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# A New Method for Ranking Fuzzy Numbers Based on Principal Component Analysis

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***Abstract***— *It is difficult to rank fuzzy numbers because of their ambiguous values. A few numbers of ranking techniques have been encountered in last few decades. However, existing techniques are situation-dependent which have drawbacks. In this paper we introduced a statistical technique to rank two types of fuzzy numbers, triangular and trapezoidal fuzzy numbers. This technique is the multi-dimensional scaling, more precisely the principal component analysis. Here, we presented each fuzzy numbers as a row in a matrix, then found the scale points in a low dimensions. The scale points are then reconfigured to have a unique representation. The results from this approach are obtained by comparison to the other ranking methods. The validity has been established by comparison with existing works.*

***Keywords: Triangular fuzzy numbers, Trapezoidal fuzzy numbers, Ranking fuzzy numbers, Principal component, Multidimensional scaling.***

## I. INTRODUCTION

Ranking fuzzy numbers is a significant aspect of decision process. Many authors establish a different method for ranking in the literature cannot discriminate a fuzzy numbers like real numbers which can be linearly ordered. A fuzzy sets theory was first introduced by Zadeh (1965) [27]. The ranking fuzzy numbers was first introduced by Jain in (1976) [9]. For decision process in fuzzy situations by representing the ill-defined quantity as a fuzzy sets. There is a various method for ranking fuzzy numbers. Yager (1981) established a method for ranking fuzzy numbers by finding a centroid of fuzzy numbers [26]. Shaun-Hu Chen [5] represented the exponential fuzzy number and obtained the functions of measurable product, addition and multiplication. For different left and right heights Shyi-Ming Chen [6] proposed a new method for ranking generalized fuzzy numbers. Salim Rezvani [20] developed a new method using location of median value for two generalized trapezoidal Fuzzy Numbers. Among all these methodologies, the statistical approaches play now a great rule for ranking fuzzy numbers. Here we will introduce some relevant papers. Parthiban and Gajivarathan [23] used NOVA method for different types of trapezoidal fuzzy numbers. Taheri and Hesamian [24] proposed a general approach for statistical non-parametric tests in fuzzy case. Gnanapriya1, et al. [7] applied statistical analysis of randomized block designs (RBD) through fuzzy ranking method. Dombia, and Jónásb [11] considered fuzzy relation as a probability-based preference intensity index. Rajeswari and Ritha [21] intended to explore a Markovain queue with two heterogeneous servers. For the first time Ramadan and Goran [17] proposed the multidimensional scaling method for ranking fuzzy numbers.

In this paper, we introduced the principal component analysis to rank two types of fuzzy numbers. This depends on the classical multidimensional method by putting each fuzzy number as a row matrix and find the configuration points (scale points) in two dimensional space.

## II. PRELIMINARIES

Definition (1) [13]

Let $X$ be a universe of discourse. A fuzzy set $\tilde{A}$ is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where $\mu_{\tilde{A}}(x): X \to [0,1]$ and $\mu_{\tilde{A}}(x)$ is called the membership function.

Definition (2) [14]

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R that satisfies the condition of normality and convexity.

Definition (3) [3]

A triangular fuzzy number (TFN) $\tilde{A}$ is a fuzzy number with a piecewise linear membership function $\mu_{\tilde{A}}(x)$ is described as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise,} \end{cases}$$

which can be denoted by an ordered triplet as $\tilde{A} = (a_1, a_2, a_3)$.

Definition (4) [22]

A trapezoidal fuzzy number $\tilde{A}$ is a fuzzy number with a membership function $\mu_{\tilde{A}}(x)$ is described as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \frac{a_3-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & \text{otherwise,} \end{cases}$$

which can be denoted by an ordered quadruple as $\tilde{A} = (a_1, a_2, a_3, a_4)$.

## III. PRINCIPAL COMPONENT ANALYSIS

Principal component analysis (PCA) is a technique for feature instruction, which combines in a way that the input variables still have most valuable information.

The represent the directions of the data with maximum variance which means the most spread out [10].

