrtxs and ‘j’ edgs, then

$$\begin{aligned} \text{ReZG}_2(\overline{G^{++}}) &= \frac{1}{2} \sum_{pq \in E_s(G)} \frac{[i+j-1-2d_G(p)].[i+j-1-2d_G(q)]}{i+j-1-(d_G(p)+d_G(q))} + \\ &\frac{1}{2} \sum_{p \in V_s(G)} \frac{(j-d_G(p))[i+j-1-2d_G(p)].[i+j-3]}{i+j-2-(d_G(p))} \\ &+ \frac{j(j-1)[i+j-3]}{4} \end{aligned}$$

Proof: Divide the edg set the $E_s(\overline{G^{++}})$ into subsets into subsets E_{s1}, E_{s2} and E_{s3} , where $E_{s1} = \{pq | pq \notin E_s(G)\}$ and $E_{s2} = \{pr | \text{the vrtx } p \text{ is not incdnt to the edg } r \text{ in } G\}$ and $E_{s3} = \{rs | r, s \in E_s(G)\}$. It is simple to inspect that $|E_{s1}| = \binom{i}{2} - j$ and $|E_{s2}| = j(i-2)$ and $|E_{s3}| = \binom{j}{2}$. By Proposition 2.1[5], if $p \in V_s(G)$ as well $d_{\overline{G^{++}}}(p) = i+j-1-2d_G(p)$ and if $r \in E_s(G)$ as well $d_{\overline{G^{++}}}(r) = i+j-3$ Therefore

$$\begin{aligned} \text{ReZG}_2(\overline{G^{++}}) &= \sum_{pq \in E_{s1}(\overline{G^{++}})} \frac{d_{\overline{G^{++}}}(p) \cdot d_{\overline{G^{++}}}(q)}{d_{\overline{G^{++}}}(p) + d_{\overline{G^{++}}}(q)} \\ &= \sum_{pq \in E_{s1}} \frac{d_{\overline{G^{++}}}(p) \cdot d_{\overline{G^{++}}}(q)}{d_{\overline{G^{++}}}(p) + d_{\overline{G^{++}}}(q)} + \sum_{pr \in E_{s2}} \frac{d_{\overline{G^{++}}}(p) \cdot d_{\overline{G^{++}}}(r)}{d_{\overline{G^{++}}}(p) + d_{\overline{G^{++}}}(r)} \\ &+ \sum_{rs \in E_{s3}} \frac{d_{\overline{G^{++}}}(r) \cdot d_{\overline{G^{++}}}(s)}{d_{\overline{G^{++}}}(r) + d_{\overline{G^{++}}}(s)} \\ &= \frac{1}{2} \sum_{pq \in E_s(G)} \frac{[i+j-1-2d_G(p)].[n+m-1-2d_G(q)]}{i+j-1-(d_G(p)+d_G(q))} + \\ &\frac{1}{2} \sum_{p \in V_s(G)} \frac{(j-d_G(p))[i+j-1-2d_G(p)].[i+j-3]}{i+j-2-(d_G(p))} \end{aligned}$$

$$+ \frac{m(m-1)[n+m-3]}{4}$$

Thrm 4.2:

Let G be a Gph with ‘i’ vrtxs and ‘j’ edgs, then

$$\text{ReZG}_2(\overline{G^{+-}}) = \sum_{pq \in E_s(G)} \frac{(i-1)}{2} + \sum_{pr \in E_{s2}} \frac{(i-1)(j+1)}{n+m} + \sum_{rs \in E_{s3}} \frac{(j+1)}{2}$$