The eigenvalues have great role to understand how PCA works. The eigenvector is the direction of the line which shows the direction of maximum variance of the dataset, while the eigenvalue is a number that shows the spread out of the dataset on the line, i.e., The eigenvector with the highest eigenvalue is the principal component [12].

## IV. MULTIDIMENSIONAL SCALING (MDS)

MDS consider as a classical technique for reducing data low dimensions, and still used widespread today.

This method starts with an $(n \times n)$ matrix $D = (\delta_{ij})$ called distance matrix. Our goal is to find $n$ points in $r$ dimensions such that the interpoint distances $d_{ij}$ in the $r$ dimensions are approximately equal to the values of $\delta_{ij}$ in $D$.

### 1.1. Algorithm of Metric Multidimensional Scaling

In the MDS approach we consider unknown points with assuming only the distances are observable [16]. MDS aims to reconstruct the global spatial configuration of this point as follows [19]:

- We start from the matrix $X$ as $X = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{pmatrix}$.
- Construct $(n \times n)$ matrix $\Delta$ by using distance $\Delta_{ij} = \|a_i - a_j\|^2$ are observable.
- $B = (b_{ij})$, is an $(n \times n)$ matrix, where

$$B = -\frac{1}{2} H.\Delta.H, \qquad (1)$$

where $H = I_n - \frac{1}{n} J_n$, where $I_n \in R^{n \times n}$ is the identity matrix and $J_n \in R^{n \times n}$ is a data-cetering matrix, and $J_n \equiv (1, \dots, 1) \in R^n$. Then, the diagonal $B$ is

$$B = VDV^r, \qquad (2)$$

where $V$ is the matrix eigenvectors of $B$ and $D$ is the diagonal matrix of eigenvalues of $B$.

- Estimate the original dimension of the point cloud, $r$.
- Finally, return $Z \in R^{n \times r}$ with columns $\sqrt{d_i} v_i$ $(i = 1,2, \dots, r)$ and embed the points into $R^r$ using the rows of this matrix,

$$Z = \left(\sqrt{\lambda_1} \mathrm{v}_1, \sqrt{\lambda_2} \mathrm{v}_2, \dots, \sqrt{\lambda_r} \mathrm{v}_r\right) = \begin{pmatrix} z'_1 \\ z'_2 \\ \cdot \\ \cdot \\ \cdot \\ z'_n \end{pmatrix} \qquad (3)$$

Is our scale points.

In this paper we defined a new scale points and chose all eigenvalues.

$$Z = \mathrm{V}_1 \Lambda_1^{1/2} = \left(\sqrt{\lambda_1} \mathrm{v}_1, \sqrt{\lambda_2} \mathrm{v}_2, \dots, \sqrt{\lambda_n} \mathrm{v}_q\right) = \begin{pmatrix} z'_1 \\ z'_2 \\ \cdot \\ \cdot \\ \cdot \\ z'_n \end{pmatrix} \text{ where } q = 1,2, \dots, n \qquad (4)$$

## V. SHIFTING TECHNIQUE OF SCALE POINTS

Since the scale points $(z'_1, z'_2, \dots, z'_n)$ belong to $R^n$, which have shortcome when finding the Euclidean distance. So, we use a new shifting technique to fix the position of these points in $R^+ \cup \{0\}$.

If the scale points are not in $R^+ \cup \{0\}$, we present the following proposition.

**Proposition (1):** If at least one of scale points $z_{i1}, z_{i2}, \dots, z_{iq}$ is less than zero and there exist a $M_1, M_2, \dots, M_q$ $\ni z_{iq} + M_q \in R^+ \cup \{0\}$, for $i, q = 1,2, \dots, n$.

**Proof:** If $\exists\, z_{i_0 1}$ $(i_0 \in i)$ such that $z_{i_0 1} < 0$.

Assume that $M_1 = \max\{|z_{i1}|\}$, for $i = 1,2,..,n$, and it to scale points $z_{i1}$, and the new scale points, $z^*_{i1}$ (say) is $z^*_{i1} = z_{i1} + M_1$.

In analogous manner we find $z^*_{i2}, z^*_{i3}, \dots, z^*_{iq}$, the new scale points is:

$$Z^* = \begin{pmatrix} z^*_{11} & z^*_{12} & \cdots & z^*_{1q} \\ z^*_{21} & z^*_{22} & \dots & z^*_{2q} \\ \vdots & \vdots & \vdots & \vdots \\ z^*_{n1} & z^*_{n2} & \dots & z^*_{nq} \end{pmatrix}$$

**Corollary (1):** If one of the scale points $z_{iq} \ge 0$, then no need to shift technique for $z_{iq}$, for $i, q = 1,2, \dots, n$.

**Proof:** If $z_{i1} \ge 0$, then the scale points in $R^+ \cup \{0\}$.

In analogous manner for $z_{i2}, z_{i3}, \dots, z_{iq}$ is hold.

**Remark (1):** $q$ is the numbers of fuzzy numbers i.e., if we have three fuzzy numbers then $q = 3$.

## VI. A PRINCIPAL COMPONENT METHOD

If $v = (v_1, v_2, .., v_q)^t$ is principal component of the matrix $B$, and the new scale points are $Z^*_{\tilde{A}_1} = (z^*_{11}, z^*_{12}, \dots, z^*_{1q})$, $Z^*_{\tilde{A}_2} = (z^*_{21}, z^*_{22}, \dots, z^*_{2q}), \dots, Z^*_{\tilde{A}_n} = (z^*_{nq}, z^*_{nq}, \dots, z^*_{nq})$. Then, the ranking function described as follow:

$$R(\tilde{A}_i, v^t) = \sqrt{(z_{i1}^* - v_1)^2 + (z_{i2}^* - v_2)^2 + \cdots + (z_{iq}^* - v_q)^2}, \text{ for } i, q = 1,2,\ldots,n$$

If $\tilde{A}_1$ and $\tilde{A}_2$ are two arbitrary fuzzy numbers, their order is defined by:

1. If $R(Z_{\tilde{A}_1}^*, v^t) < R(Z_{\tilde{A}_2}^*, v^t)$ then $\tilde{A}_1 < \tilde{A}_2$
2. If $R(Z_{\tilde{A}_1}^*, v^t) > R(Z_{\tilde{A}_2}^*, v^t)$ then $\tilde{A}_1 > \tilde{A}_2$
3. If $R(Z_{\tilde{A}_1}^*, v^t) = R(Z_{\tilde{A}_2}^*, v^t)$ then $\tilde{A}_1 \sim \tilde{A}_2$

## VII. COMPUTATIONAL RESULTS

**Example (1):** Consider the following sets of fuzzy numbers.

**Set 1:** $\tilde{A} = (0.2, 0.3, 0.5)$, $\tilde{B} = (0.17, 0.32, 0.58)$ and $\tilde{C} = (0.25, 0.4, 0.7)$.

**Set 2:** $\tilde{A} = (2, 4, 6)$, $\tilde{B} = (1, 5, 6)$ and $\tilde{C} = (3, 5, 6)$.

**Set 3:** $\tilde{A} = (0.1, 0.2, 0.3)$, $\tilde{B} = (0.15, 0.26, 0.32)$ and $\tilde{C} = (0.15, 0.3, 0.4)$.

**Set 4:** $\tilde{A} = (0, 1, 3)$, $\tilde{B} = (4, 5, 6)$ and $\tilde{C} = (7, 8, 9)$.

**Set 5:** $\tilde{A} = (0,\ 0.4,\ 0.6,\ 0.8)$, $\tilde{B} = (0.2,\ 0.5,\ 0.5, 0.9)$ and $\tilde{C} = (0.1,\ 0.6,\ 0.7, 0.8)$.