Proof: Divide the edg set the $E_s(\overline{G^{+-}})$ into subsets into subsets E_{s1}, E_{s2} and E_{s3} , where $E_{s1} = \{pq | pq \notin E_s(G)\}$ and $E_{s2} = \{pr | \text{the vrtx } p \text{ is incdnt to the edg } r \text{ in } G\}$ and $E_{s3} = \{rs | r, s \in E_s(G)\}$. It is simple to inspect that $|E_{s1}| = \binom{i}{2} - j$ and $|E_{s2}| = 2j$ and $|E_{s3}| = \binom{j}{2}$. By Proposition 2.1[6], if $p \in V_s(G)$ as well $d_{\overline{G^{+-}}}(p) = i-1$ and if $r \in E_s(G)$ as well $d_{\overline{G^{+-}}}(r) = j+1$

Therefore

$$\begin{aligned} \text{ReZG}_2(\overline{G^{+-}}) &= \sum_{pq \in E_s(\overline{G^{+-}})} \frac{d_{\overline{G^{+-}}}(p) \cdot d_{\overline{G^{+-}}}(q)}{d_{\overline{G^{+-}}}(p) + d_{\overline{G^{+-}}}(q)} \\ &= \sum_{uv \in E_{s1}} \frac{d_{\overline{G^{+-}}}(p) \cdot d_{\overline{G^{+-}}}(q)}{d_{\overline{G^{+-}}}(p) + d_{\overline{G^{+-}}}(q)} + \sum_{ue \in E_{s2}} \frac{d_{\overline{G^{+-}}}(p) \cdot d_{\overline{G^{+-}}}(r)}{d_{\overline{G^{+-}}}(p) + d_{\overline{G^{+-}}}(r)} \\ &+ \sum_{ef \in E_{s3}} \frac{d_{\overline{G^{+-}}}(r) \cdot d_{\overline{G^{+-}}}(s)}{d_{\overline{G^{+-}}}(r) + d_{\overline{G^{+-}}}(s)} \\ &= \sum_{pq \in E_s(G)} \frac{(i-1)(i-1)}{(i-1) + (i-1)} + \sum_{pr \in E_{s2}} \frac{(i-1)(j+1)}{(i-1) + (j+1)} + \\ &\sum_{rs \in E_{s3}} \frac{(j+1)(j+1)}{(j+1) + (j+1)} \\ &= \sum_{pq \in E_s(G)} \frac{(i-1)}{2} + \sum_{pr \in E_{s2}} \frac{(i-1)(j+1)}{i+j} + \sum_{rs \in E_{s3}} \frac{(j+1)}{2} \end{aligned}$$

Thrm 4.3: Let G be a Gph with ‘i’ vrtxs and ‘j’ edgs, then

$$\text{ReZG}_2(\overline{G^{-+}}) = \frac{j^2}{2} + \frac{j^2(i-2)(i+j-3)}{2j+i-3} + \binom{j}{2} \frac{(i+j-3)}{2}$$

Proof: Divide the edge set the $E_s(\overline{G^{--}})$ into subsets into subsets E_{s1} , E_{s2} and E_{s3} , where $E_{s1} = \{pq | pq \in E_s(G)\}$ and $E_{s2} = \{pr | \text{the vrtx } p \text{ is not incdnt to the edg } r \text{ in } G\}$ and $E_{s3} = \{rs | r, s \in E_s(G)\}$. It is simple to inspect that $|E_1| = j$ and $|E_2| = j(i-2)$ and $|E_3| = \binom{j}{2}$. By Proposition 2.1[7], if $p \in V_s(G)$ as well $d_{\overline{G^{--}}}(p) = j$ and if $r \in E_s(G)$ then $d_{\overline{G^{--}}}(r) = i+j-3$