**Set 6:** $\tilde{A} = (2, 7, 10, 15)$, $\tilde{B} = (3, 6, 11, 14)$ and $\tilde{C} = (4, 5, 12, 13)$.

**Table 1:** Comparison result for set 1, set 2, set 3

| Authors | Fuzzy Number | Set 1 | Set 2 | Set 3 |
|---|---|---|---|---|
| Cheng distance [3] | $\tilde{A}$ | 0.5900 | 4.0311 | 0.5385 |
| | $\tilde{B}$ | 0.6040 | 4.0348 | 0.5636 |
| | $\tilde{C}$ | 0.6620 | 4.6943 | 0.5810 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Parandin and Fariborzi [15] | $\tilde{A}$ | 0.3420 | 0.6672 | 0.0838 |
| | $\tilde{B}$ | 0.1170 | 0.6666 | 0.0400 |
| | $\tilde{C}$ | 0.0015 | 0.0206 | 0.0012 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Allahviranloo and Saneifard [1] | $\tilde{A}$ | 0.6097 | 2.0615 | 0.5385 |
| | $\tilde{B}$ | 0.5955 | 2.0688 | 0.5320 |
| | $\tilde{C}$ | 0.5462 | 1.4271 | 0.5205 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{B} < \tilde{A} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Chu and Tsao [4] | $\tilde{A}$ | 0.1620 | 2.0000 | 0.1000 |
| | $\tilde{B}$ | 0.1740 | 2.1176 | 0.1237 |
| | $\tilde{C}$ | 0.2190 | 2.3742 | 0.1437 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Asady and Zendehnam [2] | $\tilde{A}$ | 0.2500 | 2.2500 | 0.1250 |
| | $\tilde{B}$ | 0.2275 | 1.2500 | 0.1650 |
| | $\tilde{C}$ | 0.3250 | 3.2500 | 0.1750 |
| Results | | $\tilde{B} < \tilde{A} < \tilde{C}$ | $\tilde{B} < \tilde{A} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Phani and Ravi [18] | $\tilde{A}$ | 0.5180 | 4.1036 | 0.4591 |
| | $\tilde{B}$ | 0.5266 | 4.5924 | 0.4847 |
| | $\tilde{C}$ | 0.5847 | 4.3402 | 0.5080 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Yager [26] | $\tilde{A}$ | 0.3750 | 4.0000 | 0.2000 |
| | $\tilde{B}$ | 0.3475 | 4.2500 | 0.2475 |
| | $\tilde{C}$ | 1.2250 | 4.7666 | 0.2875 |
| Results | | $\tilde{B} < \tilde{A} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Wang and Luo [25] | $\tilde{A}$ | 0.2924 | 0.5750 | 0.3333 |
| | $\tilde{B}$ | 0.3349 | 0.5000 | 0.4916 |
| | $\tilde{C}$ | 0.5125 | 0.5000 | 0.6250 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{B} \sim \tilde{C} < \tilde{A}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Hari and Jayakumar [8] | $\tilde{A}$ | 0.3120 | 4.0123 | 0.2006 |
| | $\tilde{B}$ | 0.3337 | 4.6831 | 0.2547 |
| | $\tilde{C}$ | 0.4181 | 4.8945 | 0.2950 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Ayad and Goran [17] | $\tilde{A}$ | 0.2130 | 4.3687 | 0.1534 |
| | $\tilde{B}$ | 0.2901 | 2.4394 | 0.2118 |
| | $\tilde{C}$ | 0.4422 | 4.4291 | 0.2993 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{B} < \tilde{A} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Principal Component | $\tilde{A}$ | 1.0201 | 2.3798 | 1.0048 |
| | $\tilde{B}$ | 1.0751 | 1.2580 | 1.0518 |
| | $\tilde{C}$ | 1.1771 | 2.3807 | 1.1133 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{B} < \tilde{A} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |

**Table 2**: Comparison result for set 4 , set 5 ,set 6

| Authors | Fuzzy Number | Set 4 | Set 5 | Set 6 |
|---|---|---|---|---|
| Cheng distance [3] | $\tilde{A}$ | 1.4126 | 0.5864 | 8.5146 |
| | $\tilde{B}$ | 5.0249 | 0.8109 | 8.5146 |
| | $\tilde{C}$ | 8.0156 | 0.7462 | 8.5146 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{A} \sim \tilde{B} \sim \tilde{C}$ |
| Parandin and Fariborzi [15] | $\tilde{A}$ | 6.6667 | 0.2966 | 0.0000 |
| | $\tilde{B}$ | 3.0000 | 0.0383 | 0.0000 |
| | $\tilde{C}$ | 0.0000 | 0.1199 | 0.0000 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{A} \sim \tilde{B} \sim \tilde{C}$ |
| Allahviranloo and Saneifard [1] | $\tilde{A}$ | 7.6808 | 0.7173 | 6.5192 |
| | $\tilde{B}$ | 4.0311 | 0.5542 | 6.5192 |
| | $\tilde{C}$ | 1.1180 | 0.6494 | 6.5192 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{A} \sim \tilde{B} \sim \tilde{C}$ |
| Chu and Tsao [4] | $\tilde{A}$ | 0.6222 | 0.1657 | 4.2500 |
| | $\tilde{B}$ | 2.5000 | 0.3171 | 4.2500 |
| | $\tilde{C}$ | 4.0000 | 0.2780 | 4.2500 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{A} \sim \tilde{B} \sim \tilde{C}$ |
| Asady and Zendehnam [2] | $\tilde{A}$ | 0.5000 | 0.2500 | 5.7500 |
| | $\tilde{B}$ | 4.2500 | 0.4500 | 5.2500 |
| | $\tilde{C}$ | 7.2500 | 0.3750 | 4.2500 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{C} < \tilde{B} < \tilde{A}$ |
| Phani and Ravi [18] | $\tilde{A}$ | 1.1931 | 0.5830 | 30.8602 |
| | $\tilde{B}$ | 5.0006 | 0.6392 | 33.4319 |
| | $\tilde{C}$ | 8.0004 | 0.6867 | 44.4861 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |

| | | | | |
|---|---|---|---|---|
| Yager [26] | $\tilde{A}$ | 1.2500 | 0.4500 | 8.5000 |
| | $\tilde{B}$ | 5.0000 | 0.5250 | 8.5000 |
| | $\tilde{C}$ | 8.0000 | 0.5500 | 8.5000 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} \sim \tilde{B} \sim \tilde{C}$ |
| Wang and Luo [25] | $\tilde{A}$ | 0.0833 | 0.5174 | 0.5000 |
| | $\tilde{B}$ | 0.5555 | 0.5544 | 0.5960 |
| | $\tilde{C}$ | 0.8888 | 0.6142 | 0.5190 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{C} < \tilde{B} < \tilde{A}$ |
| Hari and Jayakumar [8] | $\tilde{A}$ | 1.1358 | 0.4869 | 8.6300 |
| | $\tilde{B}$ | 5.0024 | 0.5140 | 8.6586 |
| | $\tilde{C}$ | 8.0003 | 0.6096 | 8.6899 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B} < \tilde{C}$ |
| Ayad and Goran [17] | $\tilde{A}$ | 4.0226 | 0.1173 | 2 |
| | $\tilde{B}$ | 10.3936 | 0.4950 | 4 |
| | $\tilde{C}$ | 15.5823 | 0.3591 | 6 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{C} < \tilde{B} < \tilde{A}$ |
| Principal Component Analysis | $\tilde{A}$ | 1.0398 | 0.9722 | 3.3679 |
| | $\tilde{B}$ | 7.1554 | 1.2734 | 1.4736 |
| | $\tilde{C}$ | 12.3290 | 1.0107 | 0.9999 |
| Results | | $\tilde{A} < \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{C} < \tilde{B}$ | $\tilde{C} < \tilde{B} < \tilde{A}$ |

## VIII. CONCLUSION

In this paper, we present a new statistical approach called PAC, for ranking fuzzy numbers. New scale points are generated by the MDS, and then we reconfigure the scale points in $R^+ \cup \{0\}$. To fix some backwards related to the distance between the PCA and the scale point. The computational results observed to be more effective for ranking triangular and trapezoidal fuzzy numbers, and to other existing works as show in the table (1) and (2).

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