Therefore

$$\begin{aligned} \text{ReZG}_2(\overline{G^{--}}) &= \sum_{pq \in E_s(\overline{G^{--}})} \frac{d_{\overline{G^{--}}}(p) \cdot d_{\overline{G^{--}}}(q)}{d_{\overline{G^{--}}}(p) + d_{\overline{G^{--}}}(q)} \\ &= \sum_{pq \in E_{s1}} \frac{d_{\overline{G^{--}}}(p) \cdot d_{\overline{G^{--}}}(q)}{d_{\overline{G^{--}}}(p) + d_{\overline{G^{--}}}(q)} + \sum_{pr \in E_{s2}} \frac{d_{\overline{G^{--}}}(p) \cdot d_{\overline{G^{--}}}(r)}{d_{\overline{G^{--}}}(p) + d_{\overline{G^{--}}}(r)} \\ &+ \sum_{rs \in E_{s3}} \frac{d_{\overline{G^{--}}}(e) \cdot d_{\overline{G^{--}}}(f)}{d_{\overline{G^{--}}}(e) + d_{\overline{G^{--}}}(f)} \\ &= \sum_{pq \in E_s(G)} \frac{j}{2} + \sum_{pr \in E_{s2}} \frac{j(i+j-3)}{2j+i-3} + \sum_{rs \in E_{s3}} \frac{i+j-3}{2} \\ &= \frac{j^2}{2} + \frac{j^2(i-2)(i+j-3)}{2j+i-3} + \binom{j}{2} \frac{(i+j-3)}{2} \end{aligned}$$

Thrm 4.4: Let G be a Gph with 'i' vrtxs and 'j' edges, then

$$\text{ReZG}_2(\overline{G^{--}}) = 2\text{ReZG}_2(G) + \sum_{p \in V_s(G)} \frac{[2d_G(p)]^2 \cdot (j+1)}{2d_G(p) + j+1} + \frac{j(j+1)}{2}$$

Proof: Divide the edge set $E_s(\overline{G^{--}})$ into subsets into subsets E_{s1} , E_{s2} and E_{s3} , where $E_{s1} = \{pq | pq \in E_s(G)\}$ and $E_{s2} = \{pr | \text{the vrtx } p \text{ is incdnt to the edg } r \text{ in } G\}$ and $E_{s3} = \{rs | r, s \in E_s(G)\}$. It is simple to examine that $|E_{s1}| = j$ and $|E_{s2}| = 2j$ and $|E_{s3}| = \binom{j}{2}$. By Proposition 2.1[8], if $p \in V_s(G)$ as well $d_{\overline{G^{--}}}(p) = 2d_G(p)$ and if $r \in E_s(G)$ as well $d_{\overline{G^{--}}}(r) = j+1$.

Therefore

$$\text{ReZG}_2(\overline{G^{--}}) = \sum_{pq \in E_s(\overline{G^{--}})} \frac{d_{\overline{G^{--}}}(p) \cdot d_{\overline{G^{--}}}(q)}{d_{\overline{G^{--}}}(p) + d_{\overline{G^{--}}}(q)}$$

$$\begin{aligned} &= \sum_{pq \in E_{s1}} \frac{d_{\overline{G^{--}}}(p) \cdot d_{\overline{G^{--}}}(q)}{d_{\overline{G^{--}}}(p) + d_{\overline{G^{--}}}(q)} + \sum_{pr \in E_{s2}} \frac{d_{\overline{G^{--}}}(p) \cdot d_{\overline{G^{--}}}(r)}{d_{\overline{G^{--}}}(p) + d_{\overline{G^{--}}}(r)} \\ &+ \sum_{rs \in E_{s3}} \frac{d_{\overline{G^{--}}}(r) \cdot d_{\overline{G^{--}}}(s)}{d_{\overline{G^{--}}}(r) + d_{\overline{G^{--}}}(s)} \\ &= \sum_{pq \in E_{s1}} \frac{2d_G(p) \cdot 2d_G(q)}{2d_G(p) + 2d_G(q)} + \sum_{pr \in E_{s2}} \frac{2d_G(p) \cdot (j+1)}{2d_G(p) + (j+1)} \\ &+ \sum_{rs \in E_{s3}} \frac{(j+1) \cdot (j+1)}{(j+1) + (j+1)} \\ &= 2\text{ReZG}_2(G) + \sum_{p \in V_s(G)} \frac{[2d_G(p)]^2 \cdot (j+1)}{2d_G(p) + j+1} + \frac{j(j+1)}{2} \end{aligned}$$

V. CONCLUSION

In this paper, we acquire the articulations for the ReZG_2 of Generalized Transformation Graph G^{xy} and its supplements $\overline{G^{xy}}$ according to the specifications of elemental graph G . One can acquire other degree based topological indices G of generalized transformation graphs using same edge partition method and also can acquired expression for the topological indices of generalized transformation graphs G^{xyz} and its supplements.

REFERENCES

- [1]. B. Basavanagoud, I. Gutman, V. R. Desai, Zagreb indices of general-ized transformation graphs and their complements, Kragujevac J. Sci., 37(2015), 99{112.
- [2]. J. Devillers, New trends in (Q)SAR modeling with topological indices. Curr. Opin. Drug Discov. Dev. 2000; 3:275-279.
- [3]. R. B. Jummannaver, K. Narayanakar, D. Selvan, Zagreb index and coin-dex of kth Generalized transformation graphs, Bulletin of the International Mathematical Virtual Institutes, 10(2020), 389-402.
- [4]. R. B. Jummannaver, I. Gutman, R. K. Mundewadi, On Zagreb indices and coindices of Cluster graphs, Bulletin of the International Mathematical Virtual Institutes, 8(2018), 477-485.
- [5]. L. B. Kier, L. H. Hall, Molecular Connectivity in Chemistry and Drug Research, Academic Press, New York, 1976.
- [6]. L. B. Kier, L. H. Hall, Molecular Connectivity in Structure Activity Analysis, Wiley, New York, 1986.
- [7]. L. Pogliani, from molecular connectivity Indices to semiempirical connectivity terms: Recent trends in graph theoretical descriptors, Chem. Rev., 100 (2000), 3827{3858.
- [8]. H. S. Ramane, R. B. Jummannaver, S. Sedghi, Some degree based topological indices of Generalized transformation graphs and of their Complements, International Journal of

- Pure and Applied Mathematics, 109(3) (2016), 493-508. (Bulgaria) ISSN: 1311-8080 (Print) 1314-3395(Online).doi: 10.12732/ijpam.v109i3.2
- [9]. H. S. Ramane, B. Basavanagoud, R. B. Jummannaver, Harmonic index and Randic Index of generalized transformation graphs, Journal of the Nigerian Mathematics Society, 37(2) (2018), 5769.
- [10]. H. S. Ramane, R. B. Jummannaver, Note on The forgotten topological index of molecular Graphs in drugs, Applied Mathematics and Nonlinear Science, 1(2) (2016), 369-374. (Spain) ISSN: 2444-8656.
- [11]. H. S. Ramane, V. B. Joshi, R. B. Jummannaver, S. D. Shindhe, Relationship Between Randic index, Sum-connectivity Index, Harmonic index and pi-electronic energy for Benzenoid Hydrocarbons, National Academy Science Letter, 42(2019), 519-524.
- [12]. W. Nazeer, M. Akmal, M. Imran, M. Farahani, On topological indices of line graph of subdivision of star and friendship graphs. International Journal of Pure and Applied Mathematics. 2018;118(2):413-417.
- [13]. P. S. Ranjini, V. Loksha, A. Usha, Relation between phenylene and hexagonal squeeze using harmonic index. Int J Graph Theory 2013; 1: 116-21.
- [14]. E. Sampathkumar, S. B. Chikkodimath, Semi total graphs of a graph - I, J. Karnatak Univ. Sci., 18 (1973), 274-280.
- [15]. M. Ahmad, W. Nazeer, S. Kang, M. Imran, W. Gao, Calculating degree- Based topological indices of dominating david derived networks OpenPhys.2017;15:1015-1.

